FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 1 Round 1
Arithmetic: Percents

1.)

2.)

3.)

1.) When the number *x* is increased by *x* percent, the result is $\{10,20,30\}$ less than twice *x*. If that statement is represented by the equation $Ax^2 + Bx + C = 0$, where $A > 0$ and A, B, and C are integers with no common factors larger than 1, find the value of $A + B + C$.

2.) There exist specific values of *w* and *k* for which the following statement is true for all values of *x:* For constants *x*, *y*, and *z*, If $\{20,30,40\}$ percent of *x* is {4,5,3} more than *y* and {60,40,70} percent of *y* is 1 less than *z*, then *k* percent of *x* is *w* more than *z*. Find the value of 10*w* + *k*.

3.) The percent difference between p and q is defined as $\frac{|p-q|}{\frac{p+q}{2}} \times 100\%$. Two positive numbers m and n with $m > n$ have the following property: The percent difference between $2m$ and n is equal to $\left\{\frac{14}{9}, \frac{5}{2}, \frac{26}{5}\right\}$ times the percent difference between m and n. If the ratio of m to n can be expressed in simplest form by the fraction $\frac{a}{b}$, find ab.

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Match 1 Round 2
Algebra 1: Equations

1.)

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1.) If $x = \{4,3,5\}$ is a solution to the equation $(862A)x - 45987 = 749A$ for some constant A, find the value of A rounded to the nearest integer.

2.) Find the nonzero value of k such that the equation $(x + \{3,4,5\})(x^2 + k) = x^3$ has only one solution for x.

3.) For how many integer pairs (a, b) is $\{4,5,2\}a - 3b = 1$ and $0 \leq a + b \leq 2020$?

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Match 1 Round 3
Geometry: Triangles &
Quadrilaterals

1.)

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1.) An isosceles trapezoid has an area of $\{21,14,40\}$, a height of $\{3,2,4\}$, and one base of length $\{10,9,14\}$. If the perimeter of the trapezoid is $a+b\sqrt{c}$ where a, b, and c are positive integers and c has no perfect square factors greater than 1, find $a+b+c$.

2.) What is the perimeter of a rectangle with a diagonal of length $\{6,8,11\}$ and an area of $\{14,18,24\}$?

3.) A right rectangular pyramid has two lateral faces with a vertex angle of 90 degrees and two lateral faces with a vertex angle of 60 degrees. If the base of the pyramid has an area of $\{400\sqrt{2}, 576\sqrt{2}, 256\sqrt{2}\}$, find the height of the pyramid.

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Match 1 Round 4 Algebra 2: Simultaneous Equations

1.)

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1.) For a concert, tickets cost \$68 for an adult and \$31 for a child. For a particular group of {12, 14, 16} people, the cost of the tickets is {\$668, \$767, \$866}. How many adults are in the group?

2.) The graphs of $y = 4x - x^2$ and $y = kx^2$, where k is a positive constant, intersect at points M and N. If the slope between M and N is $\left\{\frac{3}{5}, \frac{2}{3}, \frac{1}{2}\right\}$, then the value of k can be written as $\frac{a}{b}$ where a and b are relatively prime integers and $b > 0$. Find $a + b$.

3.) The ordered pair $\left\{\left(2, \frac{17}{3}\right), \left(-1, \frac{5}{12}\right), \left(1, \frac{13}{6}\right)\right\}$ is one of infinite solutions of the system $\begin{cases} 4x - Ay = -9 \\ Bx + 2y = C \end{cases}$ for constants $A, B,$ and C. Find $|ABC|$.

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Match 1 Round 5
Precalculus: Right Triangle Trigonometry

1.)

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1.) In right triangle ABC with right angle C, if $\tan(A) = \{7,5,3\}$, then $\cos(B)$ can be expressed in simplest radical form as $\frac{x\sqrt{y}}{z}$ where x, y, and z are integers. Find $x + y + z$.

2.) Consider right triangle ABC with right angle A. If the hypotenuse has a length of $\{2\sqrt{5}, 2\sqrt{13}, 10\}$ units and the value of $\tan(B)$ has the same value as the area of the triangle in square units, find the area of the triangle in square units.

3.) You are standing on a straight road. You see a balloon being released from a point on the road, and a little later, at time t_1, the balloon has risen vertically, and the sine of the angle of elevation from the ground where you stand to the balloon is $\frac{4}{5}$. You run along the road in the direction away from the launch point and stop at time t_2, and find that the distance you ran is twice the height the balloon has climbed since t_1. The tangent of the new angle of elevation from the ground where you stand to the balloon is $\left\{\frac{16}{27}, \frac{4}{7}, \frac{24}{43}\right\}$. If the height of the balloon at t_1 is h_1 and the height of the balloon at t_2 is h_2, find $\frac{h_2}{h_1}$.

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Match 1 Round 6
Miscellaneous: Coordinate Geometry

1.)

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1.) A straight line intersects the x-axis at $\{(4,0), (6,0), (10,0)\}$ and the y-axis at $\{(0,8), (0,2), (0,6)\}$. The equation of the line is $Ax + By = C$, where A, B, and C are integers, $A > 0$, and the only positive integer that divides all of A, B, C is 1. Find $A + B + C$.

2.) The point P with coordinates $\{(10, 5), (11, 7), (12, 9)\}$ is reflected across the line $y = 2x$ to make the new point P'. Find the sum of the coordinates of P'.

3.) A circle centered at the origin with an area of $\{75\pi, 18\pi, 80\pi\}$ is tangent to the line $4x + 3y = k$, where k is a constant. Find the value of k^2.

Team Round

FAIRFIELD COUNTY MATH LEAGUE 2020-2021 Match 1 Team Round

1.)
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6.)

1.) Consider quadrilateral $ABCD$, inscribed in a circle, where diagonal $\overline{AC}$ is a diameter of the circle. If $\tan(\angle BAC) = \frac{4}{3}$ and $\tan(\angle CAD) = \frac{7}{24}$ and $AD = 8$, find the area of $ABCD$.

2.) The road from Ridgefield to Wilton is 5 miles uphill, then 4 miles on level ground, then 6 miles downhill. Mr. Corbishley has a consistent uphill walking speed, a consistent walking speed on level ground, and a consistent downhill walking speed. He walks from Wilton to Ridgefield in 4 hours. Later he walks the first half of the journey from Ridgefield to Wilton and returns to Ridgefield in a total of 3 hours and 55 minutes. Still later he walks from Ridgefield to Wilton in 3 hours and 52 minutes. Find Mr. Corbishley's walking speed on level ground in miles per hour. (Don't enter units.)

3.) The diagram shows two circles, each with area 288π, which are tangent to each other. Trapezoid $TRAP$ is drawn so that points T and R are the centers of the circles, $\overline{AP}$ is tangent to both circles, A lies on $\odot R$, and $AP > TR$. If the perimeter of $TRAP$ is $78\sqrt{2}$, find the area of $TRAP$.

4.) $A, B,$ and C are positive numbers. $C\%$ of B is 20 less than A. $(2A)\%$ of C is 17 more than twice B. $((2A)\%$ of $B)\%$ of $2C$ is 117. Find $A + B + C$.

5.) Find the product of all values of a such that the equation $\frac{x-4}{2x-5} = \frac{x+a}{2x+7}$ has no solutions for x.

6.) The points $(12,7)$, $\left(8 - 3\sqrt{3}, 4 + 4\sqrt{3}\right)$, and $(4,1)$ are three vertices of a regular hexagon whose area is $a\sqrt{b}$ units, where a and b are integers with b having no perfect square factors larger than 1. Find the product ab.