

# FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 5 Round 1  
Algebra I:  
Fractions and  
Exponents

1.) 1.625

2.)  $\frac{1}{5}$

3.)  $\frac{b^2}{a^2}$

1. Express as a decimal, correct to three decimal places.

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + 1}}}}$$

2) Simplify:  $\frac{(55)^3(121)^4}{(1331)^3(275)^2}$

3.) If  $x - 3y = -2$ , express the following as a single fraction without negative exponents.

$$\frac{(a^x b^{3y})^4 ((a^{3x} + b^{2y})^2 - b^{2y}(2a^{3x} + b^{2y}))(a^{-2x} b^{-3y})^{-4}}{(a^{3x})^5 (ab^7)^{3y} (a^2 b)^x}$$

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 5 Round 2  
Algebra I:  
Fractional  
Expressions and  
Equations

Assume no values of  $x$   
make any denominator equal to zero.

1.)  $\frac{-x}{x+10}$

2.)  $3, \frac{1}{33}$

3.)  $-\frac{3}{4}$

1). Multiply and simplify:  $\frac{x^2 - 18x + 80}{x^2 - 17x + 72} * \frac{9x - x^2}{x^2 - 100}$

2). Solve for  $x$ :  $\frac{3}{3x+1} - \frac{1}{4} = \frac{x-2}{7x-1}$

3.): Solve for  $x$ :  $\frac{5}{x + \frac{3}{x+4}} - \frac{4}{x + \frac{2}{x+3}} = \frac{1}{\frac{x^2 + 5x + 6}{x+1}}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2019-2020

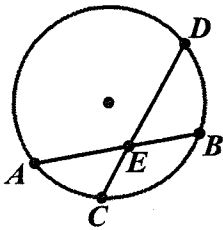
Match 5 Round 3  
Geometry:  
Circles

1.) 5

2.) 12

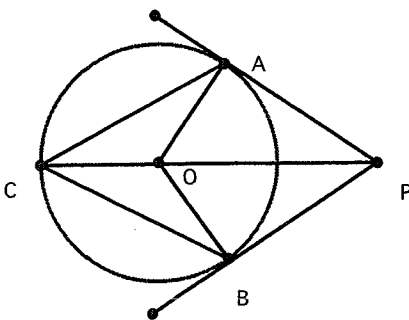
Note: Diagrams not necessarily to scale

3.)  $20\sqrt{3}$



1.) In the picture above,  $\overline{AB}$  and  $\overline{CD}$  are chords of a circle that intersect at E.  $AE=x-1$ ,  $AB=2x$ ,  $CE=x-4$ , and  $DE=3x-5$ . Find the difference  $CD - AB$ .

2) Two circles with centers P and Q are tangent to the same line at points A and B and tangent to each other. If the radius of circle P is 9 and the radius of circle Q is 4, what is the length of segment  $\overline{AB}$ ?



3.) For the circle with center O above,  $\overline{PA}$  and  $\overline{PB}$  are tangent to circle O at A and B. The line from P through O intersects the circle at C as shown above. If  $OA = x-20$ ,  $OP=x-15$ , and  $AP=\sqrt{3x}$ , find the perimeter of quadrilateral APBC.

# FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 5 Round 4  
Quadratic  
Equations and  
Complex  
Numbers

1.)  $3+2i, 3-2i$

2.)  $7i$

3.)  $-3, 4i$

- 1) Find the two complex solutions of  $x^2 + bx + c = 0$  if the sum of the solutions is 6 and the product of the solutions is 13.

2.) Simplify:  $\frac{(2+i)^3}{3-i} - \frac{(2-i)^3}{3+i}$

3) Solve for all complex  $x$ :  $x^2 + (3-4i)x - 12i = 0$

**FAIRFIELD COUNTY MATH LEAGUE 2019-2020**

Match 5 Round 5  
Solving Trig  
Equations

1.) 6, 30, 78, 102, 150, 174 degrees

2.)  $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

3.)  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$

1) Solve for all  $x$   $0^\circ \leq x \leq 180^\circ$ :  $\sin(5x) = \frac{1}{2}$

2) Solve for all  $x$ :  $0 \leq x < 2\pi$   $\tan^2(x) - \sec(x) = 1$

3.) Solve for all  $x$   $0 \leq x < \pi$ :  $\sin(4x) + \cos(2x) = 0$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2019-2020

Match 5 Round 6  
Sequences and  
Series

1.) 4

2.) 3.28

3.)  $\frac{1}{25}$

1.) For what natural number  $n$  is  $0.76 < \sum_{k=1}^n \frac{(-1)^{k+1}}{k^2} < 0.80$

2.) An arithmetic sequence has first term -2. The <sup>fourth</sup> ~~second~~ term is the square ~~root~~ of the <sup>second</sup> ~~fourth~~ term. Find all possible values of the sixth term.

3.) An infinite geometric series converges to 6.25. The second term of the original geometric sequence is 4 less than the first term. Give all possible values for the fourth term of the sequence.

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 5  
Team Round

1.)  $\frac{4}{77}$

4.)  $\frac{\sqrt{2+i\sqrt{6}}}{2}, \frac{-2-i\sqrt{6}}{2}$

Note: Diagrams not necessarily drawn to scale.

2.)  $\pm 3\sqrt{10}$

5.)  $\frac{-3\sqrt{2} \pm \sqrt{14}}{8}$

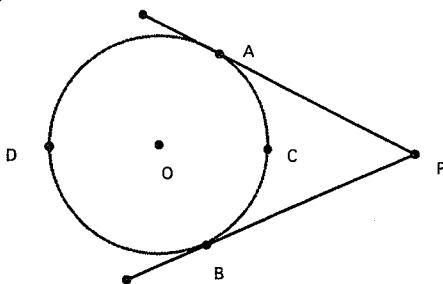
3.)  $55, \frac{589}{9}$  degrees

6.)  $-4, 2+2i\sqrt{3}, 2-2i\sqrt{3}$

1.) Simplify:  $\frac{1}{7} * \frac{1}{8} + \frac{1}{8} * \frac{1}{9} + \frac{1}{9} * \frac{1}{10} + \frac{1}{10} * \frac{1}{11}$

2.) Solve for x:

$$2 - \frac{x}{3 - \frac{x}{4 - \frac{x}{5 - \frac{x}{6}}}} = 4$$



3.) In the diagram above,  $\overleftrightarrow{PA}$  and  $\overleftrightarrow{PB}$  are tangent to the circle with center O at points A and B. The measure of arc ACB is  $2x^2 - 3$  and the measure of arc ADB is  $4x^2 - 2x - 5$ . Find the degree measure of  $\angle APB$ .

4.) Find the two complex square roots of  $-1 + i\sqrt{3}$ .

5.) What are all possible values for  $\sin(x)$  if  $\cos(x + \frac{\pi}{4}) = \frac{3}{4}$  ?

6.) A geometric sequence  $\{a_n\}$  of complex numbers has the property that  $a_4 = -64a_1$ . Find all possible values for the common ratio of the sequence.