FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 3 Round 1 Arithmetic: Scientific Notation and Bases

1.) ________1,2________________

2.) _ $6.25*10^{-6}$ ______

3.) _________14, 0.A_________

1.)_ Find all digits D such that $DD1_5$ represents a prime number.
D=0 0+0+1 =1, not prime.
D=1 25+5+1=31, prime
D=2 50+10+1=61, prime
D=3 75+15+1=91 = 13x7
D=4 100 + 20 + 1 = 121 = 11x11
D=1, D=2

2.) Simplify and express your answer in scientific notation:

$$\frac{(40*10^{6})^{4}*(50*10^{-4})^{3}}{(1250*10^{4})^{2}*(\frac{1}{8}*10^{-2})^{-5}}$$

$$\frac{(40*10^6)^4*(50*10^{-4})^3}{(1250*10^4)^2*(\frac{1}{8}*10^{-2})^{-5}}$$

$$=\frac{(2^3*5*10^6)^4*(2*5^2*10^{-4})^3}{(2*5^4*10^4)^2*(2^{-3}*10^{-2})^{-5}}$$

$$=\frac{2^{12}5^4*2^35^6*10^{12}}{2^25^8*2^{15}*10^{18}}=$$

$$=2^{-2}*5^2*10^{-6}$$

$$=\frac{25}{4}*10^{-6}$$

$$=6.25*10^{-6}$$

3.) In the hexadecimal (base 16) system, A=10, B=11, C=12, D=13, E=14, and F=15. Find the two numbers that solve the equation $x^2 - px + q = 0$ where $p=14.A_{16}$ and $q = C.8_{16}$. Express your answers in hexadecimal.

$$14A_{16}=16+4+\frac{10}{16}=20+\frac{10}{16}=\frac{165}{8}$$

$$C.8_{16}=12+\frac{8}{16}=\frac{25}{2}$$

$$Solve:\_x^2-\frac{165}{8}x+\frac{25}{2}=0$$

$$8x^2-165x+100=0$$

$$(x-20)(8x-5)=0$$

$$x=20,x=\frac{5}{8}$$

$$20=16+4=14_{16}$$

$$\frac{5}{8}=\frac{10}{16}=0.A_{16}$$

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Match 3 Round 2
Algebra: Word Problems

1.) ______35______

2.) ~~70 miles~~ 4.5 hours

3.) ~~4.5 hours~~ 70 miles

1.) Daisy has a collection of dimes and quarters that have a value of $6.80. She has 9 more quarters than she has dimes. How many coins does she have altogether?

$Let\_q = \#\_of\_quarters, d = \#\_of\_\text{dim}es$

$25q + 10d = 680$

$q = d + 9$

$25(d+9) + 10d = 680$

$25d + 225 + 10d = 680$

$35d + 225 = 680$

$35d = 455$

$d = 13$

$q = 13 + 9 = 22$

$Total\_number\_of\_coins\_is\_35$

3) ~~2.)~~ A hawk leaves the train station and flies west at H mph for 2 hours. A train then departs the station and travels east at T mph, while the hawk continues to fly west at the same speed. After the train has been traveling for one hour, the hawk turns around and flies east at a speed that is 5 mph faster than its westbound speed, while the train continues east at T mph. The time for the hawk to turn around is negligible. The train's eastbound speed is twice the hawk's eastbound speed. After 10 minutes of flying east,

the hawk and the train are the same distance from the station but in opposite directions. How far is each object from the train station?

Hawk travels at H mph, flies west for 3 hours, so its distance west of the station is 3H. Until the hawk turned around, the train went a distance of T.

In 10 minutes, the hawk flew east at $\frac{1}{6}(H+5)$ mph, so its distance west of the station was $3H-\frac{1}{6}(H+5)=\frac{17H-5}{6}$ miles. The train continued east for 10 minutes at T mph, so it went an additional $\frac{1}{6}T$ miles east, for a total of $\frac{7}{6}T$ miles. Therefore,

$$\frac{7}{6}T=\frac{17H-5}{6},$$

$$T=\frac{17H-5}{7}$$

$$7T=17H-5,\_and\_T=2(H+5),so$$

$$7(2(H+5))=17H-5$$

$$14H+70=17H-5$$

$$75=3H$$

$$H=25\_mph$$

$$Distance\_is\_\frac{17*25-5}{6}miles=$$

$$\frac{425-5}{6}=\frac{420}{6}=70\_miles$$

2) ~~3)~~ Buford and Rufus are math teachers. Rufus can correct the same pile of papers in 1.5 hours more than Buford. Buford takes the stack of papers and begins grading. One hour later, Rufus comes along and helps him until the grading is completed. Rufus works for two hours. How long would it take Buford to grade the papers working alone?

Let x=time it takes Buford to grade the stack alone, x+1.5 = time it takes Rufus to grade the stack alone. Rufus works for 2 hours and Buford works for 3 hours. Solve:

$$\frac{3}{x}+\frac{2}{x+1.5}=1$$

$$3(x+1.5)+2x=x(x+1.5)$$

$$3x+4.5+2x=x^2+1.5x$$

$$x^2-3.5x-4.5=0$$

$$2x^2-7x-9=0$$

$$(2x-9)(x+1)=0$$

$$x=\frac{9}{2}hours=4.5\_hours$$

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Match 3 Round 3
Geometry: Polygons

1.) ____________30________________

2.) ___9, 18, 36, 45, 72, 90___

3.) _______$384\sqrt{2}-384$___

1.) The number of diagonals of an N-sided convex polygon is 405. Find N.

$$\frac{N(N-3)}{2}=405$$

$$N(N-3)=810$$

$$N^2-3N-810=0$$

$$(N-30)(N+27)=0$$

$$N=30$$

2) The exterior angle of an N-sided regular polygon is an integer but not a multiple of 3. Find all values of N for which this is true if $N \leq 100$.

$\frac{360}{N}$ must be an integer, so N must be a factor of 360. In order for $\frac{360}{N}$ to not be a multiple of 9, N must have a factor of 9 since $360 = 2^3 * 3^2 * 5$. N could be 3^2, 3^2*2, 3^2*2^2, 3^2*5 , 3^2*2^3, 3^2*2*5 and then further values of N are greater than 100, which makes N = 9, 18, 36, 45,72, 90

3) A regular octagon has apothem of length 8 cm. Find the positive numerical difference between its perimeter in cm and its area in cm^2 given $\tan(67.5°)=1+\sqrt{2}$.

Let the octagon has side length x. Break up the octagon into 8 isosceles triangles with vertex angle 45 degrees and base angles 67.5 degrees. To find half of one side of the octagon, use

$$\tan(67.5) = \frac{8}{\frac{x}{2}}$$

$$1+\sqrt{2} = \frac{16}{x}$$

$$x = \frac{16}{1+\sqrt{2}} = \frac{16(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = 16\sqrt{2}-16$$

The perimeter of the octagon is $8*(16\sqrt{2}-16) = 128\sqrt{2}-128$ cm.
The area of the octagon is the sum of the areas of 8 triangles with base x and altitude 8, so area =

$$8*(\frac{1}{2})(8)(16\sqrt{2}-16) =$$

$$32*(16\sqrt{2}-16).$$

The difference is

$$32*(16\sqrt{2}-16) - 8*(16\sqrt{2}-16) =$$

$$24(16\sqrt{2}-16) = 384\sqrt{2}-384$$

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Match 3 Round 4
Algebra 2: Functions and Inverses

Note: The inverse of a function is not necessarily itself a function.

1.) ____________ $\frac{x-1}{6}$ ____________

2). Domain:____ $[-3,2)\cup(2,\infty)$ _______

Range: ______ $(-\infty,\infty)$ ___________

3.) ____________ $(\frac{-5}{4},\frac{3}{2})$ ________

1.) $f(x)=2x+5$ and $g(x)=3x-2$. Find $g^{-1}(f^{-1}(x))$

$$f(x)=2x+5$$

$$Find\_f^{-1}(x): x=2y+5, f^{-1}(x)=\frac{x-5}{2}$$

$$g(x)=3x-2$$

$$Find\_g^{-1}(x): x=3y-2, g^{-1}(x)=\frac{x+2}{3}$$

$$g^{-1}(f^{-1}(x))=\frac{\frac{x-5}{2}+2}{3}=\frac{\frac{x-5+4}{2}}{3}=\frac{x-1}{6}$$

2.) Give the domain and range of $y=\frac{\sqrt{x+3}}{2-x}$. If you use interval notation, use union and/or intersection if necessary.

x must be greater than or equal to -3 in order to take the square root, and x can not be 2. The domain can be expressed either as $[-3,2) \quad (2,\infty)$ or $x \geq -3, x \neq 2$. To find the range, when x is -3, y=0. When x is near 2, the numerator is near the square root of 5, and the denominator is near 0, so on one side of 2, the function increases without bound, and on the other side of 2, the function decreases without bound. As x gets very large, the fraction approaches 0, but since y=0 when
x=-3, the values in the range include all real numbers.

3) $h(x+1) = x^2 - x - 1$ Give the coordinates of the vertex of the relation

$y = h^{-1}(x)$

.

$$h(x+1) = x^2 - x - 1$$

$$h(x) = (x-1)^2 - (x-1) - 1 = x^2 - 2x + 1 - x + 1 - 1 = x^2 - 3x + 1$$

$$Find\_the\_vertex\_of\_h(x)$$

$$h(x) = x^2 - 3x + 1 = (x^2 - 3x + \frac{9}{4}) + 1 - \frac{9}{4}$$

$$= (x - \frac{3}{2})^2 - \frac{5}{4}$$

$$Vertex\_of\_h(x)\_is\_(\frac{3}{2}, \frac{-5}{4})$$

$$Vertex\_of\_h^{-1}(x)\_is\_(\frac{-5}{4}, \frac{3}{2})$$

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Match 3 Round 5 Advanced Math: Exponents and Logarithms

1.) ______ $18b+2$ ______________

2.) ______________ $3^{\frac{3}{5}}$ ________

3.) ________ $\sqrt{5}, 25$ ____________

1.) If $x=8^{b+4}$ and $y=32^{3b-2}$, what is $\log_2(xy)$ in terms of b?

$x=8^{b+4}=2^{3(b+4)}=2^{3b+12}$

$y=32^{3b-2}=2^{5(3b-2)}=2^{15b-10}$

$\log_2(xy)=\log_2 x+\log_2 y$

$=3b+12+15b-10=18b+2$

2.) Solve for x. You may use a fractional exponent in your answer.

$$\log_3 x+\log_{27}(x^2)=1$$

$$\log_3 x + \log_{27}(x^2) = 1$$

$$\frac{\log x}{\log 3} + \frac{\log(x^2)}{\log 27} = 1$$

$$\frac{\log x}{\log 3} + \frac{2\log x}{3\log 3} = 1$$

$$\frac{3\log x}{3\log 3} + \frac{2\log x}{3\log 3} = 1$$

$$5\log x = 3\log 3$$

$$\log x = \frac{3}{5}\log 3$$

$$x = 10^{\frac{3}{5}\log 3} = 3^{\frac{3}{5}}$$

3.)_ If $z = \log_5(y)$, solve for y: $\dfrac{(125)^{z+2}}{(0.04)^{z-3}} = (25)^{z^2+1}$ Express your answer(s) in radical form if necessary.

$$\frac{(125)^{z+2}}{(0.04)^{z-3}} = (25)^{z^2+1}$$

$$\frac{5^{3(z+2)}}{5^{-2(z-3)}} = 5^{2z^2+2}$$

$$\frac{5^{3z+6}}{5^{-2z+6}} = 5^{2z^2+2}$$

$$5^{5z} = 5^{2z^2+2}$$

$$2z^2 - 5z + 2 = 0$$

$$(2z-1)(z-2) = 0$$

$$z = \frac{1}{2}\_or\_z = 2$$

$$\frac{1}{2} = \log_5 y\_or\_2 = \log_5 y$$

$$y = 5^{\frac{1}{2}}\_or\_y = 5^2$$

$$y = \sqrt{5}, 25$$

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Match 3 Round 6
Discrete Math: Matrices

1.) $\begin{bmatrix} -20 & 16 \\ 32 & -3 \end{bmatrix}$

2) $\frac{3}{4}, -3$

3.) 0.2

1.)

$$C = \begin{bmatrix} 5 & 1 & -1 \\ 2 & -3 & 4 \end{bmatrix}, D = \begin{bmatrix} -2 & 3 \\ -4 & -5 \\ 6 & -6 \end{bmatrix}$$

,

Find CD.

$$CD_{11} = 5(-2) + 1(-4) + (-1)(6) = -20$$

$$CD_{12} = 5 * 3 + 1(-5) + (-1)(-16) = 16$$

$$CD_{21} = 2(-2) + (-3)(-4) + 4 * 6 = 32$$

$$CD_{22} = 2 * 3 + (-3)(-5) + 4(-6) = -3$$

$$CD = \begin{bmatrix} 20 & 16 \\ 32 & -3 \end{bmatrix}$$

2) Find all values of k such that

$$\begin{vmatrix} k+4 & 2 & 4 \\ k+2 & 3 & 0 \\ 2 & k & 1 \end{vmatrix} = -7$$

$$3*1*(k+4)+0+4k(k+2)-[2*3*4+2(k+2)*1+0]=-7$$

$$3k+12+4k^2+8k-[24+2k+4]=-7$$

$$4k^2+9k-16=-7$$

$$4k^2+9k-9=0$$

$$(4k-3)(k+3)=0$$

$$k=\frac{3}{4},-3$$

3.)

$$A=\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, B=\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

Find the determinant of $(AB)^{-1}+B^{-1}A^{-1}$

$$AB = \begin{bmatrix} 1+0 & -1-4 \\ 3+0 & -3+8 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ 3 & 5 \end{bmatrix}$$

$$\det(AB) = 5-(-15) = 20$$

$$(AB)^{-1} = \frac{1}{20}\begin{bmatrix} 5 & 5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.25 \\ -0.15 & 0.05 \end{bmatrix}$$

$$\det(B) = 2$$

$$B^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix}$$

$$\det(A) = 4-(-6) = 10$$

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.2 \\ -0.3 & 0.1 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 0.4 & 0.2 \\ -0.3 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.4-0.15 & 0.2+0.05 \\ 0+-0.15 & 0+0.05 \end{bmatrix} =$$

$$\begin{bmatrix} 0.25 & 0.25 \\ -0.15 & 0.05 \end{bmatrix}$$

$$(AB)^{-1} + B^{-1}A^{-1} = \begin{bmatrix} 0.25 & 0.25 \\ -0.15 & 0.05 \end{bmatrix} + \begin{bmatrix} 0.25 & 0.25 \\ -0.15 & 0.05 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ -0.3 & 0.1 \end{bmatrix}$$

Determinant is (0.5)(0.1)-(-0.3)(0.5)=(0.05)+(0.15) = (0.2)

$=$

FAIRFIELD COUNTY MATH LEAGUE 2019-20 Match 3 Team Round

Note: The inverse of a function or relation is not necessarily a function.

1.) ________320________ 4.) a=___ $18\sqrt[3]{2}$ b= $\frac{4}{3}$

2.) ______ A, B________ 5.)______________ 8__________

3.) ____________12________ 6.) ________8______________

1.) You have M ml of a solution of acid and water that is one-ninth acid. If you add N ml of acid, the solution is 20% acid. If you then add P additional ml of acid, the solution is 50% acid. If you then add 80 additional ml of acid, the solution is 60% acid. Find M+N+P.

$$M = original\_amount\_of\_solution$$

$$\frac{1}{9}M = original\_amount\_of\_acid$$

$$Add\_N\_ml\_of\_acid\_so$$

$$\frac{1}{9}M + N = 0.2(M + N)$$

$$\frac{1}{9}M + N = \frac{1}{5}M + \frac{1}{5}N$$

$$\frac{4}{5}N = \frac{4}{45}M,\_so\_M = 9N$$

$$Add\_P\_more\_ml\_of\_acid$$

$$\frac{1}{9}M + N + P = 0.5(M + N + P)$$

$$Substitute\_M = 9N$$

$$N + N + P = 0.5(9N + N + P)$$

$$2N + P = 5N + 0.5P$$

$$0.5P = 3N, P = 6N.$$

$$Add\_80\_more\_ml\_of\_acid$$

$$\frac{1}{9}M + N + P + 80 = 0.6(M + N + P + 80)$$

$$Substitute\_M = 9N\_and\_P = 6N$$

$$N + N + 6N + 80 = 0.6(N + N + 6N + 80)$$

$$8N + 80 = 0.6(8N + 80)$$

$$8N + 80 = 9.6N + 48$$

$$32 = 1.6N$$

$$N = 20, M = 9 * 20 = 180, P = 6 * 20 = 120$$

$$M + N + P = 320$$

2.) NEW: In the duodecimal (base 12) system, A=10 and B=11. All numbers in the following matrix equation are in duodecimal. Find all values of d such that

$$\left|\begin{bmatrix} d-41 & 12d+42 \\ -2 & d \end{bmatrix}\begin{bmatrix} A & B \\ 1 & 2 \end{bmatrix}\right| = -76$$

Express your answers in duodecimal.

$Convert\_numbers\_to\_base\_10$

$$\left|\begin{bmatrix} d-(4*12+1) & (1*12+2)d+(4*12+2*1) \\ -2 & d \end{bmatrix}\begin{bmatrix} 10 & 11 \\ 1 & 2 \end{bmatrix}\right| = -(7*12+6*1)$$

$$\left|\begin{bmatrix} d-49 & 14d+50 \\ -2 & d \end{bmatrix}\begin{bmatrix} 10 & 11 \\ 1 & 2 \end{bmatrix}\right| = -90$$

$Use\_\det(AB) = \det(A)\det(B)$

$$\det\begin{bmatrix} 10 & 11 \\ 1 & 2 \end{bmatrix} = 20-11 = 9, so$$

$$\det\begin{bmatrix} d-49 & 14d+50 \\ -2 & d \end{bmatrix} = -10$$

$$(d-49)d-(-2(14d+50)) = -10$$

$$d^2-49d-(-28d-100) = -10$$

$$d^2-21d+100 = -10$$

$$d^2-21d+110 = 0$$

$$(d-11)(d-10) = 0$$

$$d = 11, d = 10$$

$$In\_duodecimal, d = A, d = B$$

$$A, B$$

3. The sum of the interior angles of an N-gon is 90 less than 35 times the number of its diagonals. Find N.

$$180(n-2)=35\frac{n(n-3)}{2}-90$$

$$360(n-2)=35n^2-105n-180$$

$$360n-720=35n^2-105n-180$$

$$35n^2-465n+540=0$$

$$7n^2-93n+108=0$$

$$(7n-9)(n-12)=0$$

$$n=12$$

4. $f(x)=9^{2x+1}, g(x)=\log_{27}(2x)$. If y = f(g(x)) is expressed as $y=ax^b$ for some numbers a and b where x>0, find a and b. Express any radicals in simplest radical form.

$$f(x)=9^{2x+1}, g(x)=\log_{27}(2x)$$

$$f(g(x))=9^{2(\log_{27}(2x))+1} =$$

$$9^{2(\log_{27}(2x))} * 9 =$$

$$9 * 81^{(\log_{27}(2x))}$$

$$9 * 3^{4(\frac{\log_3(2x)}{\log_3 27})} =$$

$$9 * (3^{\log_3(2x)})^{\frac{4}{3}}$$

$$= 9 * (2x)^{\frac{4}{3}} = 9 * 2^{\frac{4}{3}} x^{\frac{4}{3}}$$

$$= 9 * \sqrt[3]{16} * x^{\frac{4}{3}}$$

$$= 9 * \sqrt[3]{8}\sqrt[3]{2}(x^{\frac{4}{3}})$$

$$= 18\sqrt[3]{2}(x^{\frac{4}{3}})$$

$$\frac{4}{3}$$

5. The amount of electricity used in 1 year in the United States is 4×10^{12} kw-hr. 1 kw-hr is equal to 3.6×10^{5} joules. The rate at which solar radiation hits the earth is 1.1×10^{3} watts/square meter. 1 joule = 1 watt/second. Solar collectors are currently about 20% efficient in converting solar radiation to electricity. There are about 3.1×10^{7} seconds in one year. The area in square meters that would need to be covered by solar collectors at the current rate of usage and efficiency is $M \times 10^{n}$ for some values of M and n. Approximate n to the nearest whole number.

$$20\%\_of\_1.1x10^{3} = 2.2x10^{2}$$

$$\frac{2.2x10^2 watts}{m^2} * AREA\_in\_m^2\_* \frac{1\frac{Joule}{\sec}}{1\_watt} * \frac{3.1x10^7 \sec}{1\_year} = \frac{4x10^{12} kw-hr}{1\_year} * \frac{3.6x10^5 Joules}{kw-hr}$$

$$AREA = \frac{3.6x10^5 * 4x10^{12}}{3.1x10^7 * 2.2x10^2} \approx 2x\frac{10^{17}}{10^9} \approx 2x10^8\_m^2$$

6. Solve for x: $\log_6(2x-10) - \log_{36}(x-2) = \frac{1}{2}$

$$\log_6(2x-10)-\log_{36}(x-2)=\frac{1}{2}$$

$$\log_6(2x-10)-\frac{\log_6(x-2)}{\log_6 36}=\frac{1}{2}$$

$$\log_6(2x-10)-\frac{\log_6(x-2)}{2}=\frac{1}{2}$$

$$\frac{2\log_6(2x-10)}{2}-\frac{\log_6(x-2)}{2}=\frac{1}{2}$$

$$\log_6(2x-10)^2-\log_6(x-2)=1$$

$$\log_6\frac{(2x-10)^2}{x-2}=1$$

$$\frac{(2x-10)^2}{x-2}=6^1$$

$$(2x-10)^2=6(x-2)$$

$$4x^2-40x+100=6x-12$$

$$4x^2-46x+112=0$$

$$2x^2-23x+56=0$$

$$(x-8)(2x-7)=0$$

$$x=8, x=\frac{7}{2}$$

$$but\_x=\frac{7}{2}\_is\_extraneous,$$

$$\log_6(2*\frac{7}{2}-10)\_is\_not\_a\_number$$

$$so\_x=8$$