

## FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 2 Round 1  
Arithmetic: Factors  
And Multiples

1.) \_\_\_\_ {4,5,6} \_\_\_\_\_

2.) \_\_\_\_ {23,20,13} \_\_\_\_\_

3.) \_\_\_\_ {504,488,512} \_\_\_\_\_

1. How many natural numbers  $N \leq \{100, 150, 200\}$  have exactly 3 distinct factors? (Note: Factors must be positive.)
  
- 2.) How many natural numbers  $N \leq \{100, 80, 50\}$  are multiples of exactly two of the following numbers: 2, 3, 5?
  
- 3.) A and B are positive integers. The greatest common factor of A and B is
4. The least common multiple of A and B is  $\{15620, 14740, 16060\}$ . What is the smallest possible value of  $A+B$ ?

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Match 2 Round 2  
Algebra: Polynomials  
And Factoring

1.) \_\_\_\_\_{300,200,400}\_\_\_\_\_

2.) \_\_\_\_ {23,22,21} \_\_\_\_\_

3.) \_\_\_\_\_26\_\_\_\_\_

1.)\_ Suppose that, for any value of  $x$ ,

$$(4x+5)(\{3,2,4\}x-20)-(x-4)(Ax+B)=-42x,-63x,-21x\}$$

Find  $AB$ .

2.) For what positive value of  $k$  does  $x^3-7x^2+(k^2 - \{22,21,20\}k)x - \{26,24,22\}=0$  have solution  $x = 2$ ?

3.) For how many distinct integers  $B$  does  $16x^2 + Bx + 81$  factor into two binomials with integer coefficients?

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Match 2 Round 3  
 Geometry:  
 Area and Perimeter

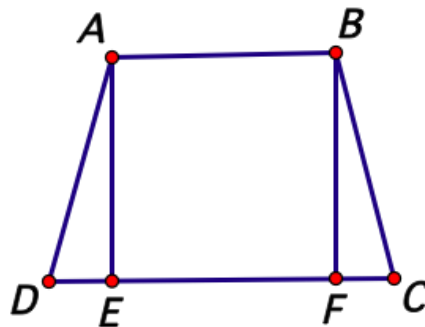
1. \_\_\_\_\_ {60,30,48} \_\_\_\_\_

2. \_\_\_\_\_ {46,60, 42} \_\_\_\_\_

3. \_\_\_\_\_ {100,25,225} \_\_\_\_\_

1. The length of a rectangular swimming pool is twice its width. The pool is surrounded by a sidewalk that is 3 feet wide. The area enclosed by the sidewalk and the pool is {416,176, 308} square feet. What is the perimeter of the pool? (Do not include a unit in your answer.)

2. The trapezoid ABCD shown in the diagram is isosceles with bases AB and DC. Segments AE and BF are drawn from A and B perpendicular to segment DC.  $AB=5$ ,  $DC=\{15,21,17\}$ , and the area of rectangle ABFE is {60,75,40}. Find the perimeter of trapezoid ABCD.



3.)  $\triangle ABC$  is inscribed in a circle with center O. Segment BC is a diameter of the circle. There is a number  $x$  such that  $AB = \{x+5, x+4, x+6\}$ ,  $AC = \{3x-5, 3x+2, 3x-12\}$  and  $BO = x+3$ . The area of the circle is  $Q\pi$ . Find Q.

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Match 2 Round 4 Algebra 2: Inequalities And Absolute value
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1)  $\{69, 77, 83\}$ \_\_\_\_\_

2.) \_\_\_\_ $\{2,4,6\}$ \_\_\_\_\_

3.)  $\{6, 4, 2\}$ \_\_\_\_\_

1.) How many integers satisfy the inequality below?

$$x^2 \leq \{1200, 1500, 1700\}$$

2.) If you solve  $\frac{3x-5}{x+2} > \left\{K, \frac{K}{2}, \frac{K}{3}\right\}$  for x, the solution is "x>9 or x<-2".

What is K?

3.) There are two values of K for which  $|x-\{3,2,1\}|+|x+K| = 5$  has infinitely many solutions. Find the absolute value of the sum of these two values.

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Match 2 Round 5 Trigonometry: Laws of Sine and Cosine
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Note: Drawings not necessarily drawn to scale. \_

1.) {169, 9, 49}

2.) \_\_\_\_\_{72, 84,96}\_\_\_\_\_

3.)\_\_\_\_\_ {164, 198, 236}\_\_\_\_\_

1.)  $\triangle XYZ$  has  $XY=8, YZ=8, XZ=\{13,12,14\}$ .  $\cos X \cos Y = -\frac{a}{b}$ , where  $a$  and  $b$  are relatively prime positive integers. Find  $a + b$ .

2.) In  $\triangle JKL$ , angle  $KJL$  is 30 degrees, angle  $JKL$  is 105 degrees, and  $KL = \{12,14,16\}\sqrt{6}$ .  $JK = A\sqrt{B}$  in simplest radical form. Find  $AB$ .

3.) The median from P to segment QR of  $\triangle PQR$  meets segment QR at S.  $PQ = 6$ ,  $RS=6$ ,  $PS=\{8,9,10\}$ . The length of segment PR is  $\sqrt{A}$ . Find A.

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Match 2 Round 6 Equations of Lines
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1.) {13, 41, 89}

2.) \_\_\_\_\_{13,26,39}\_\_\_\_\_

3.) \_\_\_\_\_{12,8,4}\_\_\_\_\_

1.) A line is given in parametric form as  $x = 2t + \frac{1}{3}$ ,  $y = \{4,10,16\}t - \frac{7}{3}$ . If the equation of the line is expressed as  $y = mx + b$ , what is the value of  $m^2 + b^2$  ?

2.) A line of slope 0.5 intersects the parabola  $y = 2x^2 + 5x + 3$  at  $(-2,1)$  and  $(A,B)$ . Find  $(8,16,24)(A+B)$ .

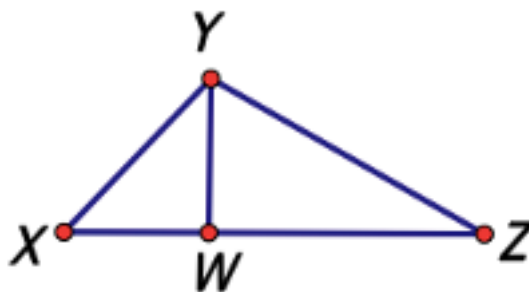
3.)\_ A circle of radius 1 is centered at  $(0,0)$ . The points of intersection of the circle with the perpendicular bisector of the segment whose endpoints are  $(2,3)$  and  $(4,-1)$  are  $(A,B)$  and  $(C,D)$ . What is the absolute value of  $(30,20,10)(A+B+C+D)$  ?

**FAIRFIELD COUNTY MATH LEAGUE 2020-21 Match 2 Team Round**

Answers:

1. 110
2. 904
3. 234
4. 12
5. 23
6. 28

1.)\_ The diagram shows  $\triangle XYZ$ , in which  $2 \cdot (XY) = XZ$ . The altitude from  $Y$  to segment  $XZ$  meets segment  $XZ$  at  $W$  and has length 12. The area of  $\triangle XYZ$  is 180. The perimeter of  $\triangle XYZ$  is  $M + 3\sqrt{N}$ , where  $M$  and  $N$  are positive integers and  $N$  is not divisible by the square of any prime. Find  $M + N$ .



2.) Find the sum of the squares of all integer values of  $n$  such that  $n^2 - 28n - 29$  is a prime number. (Note: Prime numbers must be positive.)

3.)  $3x^3 + Cx^2 + Dx - 225$  factored completely over the integers is  $3(x+A)(x+B)(x-B)$  for some values of  $A$  and  $B$ . Find the sum of all possible values of  $C$ .

4.)\_ The solution to  $5x^3 - 15x^2 - 20x + 72 < K$  is " $x < -2$  or  $2 < x < 3$ ". Find  $K$ .

5.) In triangle  $ABC$ , the ratio  $\sin A : \sin B : \sin C$  is  $5:6:7$ . The perimeter of the triangle is 27. The length of the longest side of the triangle is  $\frac{a}{b}$ , where  $a$  and  $b$  are relatively prime positive integers. Find  $a + b$ .

6.) For  $\triangle PQR$ ,  $P$  is at the origin,  $Q$  is at the

intersection of  $y = \frac{-\sqrt{3}}{3}x$  and  $y = \frac{5\sqrt{3}}{3}x - 36$ , and  $R$  is at the

intersection of  $y = \frac{\sqrt{3}}{3}x$  and  $y = \frac{5\sqrt{3}}{3}x - 36$ . The sine of angle  $PRQ$  is



$\frac{\sqrt{a}}{b}$ , where  $a$  and  $b$  are positive integers and  $a$  is not divisible by the square of any prime. Find  $a + b$ .