Teacher Packets

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 3 Round 1 Arithmetic: Scientific Notation and Bases

1.) ________ 1.503×10^{-10} ________

2.) ____________ 144 _________

3.) ________ 2,3,4,5 ____________

1.) In the famous equation $E=mc^2$, find E if $m=1.67 \times 10^{-27}$ and c is 3×10^8. Express your answer in scientific notation. (Assume these are exact numbers, so that significant figures are not an issue)

$E=(1.67 \times 10^{-27})(3 \times 10^8)^2 = (1.67 \times 10^{-27})(9 \times 10^{16}) = 15.03 \times 10^{-11} = 1.503 \times 10^{-10}$

2.) Multiply the number 121_7 by the number 121_4 and divide your answer by 121_3. Give your answer in base 8.

$121_7 = 49+14+1 = 64 \quad 121_4 = 16+8+1 = 25 \quad 121_3 = 9+6+1=16$

$\frac{64*25}{16} = 100$ in base 10, so $1*8^2 + 4*8 + 4*1 = 144_8$

3.) If d can be any of the base 6 digits 0,1,2,3,4, or 5, find all values of d such that

$\frac{5d3_6 - 3d5_6}{3d1d_6}$ is between 10^{-1} and 10^{-2}.

The d*6 part of the numerator will always cancel out, so the numerator will always be 5*36+3 minus 3*36+5 = 183-113=70.
If d=0, the denominator is 3*216+6, which is 648+6=654, and 70/654 is greater than 0.1
Whenever d increases by 1, the denominator increases by 37, so we have to consider 70/691 (d=1), 70/728 (d=2), 70/765 (d=3), 70/802 (d=4), and 70/839 (d=5). This quotient is less than 0.1 and never greater than 0.01 when d=2, 3, 4, or 5.

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Match 3 Round 2
Algebra: Word Problems

1.) 150

2.) 10:24

3.) $\frac{1200}{37}$

1.) Tickets for the theater cost $75.00 for an orchestra seat and $52.50 for a seat in the mezzanine. If 500 tickets were sold and the total revenue was for the evening was $29625, how many orchestra seats were sold?
Let x = # of orchestra seats, 500-x = #of mezzanine seats.
75x + 52.5(500-x)=29625. 75x+26250-52.5x=29625. 22.5x=3375. X=150.

2.) One train leaves Bridgeport at 9:30 heading east at 60 mph. Another train leaves Norwalk at 9:36 and heads west at 50 mph. If Norwalk is 10 miles west of Bridgeport, at what time will the two trains be 104 miles apart?
Let x = number of minutes since westbound train left Norwalk. The train from Bridgeport already has 10 mile and a 6 minute headstart. In those 6 minutes, it goes an additional 6 miles, so at 9:36 the trains are already 16 miles apart.
We need 50x+60x=104-16=88, so 110x=88, so x=0.8 hours = 48 minutes since 9:36, so the time is 10:24

3.) If Moe and Larry work together to mow the lawn, the job takes 40 minutes. If Larry and Curly work together to mow the lawn, the job takes 50 minutes. If Moe and Curly work together to mow the lawn, the job takes 60 minutes. How many minutes would it take to mow the lawn if all 3 men worked together?

Let m = the time it takes Moe to do the job by himself. Moe does $\frac{1}{m}$ lawns per minute.

Let L = the time it takes Larry to do the job by himself. Larry does $\frac{1}{L}$ lawns per minute.

Let c = the time it takes Curly to do the job by himself. Curly does $\frac{1}{c}$ lawns per minute.

$\frac{40}{m}+\frac{40}{L}=1 \therefore \frac{1}{m}+\frac{1}{L}=\frac{1}{40}$, $\frac{50}{c}+\frac{50}{L}=1 \therefore \frac{1}{c}+\frac{1}{L}=\frac{1}{50}$, $\frac{60}{c}+\frac{60}{m}=1 \therefore \frac{1}{c}+\frac{1}{m}=\frac{1}{60}$

Adding these three equations together, we get $\frac{2}{m}+\frac{2}{L}+\frac{2}{c}=\frac{1}{40}+\frac{1}{50}+\frac{1}{60}=\frac{15+12+10}{600}$.

Divide both sides by 2 to get $\frac{1}{m}+\frac{1}{L}+\frac{1}{c}$. The reciprocal of $\frac{15+12+10}{2*600}$ is the time it takes for the three men to work together, so this is $\frac{1200}{37}$

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Match 3 Round 3
Geometry: Polygons

1.) ________162____________

2.) ________ 2277__________

3.) ________ 45,16 ________

1.) The number of diagonals of a regular n-gon is 170. What is the degree measure of each of its interior angles?

$$170 = \frac{n(n-3)}{2}, n(n-3) = 340, n^2 - 3n - 340 = 0$$

$$(n-20)(n+17) = 0, \therefore n = 20$$

$$\frac{180(20-2)}{20} = 9*18 = 162 \; \frac{180(20-2)}{20} = 9*18 = 162$$

2.) The exterior angle at vertex A of a convex n-gon measures 20 degrees. All of the other exterior angles are congruent, and the interior angles associated with these angles are each 15 degrees greater than the interior angle at vertex A. How many diagonals does the n-gon have?

The interior angle at vertex A is 160 degrees, so the interior angle for all the other vertices is 175 degrees. Each of the other exterior angles measures 5 degrees, since they sum to 360, 5n+20=360, so 5n=340, so n=68. Therefore the n-gon has a total of 69 vertices, and the number of diagonals is $\frac{69*66}{2} = 69*33 = 2277$

3.)_The sum of the number of diagonals of a convex polygon and 151 times the number of its sides is equal to the numerical value in degrees of the sum of its interior angles. What are all possible values for the number of sides of the polygon?

$$\frac{n(n-3)}{2} + 151n = 180(n-2)$$

$$n(n-3) + 302n = 360n - 720$$

$$n^2 - 3n + 302n - 360n + 720 = 0$$

$$n^2 - 61n + 720 = 0$$

$$(n-45)(n-16) = 0$$

$$n = 45\_or\_n = 16$$

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Match 3 Round 4
Algebra 2: Functions and Inverses

1.) $\frac{3}{4}$

2.) x+2

3.) $(-\infty,0)\cup(2,\infty)$
(alternatively x<0 or x>2)

1.) $h(x)=5x-3$. Find all values of x such that $h^{-1}(x)=h(x)$.

$h^{-1}(x)=\frac{x+3}{5}$

$5x-3=\frac{x+3}{5}$

$25x-15=x+3$

$24x=18$

$x=\frac{3}{4}$

2.) If $f(x)=2x+3$ and $g(x)=3x+2$, find $f^{-1}(g(f(g^{-1}(x)))$.

$g^{-1}(x)=\frac{x-2}{3},\ f^{-1}(x)=\frac{x-3}{2}$

$f(g^{-1}(x))=2(\frac{x-2}{3})+3=\frac{2}{3}x+\frac{5}{3}$

$g(\frac{2}{3}x+\frac{5}{3})=3(\frac{2}{3}x+\frac{5}{3})+2=2x+5+2=2x+7$

$f^{-1}(2x+7)=\frac{2x+7-3}{2}=\frac{2x+4}{2}=x+2$

3.) $k(2x)=\dfrac{16x^3}{8x^3-2x}$. Find the domain of $k^{-1}(x)$. Express your answer using interval notation or as an inequality in terms of x.

$$k(x)=\frac{16(\frac{x}{2})^3}{8(\frac{x}{2})^3-2(\frac{x}{2})}=\frac{2x^3}{x^3-x}$$

Find the range of this and we will have the domain of $k^{-1}(x)$.

This has a removable discontinuity at x=0, and asymptotes at x=-1 and x=1.

When x is near -1 but to the left of -1, the denominator is a large negative number and the numerator is near -2, so as x approaches -1 from the left, k(x) increases without bound. As x gets more negative, k(x) approaches the line y=2 as a horizontal asymptote. When x is near 1 but to the right of 1, the numerator is near 2 and the denominator is a small positive number, so as k(x) approaches 1 from the right, k(x) increases without bound, and as x increases from 1, k(x) also approaches the line y=2. For -1<x<1, if x is close to -1 but to the right of -1, the numerator is near -2 but now the denominator is a small positive number, so k(x) decreases without bound. If x is close to 1 but to the left of 1, the numerator is near 2, but the denominator is negative, so this also decreases without bound. Both of these have a contribution to the range of (2,∞). Consider the reduced fraction $\dfrac{2x^2}{x^2-1}$ in this region. The numerator is always positive and the denominator is always negative. This would reach a maximum of 0 when x=0, but the original k(x) is undefined for x=0,

so this extends the range to also include (-∞,0). The range of the original function is the domain of the inverse. Since the question asks about domain values for $k^{-1}(x)$, if you use inequality notation, you must use x, not y. So the solution is either $(-\infty,0)\cup(2,\infty)$, alternatively $x<0$ or $x>2$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 3 Round 5
Advanced Math:
Exponents and Logarithms

1.) ______ 0.933 ________

2.) ________ 16, $\frac{1}{32}$ ________

3.) ______ 4,2 __________

1.) If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, and $\log_{10} 7 = .8451$, find $\log_{10}(\frac{60}{7})$.

$$\log_{10}(\frac{60}{7}) = \log_{10}(\frac{2*3*10}{7})$$
$$= \log(2) + \log(3) + \log(10) - \log(7)$$
$$= 0.3010 + 0.4771 + 1 - 0.8451 = 0.933$$

2.) Find all possible values of z if $\log_2(z) = y$ and $(5^{y^2-3y})(25^{y-4}) = (0.04)^{(y-6)}$

$$(5^{y^2-3y})(5^{2(y-4)}) = (5)^{-2(y-6)}$$
$$5^{y^2-3y+2y-8} = 5^{-2(y-6)}$$

$$y^2-3y+2y-8=-2y+12$$
$$y^2+y-20=0$$
$$(y+5)(y-4)=0$$
$$y=-5..or..y=4$$
$$\log_2(z)=-5..or..\log_2(z)=4$$
$$z=16..or..z=\frac{1}{32}$$

3.) Find all values of x such that

$$\log_2(x^2)+\log_x(16)=6$$
$$2\log_2(x)+\frac{1}{\log_{16}(x)}=6$$
$$2\log_2(x)+\frac{1}{\frac{\log_2(x)}{\log_2(16)}}=6$$
$$2\log_2(x)+\frac{4}{\log_2(x)}=6$$
$$2(\log_2(x))^2+4=6(\log_2(x))$$
$$2(\log_2(x))^2-6\log_2(x)+4=0$$
$$2(\log_2(x)-2)(\log_2(x)-1)=0$$
$$\log_2(x)=2..or..\log_2(x)=1$$
$$x=4..or..x=2$$

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Match 3 Round 6
Discrete Math: Matrices

1.) ____________18__________

2.) ___________2, -3________

$$B=\begin{bmatrix} -11 & 1 \\ -15.5 & 0.5 \end{bmatrix}$$

3.) ______________________

1.) Give the sum of the six entries of the matrix product:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -1 & 3 \\ 2 & 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -1 \\ -2 & 0 \\ 0 & 3 \end{bmatrix}$$

11 entry: 1+6+-6+0= 1

12 entry 2+-2+0+12 = 12

21 entry 0+-6+2+0 = -4

22 entry 0+2+0+9 = 11

31 entry 2+0+-2+0 = 0

32 entry 4+0+0+-6 = -2

Sum is 1+12+(-4)+11+0+(-2) = 18

2.) Find all values of k such that the determinants of the matrices $\begin{bmatrix} 2 & k+4 \\ k & 4 \end{bmatrix}$ and $\begin{bmatrix} 3 & 6 \\ 2 & k+2 \end{bmatrix}$ sum to -4.

8-k(k+4) + [3(k+2)-12]=-4

$8 - k^2 - 4k + 3k + 6 - 12 = -4$

$-k^2 - k + 2 = -4$

$-k^2 - k + 6 = 0$

$k^2 + k - 6 = 0$, so (k+3)(k-2) =0.

K=-3, or k=2

3.) If $ABA = \begin{bmatrix} 20 & -40 \\ -26 & 54 \end{bmatrix}$ and A= $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$, write the 2x2 matrix B. Express your entries as integers or decimals or reduced fractions. Do not leave any coefficients outside the matrix.

Find A^{-1} first. The determinant of A is -2. $A^{-1} = \begin{bmatrix} -2 & -1 \\ -1.5 & -0.5 \end{bmatrix}$.

Multiply ABA by A^{-1} on both sides to find B.

$$\begin{bmatrix} -2 & -1 \\ -1.5 & -0.5 \end{bmatrix}\begin{bmatrix} 20 & -40 \\ -26 & 54 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ -1.5 & -0.5 \end{bmatrix} =$$

$$\begin{bmatrix} -14 & 26 \\ -17 & 33 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ -1.5 & -0.5 \end{bmatrix} = \begin{bmatrix} 28-39 & 14-13 \\ 34-49.5 & 17-16.5 \end{bmatrix}$$

$$\begin{bmatrix} -11 & 1 \\ -15.5 & 0.5 \end{bmatrix}$$

FAIRFIELD COUNTY MATH LEAGUE 2014-15 Match 3 Team Round

Note: The inverse of a function or relation is not necessarily a function.

1.) __E_ _F___ _F___$_{16}$ 4.) _______(__1.5_,_3.5_)___

2.) _________ $3.125x10^{-18}$ ______ 5.) ___________5_____________

3.) __________175.5__________ 6.) ___________-360___________

1.)_ In the hexadecimal (base 16) system, A=10, B=11, C=12, D=13, E=14, and F=15. A three digit number is given in hexadecimal. If you reverse the order of the digits, the new number is FF_{16} greater than the original number. The sum of the digits is $2C_{16}$. Find the original hexadecimal number.
The sum of the digits is 2*16+10=44. This means there must be 2 F's and one E. The new number is 16*15+15=255 more than the original number. Let the number be XYZ. Then 256Z+16Y+X=256X+16Y+Z+255, so 255Z-255X=255, so Z-X=1 and X+1=Z, so X must be E, and Y and Z must be F.

2.)_ If A=2 x 10^{-4}, B=3 x 10^{6}, and C=5 x 10^{-8}, express $\dfrac{ABC}{\dfrac{6}{AC}+B^2}$ in scientific notation.

AC=10x10^{-12}=10^{-11}. $\dfrac{6}{AC}$= 6x(10^{11}) $\dfrac{6}{AC}+B^2$ = 6 x 10^{11} + 9x10^{12} = 9.6x10^{12}.

ABC=(30x10^{-6}). $\dfrac{30x10^{-6}}{9.6x10^{12}}=3.125x10^{-18}$

3.)_ A decagon has interior angles of 10 distinct measures. If you arrange the angle measurements from smallest to largest, each angle beyond the first measures 7 degrees more than the previous angle. What is the degree measure of the largest angle of the decagon?

The sum of the angles of the decagon is 180(10-2)=1440. Let x=the smallest angle. Then x+(x+7)+(x+14)+(x+21)+(x+28)+(x+35)+(x+42)+(x+49)+(x+56)+(x+63)=1440 so 10x+315=1440, so 10x=1125, so x=112.5. The largest angle is 112.5+63=175.5 degrees.

4.) $f^{-1}(x)=\frac{1}{2}x+3$ and $g^{-1}(x)=\frac{-1}{3}x+4$. The product of f(x) and g(x) is a parabola h(x). Find the vertex of $h^{-1}(x)$.

$f^{-1}(x)=\frac{1}{2}x+3, so$

$f(x)=2x-6$

$g^{-1}(x)=\frac{-1}{3}x+4, so$

$g(x)=-3x+12$

f(x)g(x)=(2x-6)(-3x+12) = $-6x^2 + 42x - 72$.
Complete the square to find the vertex of h(x) = $-6(x^2-7x+ 12.25)-72+73.5$, so the vertex of h(x) is at (3.5, 1.5), so the vertex of $h^{-1}(x)$ is at (1.5, 3.5)

5.) Find all values of x such that .

$8^{\log_2(x+3)} - x(9^{\log_3(x+4)}) = 107$

$8^{\log_2(x+3)} = 2^{3\log_2(x+3)} = (2^{\log_2(x+3)})^3 =$

$(x+3)^3.$

$9^{\log_3(x+4)} = 3^{2\log_3(x+4)} = (3^{\log_3(x+4)})^2 =$

$(x+4)^2, so$

$(x+3)^3 - x(x+4)^2 = 107$

$x^3 + 9x^2 + 27x + 27 - x^3 - 8x^2 - 16x = 107,$

$x^2 + 11x - 80 = 0, (x+16)(x-5) = 0$

$x = -16..or..x = 5$

but x=-16 gives negative arguments of logs in the original equation, so x=5 is the only correct answer.

6.) The digits 1 through 9 appear in this matrix exactly once, and it forms a “magic square”, such that the sum of all rows, columns, and diagonals of length 3 numbers is 15. Place the digits 1 through 6 in the correct positions so that it is a magic square, and write the determinant of this matrix in the answer spot.

$$\begin{vmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{vmatrix}$$

is the magic square matrix. The determinant is

8(10-63)-1(6-28)+6(27-20) = 8(-53)-1(-22)+6(7)=-424+22+42= -360