FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 2 Round 1
Arithmetic: Factors
And Multiples

1)______3,4,5,7___________

2.) _________30___________

3.) ___________126_________

1) Given N is an integer $1 \le N \le 10$, for which integers N is the expression 2^N-7 a perfect square?

$2^3 - 7 = 1 = 1^2$
$2^4 - 7 = 9 = 3^2$
$2^5 - 7 = 25 = 5^2$
$2^7 - 7 = 121 = 11^2$

2.) How many whole numbers between 1 and 100 can be factored as pq where p and q are both prime, and $p \neq q$?
2*3, 2*5, 2*7, 2*11, 2*13, 2*17, 2*19, 2*23, 2*29, 2*31, 2*37, 2*41, 2*43, 2*47, so 14 with a factor of 2.
3*5, 3*7, 3*11, 3*13, 3*17, 3*19, 3*23, 3*29, 3*31, so 9 additional with a factor of 3.
5*7, 5*11, 5*13, 5*17, 5*19, so 5 additional with a factor of 5
7*11, 7*13, so 2 additional with a factor of 7. 14+9+5+2 = 30

3) The greatest common factor of N and 540 is 18 . The least common multiple of N and 120 is 2520. Find all possible values of N.

$18 = 2*3^2$, and $540 = 2^2*3^3*5$, so N must have $2*3^2$, but not 3^3, and not 5.
$120 = 2^3*3*5$, and $2520 = 2^3*3^2*5*7$. The 7 must come from N.
N must have only one factor of 2 since there is only one factor of 2 in the GCF. N must have contain the second factor of 3, since there are two factors of 3 in the GCF. So $N = 2*3^2*7=126$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 2 Round 2
Algebra: Polynomials
And Factoring

1.) ___$-8x^2+2y^2+18z^2+12yz$____

2.) _____$2(3a-5b)(5a+b)$_

3.) ________$4c^2-d^2$______

1). Express as a polynomial of 4 terms: $(2x+y+3z)(-4x+2y+6z)$

$(2x+y+3z)(-4x+2y+6z)=$

$-8x^2+4xy+12xz-4xy+2y^2+6yz-12xz+6yz+18z^2$

$-8x^2+2y^2+18z^2+12yz$

2) Factor completely: $30a^2-44ab+10b^2$

$30a^2-44ab+10b^2$

$=2(15a^2-22ab-5b^2)$

$=2(3a-5b)(5a+b)$

3). The expression $(4c^2+d^2)^2-(4cd)^2$ is the square of some binomial involving c and d. What is that binomial?

$(4c^2+d^2)^2-(4cd)^2=$

$=16c^4+8c^2d^2+d^4-16c^2d^2$

$=16c^4-8c^2d^2+d^4$

$=(4c^2-d^2)^2$

so the answer is $4c^2-d^2$. (d^2-4c^2) is also acceptable.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 2 Round 3
Geometry:
Area and Perimeter

1.)_______60____________cm²

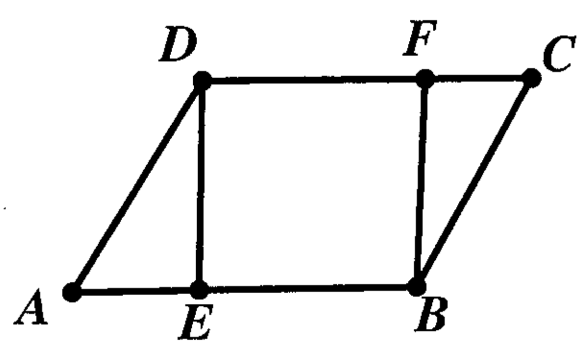
Drawings are not necessarily to scale

2.) ______$20+4\sqrt{10}$________cm_

3.) _______$2\sqrt{71}$____________cm_

1.) The lengths of the sides of a right triangle are all whole numbers. The perimeter is 40 cm. Find the area of the triangle.
The only possibilities are Pythagorean triples. The only combination that adds up to 40 cm is an 8-15-17 triangle. The two legs are 8 cm and 15 cm, so the area is $0.5(8)(15) = 60\ cm^2$

2.) Parallelogram ABCD is shown. $\overline{DE}$ and $\overline{FB}$ are drawn so that they are perpendicular to bases $\overline{AB}$ and $\overline{CD}$. The area of rectangle DEBF is $2\sqrt{71}$ of the area of parallelogram ABCD. If BF = 3x, DF = 5x-2, and AB = 2x+6, give the perimeter of ABCD.



The area of ABCD is (2x+6)(3x). The area of DEBF is (5x-2)(3x). So (5x-2)(3x) = (2x+6)(3x), so (5x-2)= $\frac{4}{5}$(2x+6), so 25x-10 = 8x + 24, so 17x=34 and x=2. Then AB and CD are both 2*2+6 = 10. The length of DF and EB are both 5*2-2= 8, so AE and FC are both 2. DE and FB are both 3*2=6, so AD and CB are both $\sqrt{2^2+6^2}=\sqrt{40}=2\sqrt{10}$. The total perimeter is $10+10+2\sqrt{10}+2\sqrt{10}$ = $20+4\sqrt{10}$.

3) A right triangle has area 30 cm^2. The length of its hypotenuse is $2\sqrt{41}$ cm. Find the sum of the two legs of the triangle.

Let the legs be x and y. Then $\frac{1}{2}xy=30, xy=60$, and

$x^2+y^2=(2\sqrt{41})^2=164$. We know

$(x+y)^2=x^2+2xy+y^2=(x^2+y^2)+2(xy)$ = 164 + 2*60 = 284, so

$x+y=\sqrt{284}=2\sqrt{71}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 2 Round 4
Algebra 2: Inequalities
And Absolute value

1) $9<x<11$

Remember to use AND or OR or the shorthand notation for a conjunction if you answer with <. >, ≤. or ≥ . You may use union and intersection symbols if you answer using interval notation.

2.) $x\geq3$ or $x\leq-5$

3.) $\frac{2}{3}, \frac{4}{3}, 2, 4$

1) Solve for x: $6 < 5x-7(x-4)<10$
   $6< 5x -7x+28< 10$
   $6< -2x + 28 < 10$
   $-22<-2x<-18$
   $18<2x<22$
   $9<x<11$

2.) Find all values of x such that $|2x+1|-|x|\geq 4$

If $x<-0.5$, we have $-2x-1-(-x)\geq4$, so $-x-1\geq4$, so $-x\geq5$, so $x\leq-5$

If $-0.5<x<0$, we have $2x+1-(-x)\geq4$, so $3x+1\geq4$, and $3x\geq3$, $x\geq1$, not in that domain.

If $x>0$, we have $(2x+1)-x\geq4$, so $x+1\geq4$, so $x\geq3$.

3.)Find all values of x such that $\left|x-|2x-3|\right|=1$.

If $x<1$, then $\left|x-|2x-3|\right|=1$, then since x is also less than 1.5, $|2x-3| = (3-2x)$, so $|x-(3-2x)|=1$, or $|3x-3|=1$, and since x is less than 1, we have $3-3x=1$, so $3x=2$, so $x=\frac{2}{3}$

If $1<x<1.5$, $|x-(3-2x)|=1$, $|3x-3|=1$, so $3x = 4$, $x=\frac{4}{3}$.

If $x>1.5$, $|x-(2x-3)|=1$, so $x-2x+3=1$, or $x-(2x-3)=-1$.

If $x-2x+3=1$, then $-x=-2$ and $x=2$. If $x-(2x-3)=-1$, then $-x+3=-1$, and $-x=-4$, and $x=4$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 2 Round 5
Trigonometry:
Laws of Sine and Cosine

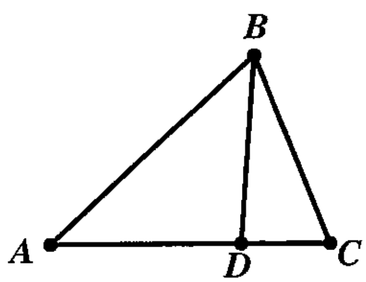
1.) $\frac{-1}{5}$

2.) $\sqrt{2}$

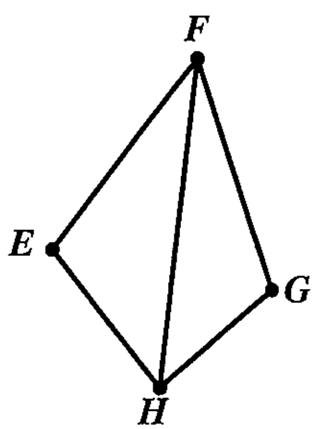
3.) 0.75

1.) The sides of a triangle measure 5 cm, 6 cm and 7 cm. Find the cosine of the largest angle of the triangle.

2.) In the figure below, $\angle ABD$ measures 45 degrees, $\angle DBC$ measures 30 degrees, and AD = 2(DC). Find the ratio of $\frac{AB}{BC}$ as a radical in simplest radical form.



3.) In the figure below, $\overline{FH}$ bisects $\angle EFG$. $\angle FEH$ measures 150 degrees. EH = 1.5(GH).
Find sin($\angle FGH$)



FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 2 Round 6
Equations of Lines

1.) ______$\frac{6}{7}$__________

2.) ________$y=x-4$_____

3) _______$y=\frac{3}{4}x+\frac{25}{4}$___

1.) What is the slope of the line given by the equation

$2x + 3(x+y)-4(2x- \frac{1}{8}y)=6$?.

$2x + 3x + 3y - 8x + \frac{1}{2}y = 6$

$-3x + \frac{7}{2}y = 6$, so $\frac{7}{2}y = 3x+ 6$, so y = $y=\frac{6}{7}x+\frac{12}{7}$, so the slope is $\frac{6}{7}$

2.) Point A has coordinates (6,3), point B has coordinates (3,-1), and point C has coordinates (7,2). Find the equation of the line that bisects $\angle ABC$. Express your answer as y=mx+b.

$\overline{AB}$ and $\overline{BC}$ form two sides of a rhombus of side 5. Create the fourth vertex of the rhombus by going right 3 and up 4 from (7,2) to get to (10,6). The diagonals of a rhombus bisect the angles, so find the equation of the diagonal containing (3,-1) and (10,6). Slope is $\frac{-1-6}{3-10}=1$. y-6=1(x-10), so y=x-4

3) A line is tangent to the circle $x^2 + y^2 = 25$ at the point (4,3). Find the equation of a line that is also tangent to $x^2 + y^2 = 25$ and intersects the original line at (1,7). Express your answer as y=mx+b.

If the line intersects the circle at (a,b), the line tangent to it will be perpendicular to the radius, which has slope $\frac{b}{a}$, so the line will have slope $-\frac{a}{b}$. We have

$y-7=-\frac{a}{b}(x-1)$ and $a^2+b^2=25$, so $b(b-7) = -a(a-1)$, $b^2-7b = -a^2+a$, and $a^2+b^2=25$,

so 25-7b = a. Then $25-7b=\pm\sqrt{25-b^2}$, square both sides to get

$$625-350b+49b^2=25-b^2$$

$$50b^2-350b+600=0$$

$$b^2-7b+12=0,$$

$$so\_b=3\_or\_b=4.$$

We already know the b=3 solution, so if b =4, then 25-7(4)=a, so a=-3.

The line passing through (-3,4) and (1,7) has slope $\frac{3}{4}$, so $y-7=\frac{3}{4}(x-1)$,

$$y-7=\frac{3}{4}x-\frac{3}{4}$$

$$y=\frac{3}{4}x+\frac{25}{4}$$

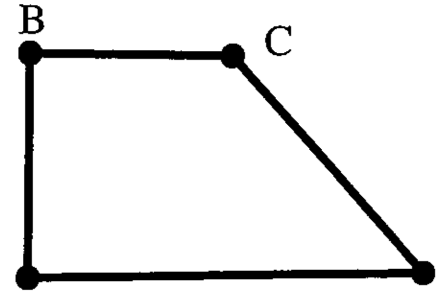
FAIRFIELD COUNTY MATH LEAGUE 2015-16 Match 2 Team Round

1.) _$\sqrt{2501}$, __$\sqrt{629}$, __$5\sqrt{5}$_ 4.) __-2.25<x<-2 or 0.75<x<1 ______

2.) _$\frac{4+3\sqrt{3}}{10}$, $\frac{-4+3\sqrt{3}}{10}$_ 5.) ____________$\frac{1}{6},\frac{5}{12},\frac{1}{2},\frac{3}{4}$__________

3.) _______$y=\frac{-1}{18}x+\frac{325}{9}$_____ 6.) _______________-20_______________

1.) . Trapezoid ABCD has bases $\overline{AD}$ and $\overline{BC}$ and has right angles at A and B. AD = 2(BC) and the area of the trapezoid is 75 cm^2. If the bases and height of the trapezoid are all whole numbers, find all possible values of CD. (Diagram not necessarily drawn to scale).



We need 75 = 0.5(AD+BC)(AB), so 150 =((2(BC)+(BC))(AB), so 150 = 3(BC)(AB) 50 = (BC)(AB). So we need factors of 50: BC and AB could be 1 and 50, 2 and 25, 5 and 10. If BC=1, AD=2, AB=50, which would be the altitude drawn from C to AD. Then CD would be $\sqrt{2501}$. If BC=2, then AD=4, and AB=25, so CD = $\sqrt{629}$. If BC=5, then AD=10 and AB=10, so CD = $\sqrt{125}=5\sqrt{5}$ $\sqrt{125}=5\sqrt{5}$.

2.)_In ΔABC, $\angle A$ = 30 degrees, AC=6, and BC=5. Given sin(X+Y)=sin(X)cos(Y)+cos(X)sin(Y), find the two possible values of $\sin(\angle C)$

$\frac{\sin(30)}{5}=\frac{\sin(\angle B)}{6}, so$

$\sin(\angle B)=\frac{3}{5}.$

Angle C is supplementary to the sum of A and B, so it has the same sine as sin(A+B). The sine of the sum of the angles is found by sin(A+B) = sinAcosB+cosAsinB =

$\frac{1}{2}*\frac{4}{5}+\frac{\sqrt{3}}{2}*\frac{3}{5}$ or

$\frac{1}{2}*\frac{-4}{5}+\frac{\sqrt{3}}{2}*\frac{3}{5}$. So $\sin(\angle C)=\frac{4+3\sqrt{3}}{10}$ or $\frac{-4+3\sqrt{3}}{10}$.

3.) A line segment has one endpoint at the origin and the other endpoint in the first quadrant. The slope of the line segment is the greatest common factor of 198 and 630. The length of the line segment is $\sqrt{N}$, where N is the least common multiple of 20, 52, and 65. Give the equation of the line which is perpendicular to the segment and passes through the endpoint that is in the first quadrant. Express your answer in the form y=mx+b.

198 = 11*2*3*3 and and 630=7*3*3*2*5, so the GCF=3*3*2 = 18, so the slope is 18. The LCM of 50, 52, and 65 is found by: 50=2*5*5, 52=2*2*13, and 65=5*13, so the LCM is 2*2*5*5*13 = 1300, and $\sqrt{1300}=2\sqrt{325}$. Since $325=18^2+1^2$, going along a slope of 18 brings you to (2,36). The slope of the desired line must be $\frac{-1}{18}$, so the equation is

$$y-36=\frac{-1}{18}(x-2)$$

$$y-36=\frac{-1}{18}x+\frac{1}{9}$$

$$y=\frac{-1}{18}x+\frac{325}{9}$$

4.) Find all real values of x such that $4<4x^2+5x-2<7$.

We have $4<4x^2+5x-2$ and $4x^2+5x-2<7$. Solving the first inequality, set one side equal to zero to get $4x^2+5x-6>0$, so (4x-3)(x+2)>0, so $x>\frac{3}{4}\_or\_x<-2$. For the second inequality, $4x^2+5x-9<0$, so (4x+9)(x-1)<0, so $x>\frac{-9}{4}\_and\_x<1$. Putting these together, we get -2.25<x<-2 or 0.75<x<1.

5.) Find all real values of k such that the line y= |4k - 1| x + 4 is perpendicular to the line y= -|4 - 6k| x + 5.

We need $|4k-1| = \frac{-1}{-|4-6k|}$, $|4k-1| = \frac{1}{|4-6k|}$. If $k > \frac{2}{3}$, we get (6k-4)(4k-1)=1, so $24k^2 - 22k + 4 = 1$,

so $24k^2 - 22k + 3 = 0$. This factors to (4k-3)(6k-1)=0, so $k = \frac{3}{4}$ or $k = \frac{1}{6}$, but only $\frac{3}{4}$ is greater than $\frac{2}{3}$.

If $\frac{1}{4} < x < \frac{2}{3}$, we have (4k-1)(4-6k)=1, so $-24k^2+22k-4=1$, so $-24k^2+22k-5=0$, and

$24k^2-22k+5=0$. This factors to (2k-1)(12k-5), so $k = \frac{1}{2}\_or\_k = \frac{5}{12}$, both of which are

in the given range. If $k < \frac{1}{4}$, we have (1-4k)(4-6k)=1, so $24k^2-22k+4=1$, so

$24k^2-22k+3=0$ as before. Now the solution of $k = \frac{1}{6}$ works. All solutions are $\frac{1}{6}, \frac{5}{12}, \frac{1}{2}, \frac{3}{4}$

6) Segments x, y, and z form a right triangle with the right angle opposite side z. If the lengths x and y stayed the same but the length of the longest side were increased by 2, the cosine of the angle opposite side z in the new triangle formed would be -0.3. Find the value of the expression 20z – 3xy.

We know $z^2 = x^2 + y^2$.
We also know $(z+2)^2 = x^2 + y^2 - 2xy(-0.3)$, or $z^2 + 4z + 4 = x^2 + y^2 + 0.6xy$.
Since $z^2 = x^2 + y^2$, subtract this from both sides to get
4z+4 = 0.6xy, so 4z-0.6xy=-4. Multiply each side by 5 to get 20z-3xy=-20. (I checked this starting with an 8-15-17 right triangle, so it's not an impossible situation.)