FAIRFIELD COUNTY MATH LEAGUE (FCML) 2016-2017

Match 1 Round 1
Arithmetic: Percents

1)______1.25____________ __

2.) _______10189_____________

3.) __________54____________%_

1) 40% of (12X+5) is two less than (2X+10)% of 80. Find X.

$$\frac{40}{100}(12x+5)=\frac{2x+10}{100}*80-2$$
$$0.4(12x+5)=0.8(2x+10)-2$$
$$4.8x+2=1.6x+8-2$$
$$3.2x+2=6$$
$$3.2x=4$$
$$x=\frac{4}{3.2}=\frac{4*10}{32}=\frac{40}{32}=1.25$$

2) 30% of 35% of 40% of 45% of N is M, where N and M are both positive integers. What is the smallest possible value of M+N?

$$(0.3)(0.35)(0.4)(0.45)N=M, so$$
$$\frac{3}{10}*\frac{7}{20}*\frac{2}{5}*\frac{9}{20}N=M, so$$
$$\frac{3*7*9}{5*20*5*20}N=M,$$

so N must be divisible by 10000, so the smallest is N=10000. M must be 3*7*9 = 189. N+M = 10189.

3) 60% of the students at Horsehide High School are girls, and 40% are boys. 35% of the girls are Red Sox fans, while 45% of the boys are Red Sox fans. To the nearest percent, what percent of the Red Sox fans are girls?

Choose a number for which these products of percents give integer values, say 300. 60% of 300 is 180 and 40% of 300 is 120. Take 35% of 180 to give 63 and 45% of 120 to give 54. The ratio of girls to the total is 63 to 117, or 7 to 13. Divide 7 by 13 to get 54%.

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Match 1 Round 2
Algebra I: Equations

1.) _________1.1________

2.) __________-90____________

3.) ______ -5, -10___________

1.) Solve for a:

$$0.4(a-0.6)=3a-(2a+0.9)$$
$$0.4a-0.24=3a-2a-0.9$$
$$0.4a-0.24=a-0.9$$
$$0.66=0.6a$$

$$a=\frac{0.66}{0.6}=1.1$$

2) Solve for b:

$$1+\frac{b}{3+\frac{b}{5}}=7$$
$$1+\frac{b}{3+\frac{b}{5}}=7$$
$$1+\frac{b}{\frac{15+b}{5}}=7$$
$$1+\frac{5b}{15+b}=7$$
$$\frac{5b}{15+b}=6$$
$$5b=6(15+b)$$
$$5b=90+6b$$
$$-b=90$$
$$b=-90$$

3) Solve for c:

$$c - 2 + \frac{c-5}{c+4} = c + 3 - \frac{c}{c+6}$$

$$(c - 2 + \frac{c-5}{c+4} = c + 3 - \frac{c}{c+6})(c+4)(c+6)$$

$$(c-2)(c+6)(c+4) + (c-5)(c+6) = (c+3)(c+4)(c+6) - c(c+4)$$

$$(c^2 + 4c - 12)(c+4) + c^2 + c - 30 = (c^2 + 7c + 12)(c+6) - (c^2 + 4c)$$

$$c^3 + 8c^2 + 4c - 48 + c^2 + c - 30 = c^3 + 13c^2 + 54c + 72 - c^2 - 4c$$

$$9c^2 + 5c - 78 = 12c^2 + 50c + 72$$

$$0 = 3c^2 + 45c + 150$$

$$0 = c^2 + 15c + 50$$

$$0 = (c+10)(c+5)$$

$$c = -10, c = -5$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2016-2017

Match 1 Round 3
Geometry: Triangles
And Quadrilaterals

1.)__________ $\sqrt{77}$ ____________cm

2.)__ 60_, _120__, 588________cm^2

3.) ________ $20+7\sqrt{2}$ _______cm

1) In quadrilateral ABCD, $\angle ABC$ is a right angle. If the diagonal is drawn from A to C, $\angle ACD$ is also a right angle. If AB=6, BC=5, and CD=4, find AD.

   By Pythagorean theorem, $AC^2 = AB^2 + BC^2$, so AC = $\sqrt{36+25}$ = $\sqrt{61}$. Then ΔACD is a right triangle, so $AD^2 = AC^2 + CD^2$, so AD = $\sqrt{(\sqrt{61})^2+4^2}=\sqrt{77}$

2). The length and width of a rectangle differ by 7 cm. Give the 3 smallest values of areas of rectangles that also meet the condition that the length, width, and diagonal measurement are all integers.

From Pythagorean triples, two basic triples for which the first two numbers differ by 7 are 5-12-13 and 8-15-17, so the areas for those are 5*12=60 and 8*15=120. A 3-4-5 triangle can be scaled up to a 21-28-35, so the area of that triangle is 21*28=588. All other such rectangles have larger areas.

3.) The median of an isosceles trapezoid measures 10 cm. The area of the trapezoid is 35 cm^2. One of the angles of the trapezoid measures 45 degrees. Find the perimeter of the trapezoid and express it as a single fraction.

   Let a = length of shorter base, and a+2x=length of median. Then a+4x= length of longer base. Since one angle is 45 degrees, the perpendicular lines from the edges of the shorter base to the longer base separate the trapezoid into a rectangle and 2 isosceles triangles. Each triangle has base 2x, so the height of the triangle and therefore the height of the trapezoid is 2x. Area = $\frac{1}{2}(a+(a+4x))2x$ = $\frac{1}{2}(2a+4x))2x = \frac{1}{2} * 2 * (a+2x)(2x) = \frac{1}{2} * 2 * 10 * 2x$ = 35, so if 20x=35, then x=1.75 cm and 2x=3.5 cm. The longer base is 2*(3.5) longer than the shorter base, and since the median is 10 cm, they must add to 20, so the shorter base is 6.5 cm and the longer base is 13.5 cm. The two sides of the trapezoid that are not bases have length $\sqrt{(3.5)^2 + (3.5)^2} = \sqrt{24.5} = \sqrt{\frac{49}{2}} = \frac{7\sqrt{2}}{2}$. Twice that amount is $7\sqrt{2}$, so the perimeter is 20 + $7\sqrt{2}$

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Match 1 Round 4
Algebra 2:
Simultaneous Equations

1.) _____a= 32 b= 29__

2.) _____k=-6_ _____

3.) (5, -1) or (6,-1.5)___

1.) Solve for a and b:

$10a - 11b = 1$

$9a - 10b = -2$

$(10a - 11b = 1)9$

$(9a - 10b = -2)(-10)$

$90a - 99b = 9$

$-90a + 100b = 20$

Add to get b=29.

Then 9a-290=-2, so 9a=288, and a=32

2.) What must be the value of k if the system

$kx + 4y = -12$

$9x + ky = 18$

has infinitely many solutions?

The determinant of the coefficient matrix is $k^2 - 36$, and that would have to be zero, so k= or k=-6.

k=6 yields no solution, but k=-6 gives infinitely many solutions.

3. Solve for the two possible ordered pairs (x,y):

$$x - \frac{1}{y+2} = 4$$

$$x(y+4) = 15$$

$$x - \frac{1}{y+2} = 4$$

$$x(y+4) = 15$$

$$x = \frac{15}{y+4}, so... \frac{15}{y+4} - \frac{1}{y+2} = 4$$

$$15(y+2) - y + 4 = 4(y+4)(y+2)$$

$$14y + 26 = 4y^2 + 24y + 32$$

$$4y^2 + 10y + 6 = 0$$

$$2y^2 + 5y + 3 = 0$$

$$(y+1)(2y+3) = 0$$

$$y = -1..or..y = -1.5$$

$$x = \frac{15}{y+4}, so..if..y = -1, x = 5$$

$$if..y = -1.5, x = 6$$

FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 1 Round 5
Trig: Right Triangles

1.) ______ $\frac{9\sqrt{106}}{106}$ _____

2.) ____12_______ feet

3.) __________15 _______meters

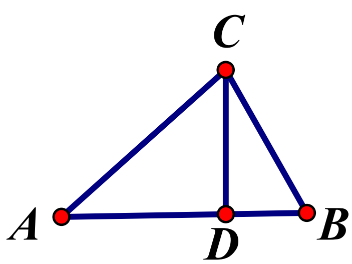
1.), NEW: In right triangle XYZ, the right angle is at Y. XY=5 and YZ=9. What is the sine of angle X? Express your answer in simplest radical form.

$$YZ = \sqrt{5^2 + 9^2} = \sqrt{106}$$

$$\frac{YZ}{XZ} = \frac{9}{\sqrt{106}} = \frac{9\sqrt{106}}{106}$$

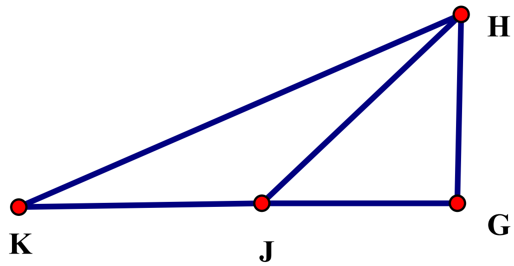
$$YZ = \sqrt{5^2 + 9^2} = \sqrt{106}.$$

2.) In right triangle ABC, the right angle is at C. An altitude from C intersects what is the length of $\overline{CD}$ in feet?



Let AD=4x and AC=5x. Then CD=3x. Since, then BC would be $\frac{3x}{\frac{4}{5}} = \frac{15}{4}x$, and

BD= $\frac{3x}{\frac{3}{4}} = \frac{9x}{4}$ = Then $75x^2 = 1200, x^2 = 16, x = 4$, so the triangle is a 15-20-25 triangle, AD=16, BD=9, and CD is the geometric mean between 16 and 9, so CD = 12.



3.) Points G, J, and K are at the same horizontal level. An observer at point J looks at the top of a vertical tower $\overline{GH}$ and finds that $\tan(\angle GJH) = 0.3$ The observer moves back 50 meters to point K and finds that $\tan(\angle GKH) = 0.15$. How tall is the tower?

$0.3 = \frac{GH}{GJ} and\_0.15 = \frac{GH}{GJ+50}$, so 0.3(GJ) = 0.15(GJ+50), so 0.15(GJ) =7.5, and GJ = 50 meters. GH = 0.3(50) = 15 meters

FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 1 Round 6
Coordinate Geometry

1)_______ $y = \frac{-2}{7}x - \frac{29}{7}$ _________

2.) __________ $\sqrt{109}$ _______________

3.) _________-10, -2___________

1.) Find the equation of the line that is the perpendicular bisector to the segment connecting (-2,4) and (-6,-10). Express your answer in the form y=mx+b

Slope of original line is $\frac{-10-4}{-6-(-2)} = \frac{-14}{-4} = \frac{7}{2}$.

Slope of line perpendicular is $\frac{-2}{7}$

Midpoint of original segment is (-4,-3).

y+3= $\frac{-2}{7}$ (x+4)

$$y+3 = \frac{-2}{7}(x+4)$$

$$y+3 = \frac{-2}{7}x - \frac{8}{7}$$

$$y = \frac{-2}{7}x - \frac{29}{7}$$

2.) Point A is in quadrant III, point B is in quadrant I, and point C is in quadrant II. $\overline{AB}$ has slope $\frac{3}{2}$ and has length $3\sqrt{13}$. $\overline{AC}$ has slope -3 and has length $4\sqrt{10}$. Find the length of $\overline{BC}$.

Say A is (-1,-1), to get a starting place and put B and C in the correct quadrants. Let the horizontal distance from A to B be 2x, then the vertical distance is 3x. So

$$\sqrt{(2x)^2+(3x)^2}=3\sqrt{13}$$
$$\sqrt{13x^2}=3\sqrt{13}$$
$$\sqrt{13}\sqrt{x^2}=3\sqrt{13}$$
$$x=3, 2x=6, 3x=9$$

Go over 6, up 9 from (-1,-1) to get to (5,8). Let y=horizontal displacement from C to A, then -3y = vertical displacement from C to A

$$\sqrt{(3y)^2+(y)^2}=4\sqrt{10}$$
$$\sqrt{10y^2}=4\sqrt{10}$$
$$\sqrt{10}\sqrt{y^2}=4\sqrt{10}$$
$$y=4, -3y=-12$$

So C is at (-5,11). Length of $\overline{BC}$ is $\sqrt{(8-11)^2+(5-(-5))^2}=\sqrt{109}$

3) Find all values of k such that the line segment connecting (-8,k) and (k,-4) has length $2\sqrt{10}$.

We need

$$\sqrt{(k-(-8))^2+(-4-k)^2}=2\sqrt{10}$$

$$(k+8)^2+(4+k)^2=40$$

$$k^2+16k+64+16+8k+k^2=40$$

$$2k^2+24k+80=40$$

$$k^2+12k+40=20$$

$$k^2+12k+20=0$$

$$(k+10)(k+2)=0$$

$$k=-10\_or\_k=-2$$

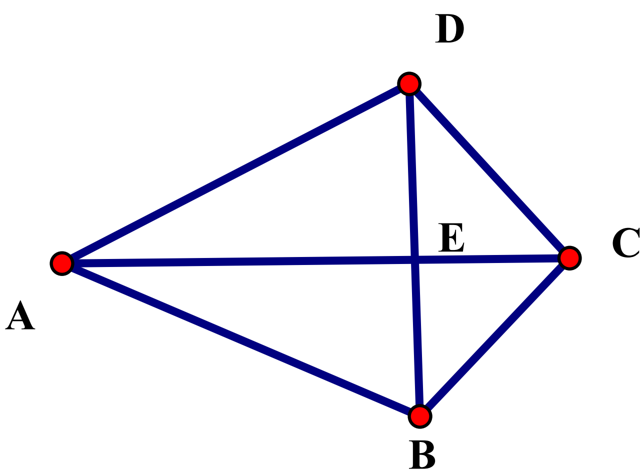
FAIRFIELD COUNTY MATH LEAGUE (FCML) 2016-17 Match 1 Team Round

1.) ____________ $\frac{\sqrt{2}-1}{2}$ _________ 4.) ___(_2__,__3____)_____

2.) _____________621____________ 5.) _____10, 0.4____________________

3.) ________ $75\sqrt{3}-100$_______ 6.) ____________45______________

1.)_ The ratio of the 4 interior angles of kite ABCD is 4:7:6:7, where the 4 corresponds with angle A and the 6 corresponds with angle C. The diagonals intersect at E. Find the positive difference between $\sin(\angle BCE)$ and $\sin(\angle BAE)$.



We must have 4x+7x+6x+7x=360, so 24x=360, and x=15. Angle A is 60 degrees and angle C is 90 degrees. The long diagonal bisects the angles, and the diagonals are perpendicular, so $\angle BCE$ is 45 degrees and $\angle BAE$ is 30 degrees. Their sines are $\frac{\sqrt{2}}{2}$ and $\frac{1}{2}$, so the difference is $\frac{\sqrt{2}-1}{2}$.

2.) x% of y is z. z% of x is 36y. y% of z is $\frac{x-60}{1000}$. Give the value of x+y+z, given that x, y, and z are all positive.

$$\frac{x}{100}y = z, \frac{z}{100}x = 36y, \frac{y}{100}z = \frac{x - 60}{1000}$$

$$y = \frac{100z}{x}, substitute$$

$$\frac{z}{100}x = 36(\frac{100z}{x}), so\_x^2 = 360000, x = 600.$$

$$Then\_\frac{\frac{100z}{600}}{100}z = \frac{600 - 60}{1000}$$

$$\frac{z^2}{600} = \frac{540}{1000}, \frac{z^2}{3} = \frac{540}{5} = 108, z^2 = 324, z = 18.$$

$$Then\_y = \frac{100 * 18}{600} = 3$$

$$x + y + z = 600 + 3 + 18 = 621$$

3.) Ten years ago, Joe walked back a certain horizontal distance from the base of a tree and found that the cosine of the angle of elevation to the top of the tree was 0.6. The tree grew over time, so today, Joe walked back the same horizontal distance and found the cosine of the angle of the elevation was 0.5. What was the percent increase in the height of the tree? Give your answer in simplest radical form.

Let x be the distance walked in each case. 10 years ago, the triangle formed had adjacent side x, hypotenuse $\frac{x}{0.6} = \frac{5}{3}x$, so the height was $\sqrt{(\frac{5}{3}x)^2 - x^2} = \frac{4}{3}x$. Now the triangle has adjacent side x and hypotenuse $\frac{x}{0.5} = 2x$, so the new height is $\sqrt{(2x)^2 - x^2} = \sqrt{3}x$. The ratio of the increase is

$$\frac{\sqrt{3} - \frac{4}{3}}{\frac{4}{3}} = \frac{\frac{3\sqrt{3} - 4}{3}}{\frac{4}{3}} = \frac{3\sqrt{3} - 4}{4}$$ so the percent increase is $\frac{3\sqrt{3} - 4}{4} = 75\sqrt{3} - 100$

4.) The circumcenter of a triangle is the point where its perpendicular bisectors intersect. Give the coordinates of the circumcenter of the triangle with vertices (5,7), (6,6), and (2,-2)

It is sufficient to find the intersection of two of the equations of the perpendicular bisectors. Since the segment between (5,7) and (6,6) has slope -1, its perpendicular bisector has slope 1, and passes through (5.5,6.5). The segment between (6,6) and (2,-2) has slope 2, so its perpendicular bisector has slope -0.5, and passes through (4,2). The equations of the two lines are y-6.5=1(x-5.5) and y-2=-0.5(x-4), so y=x+1 and y=-0.5x+4. If x+1=-0.5x+4, then 1.5x=3, x=2 and y=2+1=-3.

5.) Solve for c:

$$\frac{1}{c-4}+\frac{1}{c+14}=\frac{1}{c+2}+\frac{1}{c-2}$$

$$\frac{1}{c-4}+\frac{1}{c+14}=\frac{1}{c+2}+\frac{1}{c-2}$$
$$(c+14)(c+2)(c-2)+(c-4)(c+2)(c-2)=$$
$$(c-4)(c+14)(c-2)+(c-4)(c+14)(c+2);$$
$$(c+14)(c^2-4)+(c-4)(c^2-4)=(c+14)(c^2-6c+8)+(c+14)(c^2-2c-8)$$
$$(c^2-4)((c+14)+(c-4))=(c+14)((c^2-6c+8)+(c^2-2c-8))$$
$$(c^2-4)(2c+10)=(c+14)(2c^2-8c)$$
$$2c^3+10c^2-8c-40=2c^3+20c^2-112c$$
$$-10c^2+104c-40=0$$
$$10c^2-104c+40=0$$
$$5c^2-52c+20=0$$
$$(5c-2)(c-10)=0$$
$$c=10, c=0.4$$

6. The lines x+3y=5 and 3x-y=10 contain two diagonals of a rhombus. Two of the vertices of the rhombus are (-1,2) and (2,-4). Find the area of the rhombus.

The lines x+3y=5 and 3x-y=10 intersect at (3.5,0.5). The area is half the product of the diagonals and the diagonals bisect each other, so the length of one diagonal is two times

$$\sqrt{(-1-3.5)^2+(2-0.5)^2}=\sqrt{20.25+2.25}=$$

$$\sqrt{22.5}=\frac{15}{\sqrt{10}}$$

so the length of one diagonal is $\frac{30}{\sqrt{10}}$.

The length of the other diagonal is 2 times

$$\sqrt{(2-3.5)^2+(-4-0.5)^2}=\sqrt{20.25+2.25}=$$

$$\sqrt{22.5}=\frac{15}{\sqrt{10}}$$

, so the rhombus is actually a square

with both diagonals of length $\frac{30}{\sqrt{10}}$. $\frac{1}{2}*\frac{30}{\sqrt{10}}*\frac{30}{\sqrt{10}}=\frac{900}{20}=45$