FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 1 Round 1
Arithmetic: Percents

1)__________5________

2.) ______16__________

3.) ________7.5___________

1) X% of 60 is equal to 60% of (4X-15). Find X.

$$\frac{x}{100}*60=\frac{60}{100}(4x-15)$$
$$x=4x-15$$
$$-3x=-15$$
$$x=5$$

2) How many integers N between 1 and 1000 satisfy the condition that 20% of N, 25% of N, and $33\frac{1}{3}$% of N are all integers?

These percents are $\frac{1}{5},\frac{1}{4},\frac{1}{3}$, so N must be divisible by 60.
60,120,180,240,300,360,420,480,540,600,660,720,780,840,900, and 960 all work.

3) The price of an item increased by x% and then decreased by y%. The final price was 90% of the original price. If the item had been increased by y% and then decreased by x%, the final price would have been 105% of the original price. Find y-x.

Let a=x% and b=y%
(1+a)(1-b)=0.9 and (1+b)(1-a) = 1.05.
1+a-b-ab=0.9 and 1-a+b-ab=1.05
Subtract the second equation from the first to get 2a-2b=.-15, so b-a=0.075.
so y-x=7.5.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 1 Round 2
Algebra I: Equations

1.) $-\frac{5}{4}$

2.) $\frac{50}{17}$

3.) $10, -\frac{18}{13}$

1.) Solve for a:

$$5-0.2a=6-(a+2)$$

$5-0.2a=6-a-2$

$5+0.8a=4$

$0.8a=-1$

$a=\frac{-1}{0.8}=-\frac{5}{4}$

2) Solve for b:

$\frac{1}{b-5}+\frac{5}{b}=\frac{6}{b+2}$

$Mult\_each\_side\_by\_(b-5)b(b+2)$

$b(b+2)+5(b-5)(b+2)=6(b-5)b$

$b^2+2b+5b^2-15b-50=6b^2-30b$

$2b-15b-50=-30b$

$-13b-50=-30b$

$-50=-17b$

$b=\frac{50}{17}$

3) Solve for c:

$$c(c-6)(c-4)=(c+2)(c-9)(c+10)$$

$c(c-6)(c-4)=(c+2)(c-9)(c+10)$

$c(c^2-10c+24)=(c^2-7c-18)(c+10)$

$c^3-10c^2+24c=c^3+3c^2-88c-180$

$13c^2-112c-180=0$

$(c-10)(13c+18)=0$

$c=10\_or\_c=-\frac{18}{13}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 1 Round 3
Geometry: Triangles
And Quadrilaterals

1.)__ 36 ______cm

2.)_____ 48 ______cm^2

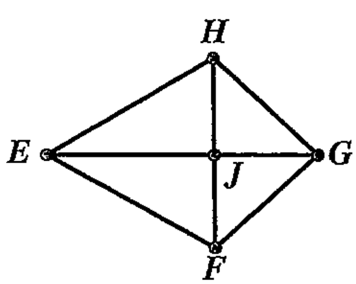
3.) _Perimeter: ____$10\sqrt{5}+10\sqrt{2}$ _ cm

Area: ________75__________cm^2

1) ΔABC is equilateral. The altitude drawn from A to BC has length $6\sqrt{3}$ cm. Find the perimeter of ΔABC.
   The altitude cuts the triangle into two 30-60-90 Δ's. The altitude $6\sqrt{3}$ is opposite one of the 60 degree angles, so the side opposite the 30 degree angle is 6, and the hypotenuse is 12. The perimeter is $3*12 = 36$ cm.

2.) An isosceles trapezoid has one angle of 45 degrees. The difference in length between the two bases is 12 cm, and one base is 7 times longer than the other. Find the area of the trapezoid.

If the two bases are a and b, a=7b and a-b=12, so 7b-b=12, and 6b=12, so b=2 cm The longer base is 14 cm. That difference of 12 cm must be split equally on either side of the smaller base, since the trapezoid is isosceles, so the trapezoid consists of a rectangle with one side equal to 2, and two isosceles right triangles with one leg equal to 6. Since the triangles are isosceles, the height of the trapezoid is also 6, and the area is (1/2)(2+14)*6=48 cm^2



3. In kite EFGH, FJ=JG and EJ = 2(FJ). If EH=$5\sqrt{5}$ cm, give the perimeter and the area of EFGH. $2\sqrt{13}$ By definition of a kite, EF=EH=$5\sqrt{5}$. The diagonals are perpendicular to each other so we have 4 right triangles. Let HJ=JF=GJ=x. Then EJ=2x. If $(5\sqrt{5})^2 = x^2 + (2x)^2$, then $125=5x^2$ and x=5. Then GH = FG = $\sqrt{5^2+5^2} = \sqrt{50} = 5\sqrt{2}$, so the total perimeter is $10\sqrt{5}+10\sqrt{2}$ cm. The area is half the product of the diagonals, so 0.5(10)(15)=75 cm^2

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 1 Round 4
Algebra 2:
Simultaneous Equations

1.) ____a= -0.2 b= -2__

2.) _____k=-3______

3.) _$x=\frac{1}{3}$, $y=\frac{1}{2}$, z=4__

1.) Solve for a and b:

$5a-3b=5$

$b=15a+1$

$5a-3(15a+1)=5$

$5a-45a-3=5$

$-40a=8\_so\_a=-0.2$

$b=15(-0.2)+1=-2$

2.) Find all values of k such that the system

2kx = 5– 0.5y
ky= 30 – 36x
has no solution (x,y).

2kx+0.5y=5
36x+ky=30 Multiply the first equation by 2k and subtract the second equation

$4k^2x + ky = 10k$

36x+ky=30 , so $4k^2x-36x = 10k-30$, so $x=\dfrac{10k-30}{4k^2-36}$. So there is a unique solution to the original system if $k \neq 3$ and $k \neq -3$.

but if k=3, we have 6x + 0.5y = 5
36x+6y=30 and there are infinitely many solutions. But k=-3 gives the system -6x+0.5y=5
-3y=30-36x, and this has no solution.

Or find the determinant and set it equal to zero, and do the same analysis.

3) Solve for (x,y,z):

$$\frac{2}{x}+\frac{3}{y}+z=16$$

$$\frac{3}{x}-\frac{1}{y}+2z=15$$

$$\frac{1}{x}+\frac{2}{y}-3z=-5$$

Let $a=\frac{1}{x}, b=\frac{1}{y}$. Then we have

$2a + 3b + z = 16$
$3a - b + 2z = 15$
$a + 2b - 3z = -5$.
Lots of choices here, eliminate z by multiplying the first equation by -2 and then adding to the second equation, then multiply the first equation by 3 and add to the third equation

$-4a - 6b - 2z = -32$
$3a - b + 2z = 15$
so $-a - 7b = -17$

$6a + 9b + 3z = 48$
$a + 2b - 3z = -5$
so $7a + 11b = 43$

$-a - 7b = -17$
$7a + 11b = 43$

$a = 17-7b$, so $7(17-7b)+11b=43$, $119 - 49b + 11b = 43$,

$-38b = -76$, so $b=2$, so $y=\frac{1}{2}$. $a = 17-7(2)$, so $a=3$, $x=x=\frac{1}{3}$. $2a+3b+z=16$, so

$6+6+z=16$, so $z=4$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 1 Round 5
Trig: Right Triangles

1.) $\frac{\sqrt{33}}{7}$

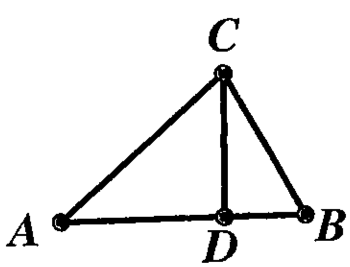
2.) $\frac{1}{3}$

3.) $250\sqrt{3}$

1.), In right triangle XYZ, the right angle is at Y. XY=4 and XZ=7. What is the sine of angle X?

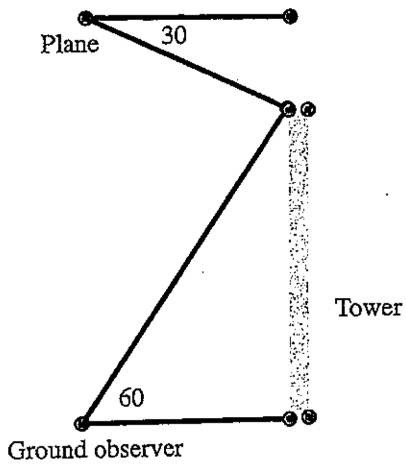
$YZ = \sqrt{7^2 - 4^2} = \sqrt{33}$. Sin X = $\frac{YZ}{XZ} = \frac{\sqrt{33}}{7}$

2.) In right triangle ABC, the right angle is at C. An altitude from C intersects $\overline{AB}$ at D such that AD = 2(DB). Find the cosine of angle B.



Let CD=y, BD=x, and AD=2x and BC=z The square of the altitude is the product of the two parts of the base, so $y^2 = 2x^2$. By Pythagorean theorem $(x)^2 + 2x^2 = (z)^2$, so $x^2+2x^2 = z^2$. Then $3x^2 = z^2$ so $x= \frac{z}{\sqrt{3}}$. Then the cosine of B is $\frac{\frac{z}{\sqrt{3}}}{z} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

3.) As viewed from an incoming plane, the angle of depression to the top of a tower is 30 degrees. As viewed by a ground observer, the angle of elevation to the top of the tower is 60 degrees. If the plane is 1000 feet directly above the observer, what is the horizontal distance from the plane to the tower in feet?



Let y=height of tower and x=horizontal distance from either the plane or the observer to

$$\tan(30) = \frac{\sqrt{3}}{3} = \frac{1000 - y}{z}$$

the tower. Then $\tan(60) = \sqrt{3} = \frac{y}{z}, y = z\sqrt{3}$

$$Solve: \frac{\sqrt{3}}{3} = \frac{1000 - z\sqrt{3}}{z}$$

$z\sqrt{3} = 3000 - 3z\sqrt{3}$

$4z\sqrt{3} = 3000$

$z = \frac{3000}{4\sqrt{3}} = \frac{750}{\sqrt{3}} = \frac{750\sqrt{3}}{3} = 250\sqrt{3}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 1 Round 6
Coordinate Geometry

1.) (______4_____, _____4_______)

2.) ______10, 3______________

3.) ________ $\frac{14\sqrt{5}}{5}$ _______

1.) A parallelogram has three of its vertices at (-3,2), (1,10), and (-6,8). If the fourth vertex is in the first quadrant, find its coordinates.

Move from (1,10) down 6 and right 3 to correspond with the movement from (-6,8) to (-3,2). You end up at (4,4). The slope from (-3,2) to (4,4) is the same as the slope from (-6,8) to (1,10).

2.) Find all values of k such that the segment connecting (k,4) and (9,k) has length $\sqrt{37}$. We need:

$$\sqrt{(9-k)^2+(k-4)^2}=\sqrt{37}$$

$$(9-k)^2+(k-4)^2=37$$

$$81-18k+k^2+k^2-8k+16=37$$

$$2k^2-26k+60=0$$

$$k^2-13k+30=0$$

$$(k-10)(k-3)=0$$

$$k=10, k=3$$

3.) The line x+2y=15 is the perpendicular bisector of line segment $\overline{AB}$ where A has coordinates (2,3). Find the length of $\overline{AB}$.

The line x+2y=15 has slope $\frac{-1}{2}$, so the slope of the segment from (2,3) has slope 2.

x+2y=15 must meet y-3=2(x-2) at the midpoint of $\overline{AB}$. Solving the system gives x=-2y+15, so y-3=2(-2y+15-2), y-3=-4y+26, 5y=29, y=5.8, then x=3.4. Since this is the midpoint of the segment with one endpoint (2,3), the other endpoint must be (4.8, 8.6). The length of the segment is $\sqrt{(4.8-2)^2+(8.6-3)^2}$ =

$$\sqrt{(4.8-2)^2+(8.6-3)^2} = \sqrt{7.84+31.36} = \sqrt{39.2} = \sqrt{\frac{196}{5}} = \frac{14}{\sqrt{5}} = \frac{14\sqrt{5}}{5}$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-16 Match 1 Team Round

1.) $\frac{120}{169}$ 4.) $\frac{2}{3}$

2.) \$14500 5.) $\frac{-421}{1326}$

3.) 15 6.) $2\sqrt{5}$

1.) The diagonals of a rhombus are 10 cm and 24 cm long. Give the sine of any of the angles of the rhombus.

We know the diagonals bisect each other and are perpendicular, so they form two right triangles that have legs 5 cm and 12 cm each. The area of each triangle is 30 cm^2, so the total area of the rhombus is 120 cm^2 (or use area = (1/2)(10)(24) = 120 cm^2). The length of each side is 13 cm, since each side is the hypotenuse of the right triangle. Area = base x height. Since the base = 13 cm, the height h must be such that 13*h=120, so h=120/13 cm. The sine of any of the acute angles is the same as the sine of any of the obtuse angles by the symmetry of the sine function. The sine of an acute angle is the height of the rhombus divided by one of the sides, so the sine is $\frac{\frac{120}{13}}{13} = \frac{120}{169}$

2.) Carly had \$24,500 to invest. She divided the money into three different accounts. At the end of the year, she had made \$1,300 in interest. The annual yield on each of the three accounts was 4%, 5.5%, and 6%. If the amount of money in the 4% account was four times the amount of money in the 5.5% account, how much money had she placed in the account earning 6% interest?

Let A = amount of money invested at 4% interest, B = amount of money invested at 5.5% interest, and C=amount of money invested at 6% interest. We have

A+B+C = 24500

0.04A + 0.055B + 0.06C =1300

A=4B, so

4B+B = 24500

0.04(4B)+0.055B + 0.06C = 1300

5B+C=24500
0.215B+0.06C=1300

-0.3B + -0.06C = -1470
0.215B + 0.06C = 1300

-0.085B = -170
B=2000, so A=8000, and C=14500.

3.) A surveyor sighted a tree such that the angle of elevation to the top of the tree was 16.7 degrees. If he moved back 100 feet, the angle of elevation to the top of the tree would be 5.71 degrees. If the tangent of 5.71 degrees is 0.1 and the tangent of 16.7 degrees is 0.3, what is the height of the tree in feet?

Let y=height of tree. x=original distance. Then tan 16.7=0.3=$\frac{y}{x}$, so y=0.3x. We also have tan(5.71)= $\frac{y}{x+100}$. Solve $0.1=\frac{0.3x}{x+100}$.

$$0.1=\frac{0.3x}{x+100}$$

$$0.1x+10=0.3x$$

$$10=0.2x$$

$$x=50$$

$$y=0.3(50)=15$$

4.) The perpendicular bisectors of a triangle intersect at the circumcenter A. The medians of a triangle intersect at the centroid B. Find the length of $\overline{AB}$ for the triangle whose vertices are (6, 4), (8,10), and (10,4).

Since the triangle is isosceles with line of symmetry x=8, both points will be on the line x=8. The slope between (6,4) and (8,10) is 3, and the midpoint of that line is (7,7), so the perpendicular bisectors lie on the line y-7=$\frac{-1}{3}$ (x-7), and if x=8, then y=$\frac{20}{3}$. The median drawn from, say, (10,4) to (7,7) has slope -1, so its equation is y-7=-1(x-7), so y=-x+14. If x=8, then y=6. The distance between 6 and $\frac{20}{3}$ is $\frac{2}{3}$.

5.) Solve for all possible values of x:

$$x-\frac{2}{3}(x-\frac{4}{5}(x-\frac{5}{6}))=\frac{1}{2}x-\frac{3}{4}(x-\frac{7}{8}(x-\frac{9}{10}))$$

$$x-\frac{2}{3}(x-\frac{4}{5}x+\frac{2}{3})=\frac{1}{2}x-\frac{3}{4}(x-\frac{7}{8}x+\frac{63}{80})$$

$$x-\frac{2}{3}(\frac{1}{5}x+\frac{2}{3})=\frac{1}{2}x-\frac{3}{4}(\frac{1}{8}x+\frac{63}{80})$$

$$x-\frac{2}{15}x+\frac{4}{9}=\frac{1}{2}x-\frac{3}{32}x+\frac{189}{320}$$

$$\frac{13}{15}x+\frac{4}{9}=\frac{13}{32}x+\frac{189}{320}$$

$$\frac{416}{480}x-\frac{1280}{2880}=\frac{195}{480}x-\frac{1701}{2880}$$

$$\frac{221}{480}x=\frac{-421}{2880}$$

$$x=\frac{-421}{2880}*\frac{480}{221}=\frac{-421}{6}*\frac{1}{221}=\frac{-421}{1326}$$

6.) A right triangle ΔABC has its right angle at C. If BC were 20% greater than it is and $\overline{AC}$ remained the same length, the tangent of angle A would be 0.2 larger than if BC were 20% less than it is and $\overline{AC}$ remained the same length . If AB=10 cm, find the length of BC.

$$\frac{1.2(BC)}{AC}-\frac{0.8(BC)}{AC}=0.2 \quad \frac{1.2(BC)}{AC}-\frac{0.8(BC)}{AC}=0.2$$

$$\frac{0.4(BC)}{AC}=0.2 \qquad \frac{0.4(BC)}{AC}=0.2$$

$$\frac{BC}{AC}=\tan(A)=0.5 \qquad \frac{BC}{AC}=\tan(A)=0.5$$

Then $\sin(A)=\frac{\sqrt{5}}{5}$ and $\cos(A)=\frac{2\sqrt{5}}{5}$, so $\sin(A)=\frac{BC}{AB}$, so $\frac{\sqrt{5}}{5}=\frac{BC}{10}$

$\frac{\sqrt{5}}{5}=\frac{BC}{10}$, so

BC = $2\sqrt{5}$ cm.