FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 5 Round 1
Algebra I:
Fractions and
Exponents

1.) ________1.625_____

2.) ________ $\frac{1}{5}$ ________

3.) ________ $\frac{b^2}{a^2}$ ________

1. Express as a decimal, correct to three decimal places. .

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}}$$

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}}=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\frac{3}{2}}}}+=1+\cfrac{1}{1+\cfrac{1}{1+\frac{2}{3}}}$$

$$=1+\cfrac{1}{1+\cfrac{1}{\frac{5}{3}}}=1+\cfrac{1}{1+\frac{3}{5}}=1+\cfrac{1}{\frac{8}{5}}=1+\frac{5}{8}=\frac{13}{8}=1.625$$

2) Simplify: $\frac{(55)^3(121)^4}{(1331)^3(275)^2}$

$$\frac{(55)^3(121)^4}{(1331)^3(275)^2}=\frac{(11)^3(5)^3(11)^8}{(11)^9(11)^2(5)^4}=\frac{1}{5}$$

3.)_ If x – 3y = -2, express the following as a single fraction without negative exponents.

$$\frac{(a^x b^{3y})^4((a^{3x}+b^{2y})^2 - b^{2y}(2a^{3x}+b^{2y}))(a^{-2x}b^{-3y})^{-4}}{(a^{3x})^5(ab^7)^{3y}(a^2b)^x}$$

$$\frac{(a^x b^{3y})^4((a^{3x}+b^{2y})^2 - b^{2y}(2a^{3x}+b^{2y}))(a^{-2x}b^{-3y})^{-4}}{(a^{3x})^5(ab^7)^{3y}(a^2b)^x} =$$

$$(a^{4x}b^{12y})((a^{6x}+2a^{3x}b^{2y}+b^{4y}-b^{2y}(2a^{3x})-b^{4y})(a^{8x}b^{12y})$$

$$\frac{(a^{4x}b^{12y})((a^{6x}+2a^{3x}b^{2y}+b^{4y}-b^{2y}(2a^{3x})-b^{4y})(a^{8x}b^{12y})}{a^{15x}a^{3y}b^{21y}a^{2x}b^x} =$$

$$\frac{(a^{4x}b^{12y})((a^{6x})(a^{8x}b^{12y})}{a^{17x}a^{3y}b^{21y}b^x} = \frac{a^{12x}b^{24y}(a^{6x})}{a^{17x}a^{3y}b^{21y}b^x} = \frac{a^x b^{3y}}{a^{3y}b^x} = a^{x-3y}b^{3y-x} =$$

$$a^{-2}b^{-(-2)} = b^2a^{-2} = \frac{b^2}{a^2}$$

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Match 5 Round 2 Algebra I: Fractional Expressions and Equations

1.) $-\dfrac{x}{x+10}$

2.) $\dfrac{1}{33}, 3$

Assume no values of x

make any denominator equal to zero.

3.) $-\dfrac{3}{4}$

1). Multiply and simplify: $\dfrac{x^2-18x+80}{x^2-17x+72} * \dfrac{9x-x^2}{x^2-100}$

$$\frac{x^2-18x+80}{x^2-17x+72} * \frac{9x-x^2}{x^2-100} =$$

$$\frac{(x-8)(x-10)}{(x-8)(x-9)} * \frac{x(9-x)}{(x+10)(x-10)} =$$

$$-\frac{x}{x+10}$$

2). Solve for x: $\dfrac{3}{3x+1} - \dfrac{1}{4} = \dfrac{x-2}{7x-1}$

$$\frac{3}{3x+1}-\frac{1}{4}=\frac{x-2}{7x-1}$$

$$Multiply\_by\_(3x+1)*4*(7x-1)$$

$$3*4*(7x-1)-(3x+1)(7x-1)=4(3x+1)(x-2)$$

$$84x-12-(21x^2+4x-1)=4(3x^2-5x-2)$$

$$84x-12-21x^2-4x+1=12x^2-20x-8$$

$$-21x^2+80x-11=12x^2-20x-8$$

$$33x^2-100x+3=0$$

$$(33x-1)(x-3)=0$$

$$x=\frac{1}{33},3$$

3.): Solve for x: $\dfrac{5}{x+\dfrac{3}{x+4}}-\dfrac{4}{x+\dfrac{2}{x+3}}=\dfrac{1}{\dfrac{x^2+5x+6}{x+1}}$

$$\frac{5}{x+\frac{3}{x+4}}-\frac{4}{x+\frac{2}{x+3}}=\frac{1}{\frac{x^2+5x+6}{x+1}}$$

$$\frac{5}{\frac{x^2+4x+3}{x+4}}-\frac{4}{\frac{x^2+3x+2}{x+3}}=\frac{(x+1)}{x^2+5x+6}$$

$$\frac{5x+20}{(x+1)(x+3)}-\frac{4x+12}{(x+2)(x+1)}=\frac{(x+1)}{(x+2)(x+3)}$$

$Multiply\_by\_(x+1)(x+3)(x+2)$

$$(5x+20)(x+2)-(4x+12)(x+3)=(x+1)(x+1)$$

$$(5x^2+30x+40)-(4x^2+24x+36)=x^2+2x+1$$

$$x^2+6x+4=x^2+2x+1$$

$$4x=-3$$

$$x=\frac{-3}{4}$$

$$\frac{(x-1)(x-2)(x-3)-(9-2x)(x+2)}{(4x-3)(x-5)+(2x+5)(x-3)}$$

$$\frac{(x^2-3x+2)(x-3)-(-2x^2+5x+18)}{(4x^2-23x+15)+(2x^2-x-15)}$$

$$\frac{(x^3-6x^2+11x-6)+(2x^2-5x-18)}{6x^2+24x}=$$

$$\frac{(x^3-4x^2+6x-24)}{6x^2-24x}=\frac{x^2(x-4)+6(x-4)}{6x(x-4)}=\frac{(x^2+6)(x-4)}{6x(x-4)}=\frac{x^2+6}{6x}$$

$=$

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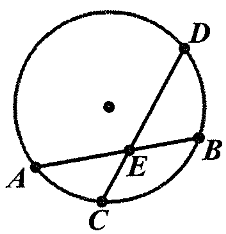
Match 5 Round 3
Geometry:
Circles

1.) ________5____________

2.) ________12_____

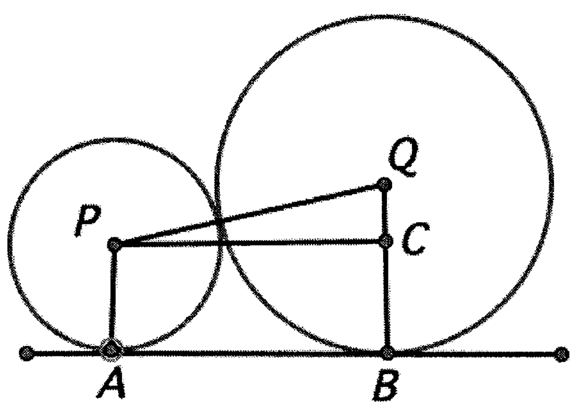
Note: Diagrams not necessarily to scale

3.) _______ $20\sqrt{3}$ ______________

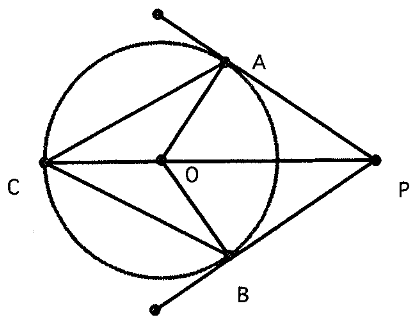


1.)_ In the picture above, $\overline{AB}$ and $\overline{CD}$ are chords of a circle that intersect at E. AE=x-1, AB=2x, CE=x-4, and DE=3x-5. Find the difference CD –AB.

AE*BE=CE*DE. BE=2x-(x-1)=x+1. (x-1)(x+1)=(x-4)(3x-5)
$x^2-1 = 3x^2-17x+20$, $2x^2-17x+21=0$, (x-7)(2x-3)=0. x=7 or x=1.5, but if x=1.5, CE and DE are negative, so x=7. AE=6, AE=14, so BE=8. CE=3, ED=16. (3+16)-(6+8)=5



Draw a line through P parallel to $\overline{AB}$ intersecting $\overline{QB}$ at C. PQ = 4+9 = 13. PA=4, so CB=4, and QC=9-4 = 5. Then PC = $\sqrt{13^2-5^2} = \sqrt{144} = 12$. PC=AB=12.



3.) For the circle with center O above, $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$ are tangent to circle O at A and B. The line from P through O intersects the circle at C as shown above. If OA = x-20, OP=x-15, and AP=$\sqrt{3x}$, find the perimeter of quadrilateral APBC.

Since $\overline{OA}$ is perpendicular to $\overline{AP}$, $OA^2 + AP^2 = OP^2$.

$$(x-20)^2 + (\sqrt{3x})^2 = (x-15)^2$$

$$x^2 - 40x + 400 + 3x = x^2 - 30x + 225$$

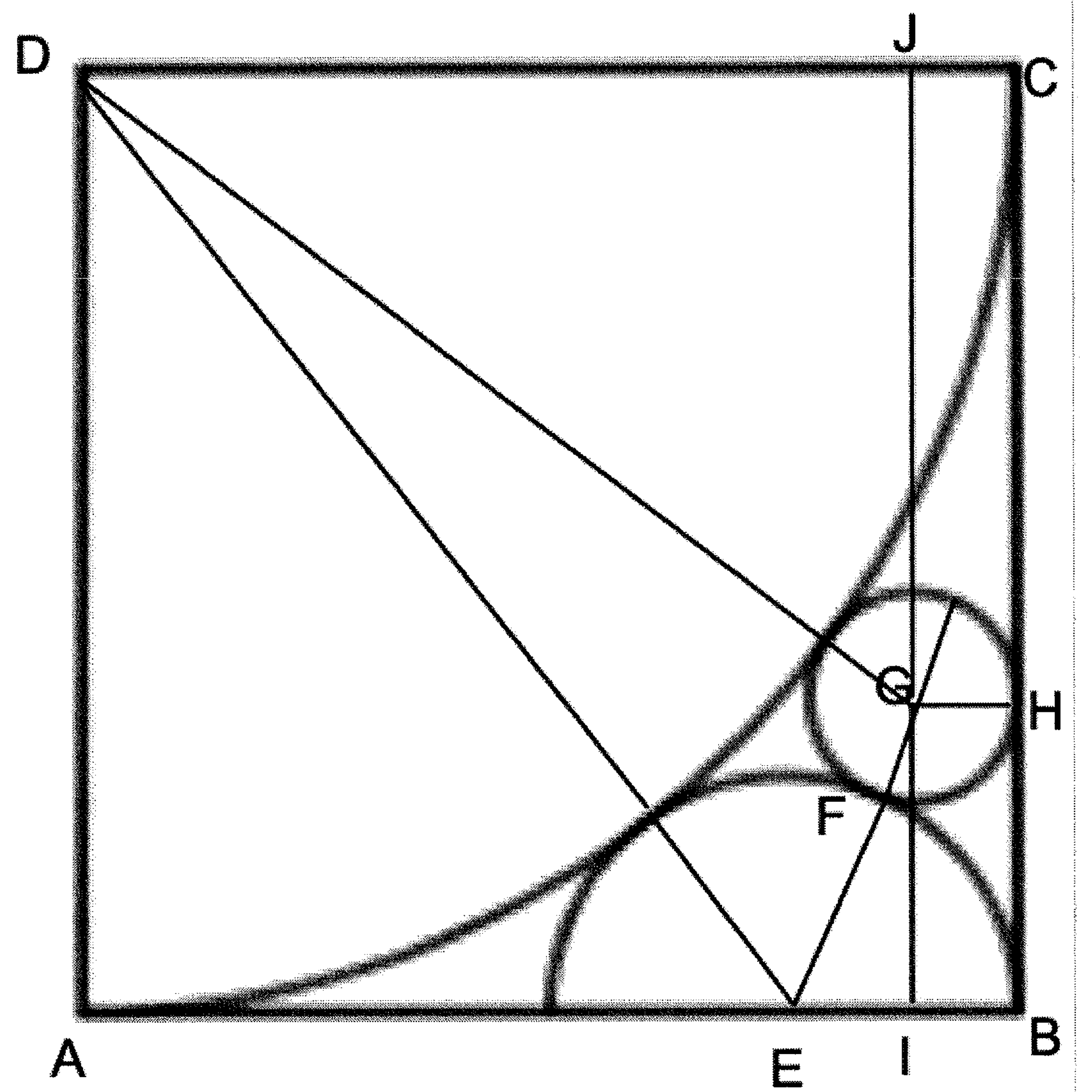
$$-7x + 175 = 0$$

$$x = 25$$

$$x - 20 = 25 - 20 = 5 = radius\_of\_circle$$

$$x - 15 = 25 - 15 = 10 = OP$$

AP = $\sqrt{3x}$ = $5\sqrt{3}$, BP=$5\sqrt{3}$, since tangent lengths are equal. ΔAOP is a 30-60-90 triangle. $\angle AOP = \angle BOP = 60°$, so the measure of minor arc AB is $120°$. Since $\angle ACP$ is an inscribed angle, $\angle ACP = 60°$. Draw the altitude from O in ΔAOC to get two 30-60-90 triangles with hypotenuse 5. Say the altitude meets $\overline{AC}$ at point D. Then OD=2.5, and AD= $\frac{5\sqrt{3}}{2}$, and AC=$5\sqrt{3}$. Similarly, CB=$5\sqrt{3}$, so AC+CB+AP+BP = $5\sqrt{3}+5\sqrt{3}+5\sqrt{3}+5\sqrt{3}=20\sqrt{3}$



Large circle has radius 2. Let medium-sized circle have radius x and small circle have radius y.

AB=2-x and DE=2+x, so for right ΔADE,

$(2-x)^2 + 2^2 = (2+x)^2$, $4-4x+x^2 + 4 = 4+4x+x^2$

$8x=4$, so $x=0.5$. Draw GH parallel to AB and Draw GI parallel to HB. $IB=y=GH$. $IE = 0.5-y$, since $EB=0.5$ and $EB-IB=EI$. $EG=0.5+y$. Find IG by $IE^2 + IG^2 = EG^2$ $(0.5-y)^2 + IG^2 = (0.5+y)^2$, so $0.25-y+y^2 + IG^2 = 0.25+y+y^2$, so $IG^2 = 2y$, and From ΔDGJ, $DJ=2-y$ and $DG=2+y$, so $JG^2+DJ^2 = DG^2$. $JG^2 + (2-y)^2 = (2+y)^2$. $JG^2 + 4-4y+y^2 = 4+4y+y^2$, so $JG^2=8y$. Now $IG+JG=BH+CH=2$.

$$\sqrt{2y} + \sqrt{8y} = 2$$
$$\sqrt{2y} + 2\sqrt{2y} = 2$$
$$3\sqrt{2y} = 2$$
$$\sqrt{2y} = \frac{2}{3}$$
$$2y = \frac{4}{9}$$
$$y = \frac{2}{9}$$

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Match 5 Round 4 Quadratic Equations and Complex Numbers

1.) ______ $3+2i, 3-2i$ ______

2.) _____ $7i$ ____

3.) __ $-3, 4i$ ________

1) Find the two complex solutions of $x^2 + bx + c = 0$ if the sum of the solutions is 6 and the product of the solutions is 13.

$$-b = r_1 + r_2 = 6$$

$$c = r_1 r_2 = 13$$

$$r_1(6 - r_1) = 13$$

$$-r_1^2 + 6r_1 - 13 = 0$$

$$r_1^2 - 6r + 13 = 0$$

$$r = \frac{6 \pm \sqrt{36-52}}{2} = \frac{6 \pm 4i}{2} = 3+2i, 3-2i$$

2.) Simplify: $\frac{(2+i)^3}{3-i} - \frac{(2-i)^3}{3+i}$

$$= \frac{(2+i)(2+i)(2+i)}{3-i} - \frac{(2-i)(2-i)(2-i)}{3+i}$$

$$= \frac{(3+4i)(2+i)}{3-i} - \frac{(3-4i)(2-i)}{3+i}$$

$$= \frac{2+11i}{3-i} - \frac{2-11i}{3+i}$$

$$= \frac{(2+11i)(3+i)}{(3-i)(3+i)} - \frac{(2-11i)(3-i)}{(3+i)(3-i)}$$

$$= \frac{-5+35i}{(3-i)(3+i)} - \frac{-5-35i}{(3+i)(3-i)}$$

$$= \frac{70i}{10} = 7i$$

3) Solve for all complex x: $x^2 + (3-4i)x - 12i = 0$

$$x^2 + (3-4i)x - 12i = 0$$

$$x = \frac{(-3+4i) \pm \sqrt{(3-4i)^2 - 4(-12i)}}{2}$$

$$= \frac{(-3+4i) \pm \sqrt{-7-24i+48i}}{2} = \frac{(-3-4i) \pm \sqrt{-7+24i}}{2}$$

$$Find\_\sqrt{-7+24i} = a+bi$$

$$-7+24i = (a^2-b^2)+2abi$$

$$a^2-b^2 = -7\_and\_2ab = 24$$

$$a = \pm 3, b = \pm 4$$

$$x = \frac{(-3+4i) \pm (3+4i)}{2}$$

$$x = \frac{(-3+4i)+(3+4i)}{2}\_or\_\frac{(-3+4i)-(3+4i)}{2}$$

$$x = \frac{8i}{2} = 4i\_or\_x = \frac{-6}{2} = -3$$

Can also be solved by grouping.

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Match 5 Round 5
Solving Trig
Equations

1.)__6, 30, 78, 102, 150_degrees , 174°

2.) __________ $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$ ________

3.) _____ $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$ ______

1) Solve for all x $0° \leq x \leq 180°$: $\sin(5x) = \frac{1}{2}$

5x=30, 390, 750,…
Or 5x=150, 510,870
x=6, 30, 78, 102, 150, and then x>180.

2) Solve for all x $0 \leq x < 2\pi$: $\tan^2(x) - \sec(x) = 1$

$\tan^2(x)-\sec(x)=1$

$(\sec^2(x)-1)-\sec(x)=1$

$\sec^2(x)-\sec(x)-2=0$

$(\sec(x)-2)(\sec(x)+1)=0$

$\sec(x)=2$ or $\sec(x)=-1$

$\cos(x) = \frac{1}{2}\_or\_\cos(x) = -1$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

3.) Solve for all x $0 \leq x < \pi$: $\sin(4x) + \cos(2x) = 0$

$$\sin(4x)+\cos(2x)=0$$

$$2\sin(2x)\cos(2x)+\cos(2x)=0$$

$$(\cos(2x))[2\sin(2x)+1]=0$$

$$\cos(2x)=0\_or\_\sin(2x)=\frac{-1}{2}$$

$$If\ \cos(2x)=0, 2x=\frac{\pi}{2}\_or\_2x=\frac{3\pi}{2}$$

$$x=\frac{\pi}{4},\frac{3\pi}{4}$$

$$If\ \sin(2x)=\frac{-1}{2}, 2x=\frac{7\pi}{6}\_or\_2x=\frac{11\pi}{6}$$

$$x=\frac{\pi}{4},\frac{3\pi}{4},\frac{7\pi}{12},\frac{11\pi}{12}$$

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Match 5 Round 6
Sequences and Series

1.) ____4____

2.) ____3, 28____

3.) ____$\frac{1}{25}$____

1.) For what natural number n is $0.76 < \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k^2} < 0.80$

$$0.76 < \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k^2} < 0.80$$

$$n = 1, sum\_is\_1$$

$$n = 2, sum\_is\_1 - \frac{1}{4} = 0.75$$

$$n = 3, sum\_is\_1 - \frac{1}{4} + \frac{1}{9} = 0.8611...$$

$$n = 4, sum\_is\_1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} = 0.8611 - 0.0625$$

Which is just smaller than 0.8

2.) An arithmetic sequence has first term -2. The second term is the square root of the fourth term. Find all possible values of the sixth term.

The square of the second term is the fourth term, so if the sequence is $\{a_n\}$ with common difference d,

$$(-2+d)^2=-2+3d$$

$$4-4d+d^2=-2+3d$$

$$d^2-7d+6=0$$

$$(d-6)(d-1)=0$$

$$d=1\_or\_d=6$$

$$If\_d=1,a_6=-2+5*1=3$$

$$If-d=a_6=-2+5*6=28$$

3. An infinite geometric series converges to 6.25. The second term of the original geometric sequence is 4 less than the first term. Give all possible values for the fourth term of the sequence.

$$\frac{a_1}{1-r}=\frac{25}{4}\_and\_a_1r=a_1-4,\_so\_r=\frac{a_1-4}{a_1}$$

$$\frac{a_1}{1-\frac{a_1-4}{a_1}}=\frac{a_1}{\frac{a_1-a_1-4}{a_1}}=\frac{a_1^2}{4}=\frac{25}{4},so$$

$$a_1^2=25,a_1=\pm5$$

$$If\_a_1=5,r=\frac{5-4}{5}=\frac{1}{5},a_4=5\left(\frac{1}{5}\right)^3=\frac{1}{25}$$

$$If\_a_1=-5,r=\frac{-5-4}{-5}=\frac{9}{5},series\_does\_not\_converge$$

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 5 Team Round

1.) $\frac{4}{77}$

4.) $\frac{\sqrt{2}+i\sqrt{6}}{2}$, $\frac{-\sqrt{2}-i\sqrt{6}}{2}$

2.) $\pm 3\sqrt{10}$

5.) $\frac{-3\sqrt{2}\pm\sqrt{14}}{8}$

Note: Diagrams not necessarily drawn to scale

3.) $55, \frac{589}{9}$ degrees

6.) $-4, 2+2i\sqrt{3}, 2-2i\sqrt{3}$

1.) Simplify: $\frac{1}{7}*\frac{1}{8}+\frac{1}{8}*\frac{1}{9}+\frac{1}{9}*\frac{1}{10}+\frac{1}{10}*\frac{1}{11}$

Use the sequence-related method of telescoping:

$$\frac{1}{7}*\frac{1}{8}+\frac{1}{8}*\frac{1}{9}+\frac{1}{9}*\frac{1}{10}+\frac{1}{10}*\frac{1}{11}=$$

$$(\frac{1}{7}-\frac{1}{8})+(\frac{1}{8}-\frac{1}{9})+(\frac{1}{9}-\frac{1}{10})+(\frac{1}{10}-\frac{1}{11})$$

$$=\frac{1}{7}-\frac{1}{11}=\frac{11}{77}-\frac{7}{77}=\frac{4}{77}$$

2.) Solve for x:

$$2-\cfrac{x}{3-\cfrac{x}{4-\cfrac{x}{5-\cfrac{x}{6}}}}=4$$

$$2-\cfrac{x}{3-\cfrac{x}{4-\cfrac{x}{5-\cfrac{x}{6}}}}=4$$

$$2-\cfrac{x}{3-\cfrac{x}{4-\cfrac{x}{5-\cfrac{x}{6}}}}=2-\cfrac{x}{3-\cfrac{x}{4-\cfrac{x}{\cfrac{30-x}{6}}}}=$$

$$2-\cfrac{x}{3-\cfrac{x}{4-\cfrac{6x}{30-x}}}=2-\cfrac{x}{3-\cfrac{x}{\cfrac{120-4x-6x}{30-x}}}=$$

$$2-\cfrac{x}{3-\cfrac{x}{\cfrac{120-10x}{30-x}}}=2-\cfrac{x}{3-\cfrac{30x-x^2}{120-10x}}=$$

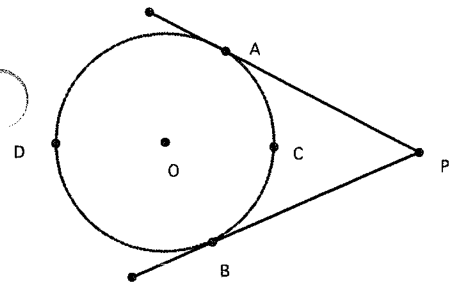
$$2-\cfrac{x}{\cfrac{360-30x-(30x-x^2)}{120-10x}}=2-\cfrac{x}{\cfrac{360-60x+x^2}{120-10x}}=$$

$$2-\frac{120x-10x^2}{360-60x+x^2}=\frac{720-120x+2x^2-120x+10x^2}{360-60x+x^2}=$$

$$\frac{720-240x+12x^2}{360-60x+x^2}$$

$$\frac{720-240x+12x^2}{360-60x+x^2}=4:720-240x+12x^2=1440-240x+4x^2:$$

$$8x^2-720=0:x^2-90=0,x=\pm3\sqrt{10}$$



3.) In the diagram above, $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$ are tangent to the circle with center O at points A and B. The measure of arc ACB is $2x^2$-3 and the measure of arc ADB is $4x^2 - 2x$-5. Find the degree measure of $\angle APB$.

$2x^2 - 3 + 4x^2 - 2x - 5 = 360$
$6x^2 - 2x - 368 = 0$
$3x^2 - x - 184 = 0$
$(x-8)(3x+23)=0$

$$x = 8\_or\_x = \frac{-23}{3}$$

If x=8, $2x^2 - 3 = 125$, $4x^2 - 2x - 5 = 235$, $\angle APB = \frac{235-125}{2} = 55$

If $x = \frac{-23}{3}$, then

$$x = \frac{-23}{3}$$

$$2(\frac{-23}{3})^2 - 3 = 2*\frac{529}{9} - \frac{27}{9} = \frac{1058-27}{9} = \frac{1031}{9}$$

$$Arc\_ADB\_has\_measure\_360 - \frac{1031}{9} = \frac{3240-1031}{9} = \frac{2209}{9}$$

$$\frac{1}{2}(\frac{2209}{9} - \frac{1031}{9}) = \frac{1}{2}(\frac{1178}{9}) = \frac{589}{9}$$

4. Find the two complex square roots of $-1+i\sqrt{3}$.

$$-1+i\sqrt{3}=(a+bi)^2=a^2-b^2+2abi$$

$$2ab=\sqrt{3}, b=\frac{\sqrt{3}}{2a}$$

$$-1=a^2-b^2=a^2-(\frac{\sqrt{3}}{2a})^2=a^2-\frac{3}{4a^2}$$

$$-1=a^2-\frac{3}{4a^2}$$

$$-4a^2=a^4-3$$

$$4a^4+4a^2-3=0$$

$$(2a^2-1)(2a^2+3)=0$$

$$a^2=\frac{1}{2}, a=\pm\frac{\sqrt{2}}{2}$$

$$b=\frac{\sqrt{3}}{2(\pm\frac{\sqrt{2}}{2})}=\pm\frac{\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$$

$$a+bi=\frac{\sqrt{2}+i\sqrt{6}}{2}\_or\_\frac{-\sqrt{2}-i\sqrt{6}}{2}$$

5.) What are all possible values of sin(x) if $\cos(x+\frac{\pi}{4})=\frac{3}{4}$?

$$\cos(x+\frac{\pi}{4})=\frac{3}{4}$$

$$\cos(x)\cos(\frac{\pi}{4})-\sin(x)\sin(\frac{\pi}{4})=\frac{3}{4}$$

$$\frac{\sqrt{2}}{2}(\cos(x)-\sin(x))=\frac{3}{4}$$

$$\cos(x)-\sin(x)=\frac{3\sqrt{2}}{4}$$

$$\cos(x)=\frac{3\sqrt{2}}{4}+\sin(x)$$

$$\cos^2(x)=\frac{9}{8}+\frac{3\sqrt{2}}{2}\sin(x)+\sin^2(x)$$

$$1-\sin^2(x)=\frac{9}{8}+\frac{3\sqrt{2}}{2}\sin(x)+\sin^2(x)$$

$$2\sin^2(x)+\frac{3\sqrt{2}}{2}\sin(x)+\frac{1}{8}=0$$

$$x=\frac{-\frac{3\sqrt{2}}{2}\pm\sqrt{\frac{9}{2}-4*2*\frac{1}{8}}}{4}=\frac{-\frac{3\sqrt{2}}{2}\pm\sqrt{\frac{7}{2}}}{4}=$$

$$\frac{-\frac{3\sqrt{2}}{2}\pm\frac{\sqrt{14}}{2}}{4}=\frac{-3\sqrt{2}\pm\sqrt{14}}{8}$$

6.) A geometric sequence $\{a_n\}$ of complex numbers has the property that $a_4 = -64a_1$. Find all possible values for the common ratio of the sequence.

$$a_4 = a_1 r^3 = -64a_1, so\_r^3 = -64$$

$$r^3 + 64 = 0\_has\_real\_solution\_r = -4$$

$$\frac{r^3+64}{r+4} = r^2 - 4r + 16$$

$$Solve\_r^2 - 4r + 16 = 0$$

$$r = \frac{4 \pm \sqrt{16-64}}{2} = \frac{4 \pm i\sqrt{48}}{2} = \frac{4 \pm 4i\sqrt{3}}{2} = 2 \pm 2i\sqrt{3}$$

$$-4, 2+2i\sqrt{3}, 2-2i\sqrt{3}$$