FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 1 Round 1
Arithmetic: Percents

1) 60

2.) 6

3.) 13, 26

1) A number N is increased by 20% and 5 is added to the result. If this sum is decreased by 30%, the result is 53.9. Find N.

$0.7(1.2N + 5) = 53.9$
$0.84N + 3.5 = 53.9$
$0.84N = 50.4$
$N=60$

2) 60% of (X+9) is equal to (10X+3)% of $\frac{100}{7}$. Find X.

$0.6(X+9) = ((10X+3)/100)(100/7)$
$0.6(X+9) = (10X+3)/7$
$0.6X + 5.4 = (10X+3)/7$
$4.2X + 37.8 = 10X + 3$
$34.8 = 5.8X$
$X=6$

3) 20% of the students who applied to State U. from Rufus High School were accepted. 26% of the students who applied to State U. from Buford High School were accepted. If there were 150 students in total who applied from the two high schools, Y% of the total number of students who applied were accepted, and Y is an integer, what are all possible values for the number of students who were accepted to State U. from Buford High School.

Let x = # of students who applied from Rufus HS, z = # of students who applied from Buford HS. $0.2x + 0.26z = 0.01Y(150) = 1.5Y$, and $x+z=150$. Solve $x+z=150$ for x to get $x=150-z$, and substitute into the first equation to get $0.2(150-z) + 0.26z = 1.5Y$, so $30-0.06z=1.5Y$.

We know Y must be between 20 and 26. Both Y and z must be integers, and 26% of z must also be an integer If Y=21, 0.06z=1.5, so z is 25, but 0.26(25) is not an integer. . If Y=22, 0.06z=3, so z=50, and 0.26(50)=13. OK. If Y=23, 0.06z=4.5, and z=75, but 0.26(75) is not an integer. If Y=24, 0.06z=6, so z=100, and 0.26(100)=26. OK. If Y=25, 0.06z=7.5, and z=125, but 0.26(125) is not an integer. So 13 or 26 are the possible values.

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Match 1 Round 2
Algebra I: Equations

1.) __________-9__________

2.) __________1__________

3.) ________10, 45__________

1.) Solve for a:

$$3-2(a-4(a-1))=1-4(a-2(a-3))$$
$$3-2(a-4a+4))=1-4(a-2a+6)$$
$$3-2(-3a+4)=1-4(-a+6)$$
$$3+6a-8=1+4a-24$$
$$6a-5=4a-23$$
$$2a=-18$$
$$a=-9$$

2) Solve for b:

$$(2b+5)(3b-4)-(b+4)(5b-2)=(b+10)(b-3)$$
$$(6b^2+7b-20)-(5b^2+18b-8)=b^2+7b-30$$
$$b^2-11b-12=b^2+7b-30$$
$$-11b-12=7b-30$$
$$18=18b$$
$$b=1$$

3) Solve for c:

$$0.3c-\frac{c+6}{4}=\frac{c-15}{c-5}$$

$$4(c-5)0.3c-(c-5)(c+6)=4(c-15)$$

$$1.2c^2-6c-(c^2+c-30)=4c-60$$

$$0.2c^2-7c+30=4c-60$$

$$0.2c^2-11c+90=0$$

$$c^2-55c+450=0$$

$$(c-10)(c-45)=0$$

$$c=10, c=45$$

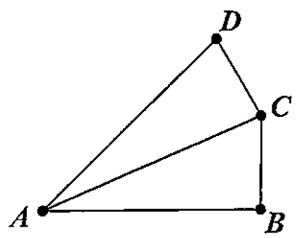
FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 1 Round 3
Geometry: Triangles
And Quadrilaterals

1.) ___ $3\sqrt{3}+3\sqrt{5}+6$ ______ cm

2.) ________ $16\sqrt{3}$ ______ cm

3. __________ 54 cm _____

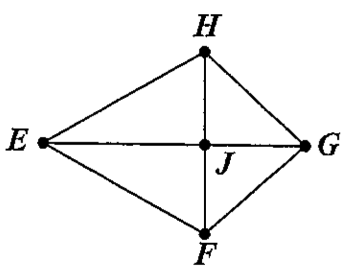


1) If angles ABC and ACD are right angles, AC=6 cm, and BC=CD=3 cm, find the perimeter of quadrilateral ABCD.

AB = $\sqrt{(AC^2 - BC^2)} = \sqrt{6^2-3^2} = 3\sqrt{3}$ $\sqrt{6^2+3^2} = 3\sqrt{5}$. AD = $\sqrt{(AC^2 + CD^2)} = \sqrt{6^2+3^2} = 3\sqrt{5}$. Perimeter is $3\sqrt{3}+3\sqrt{5}+6$

2.) A rhombus with side length 4 cm has one angle of 60 degrees. What is the product of the diagonals of the rhombus?

One diagonal of the rhombus splits the rhombus into 2 equilateral triangles of side 4. The area of each is $\frac{4^2\sqrt{3}}{4} = 4\sqrt{3}$, so the total area is $16\sqrt{3}$ $8\sqrt{3}$. The total area is half the product of the diagonals, so the product of the diagonals is $16\sqrt{3}$



3. The area is 168 cm^2, so the diagonals must multiply to 336. There must be 4 integers that multiply to 336, 2 of which are identical, since one diagonal bisects the other. Suppose HF is the diagonal that is bisected, so since 336 $=2^4*3*7$, the possibilities for diagonal HJ and JF are either 1, 1 or 2,2 or 4,4 or 8,8. Since the sides also must be integers and the diagonals are perpendicular, we need Pythagorean triples. There are no Pythagorean triples with 1 and 2. There is only one Pythagorean triple with 4, $3^2+4^2= 5^2$. If HJ and JF are 8 and 8, then JG could be 6 and EJ could be 15, which adds to the required 21 remaining if we are to get 336 = (8+8)(6+15). $8^2+6^2=10^2$, and $8^2+15^2=17^2$, so the sides of the kite are 10, 10, 17, and 17, which adds to 54 cm.

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Match 1 Round 4 Algebra 2: Simultaneous Equations

1.) ______a= 4 b= __10__

2.) _______c= __-6_ ____d= -4__

3.) ___k = __-4____________

1.) Solve for a and b:

$1.25a - 0.4b = 1$

$0.75a + 0.01b = 3.1$

Multiply first equation by 40, second equation by 10

$50a - 16b = 40$

$75a + b = 310$

Multiply first equation by 3 and second equation by -2

$150a - 48b = 120$

$-150a - 2b = -620$

add these together to get -50b = -500, so b=10. 1.25a-4=1, so 1.25a=5, so a=4.

2.) Solve for c and d:

$$\frac{2}{c-4} + \frac{4}{d+3} = \frac{-21}{5} \quad Let\_u = \frac{1}{c-4}, v = \frac{1}{d+3} \qquad 2u + 4v = \frac{-21}{5} \qquad 5u - 6v = \frac{11}{2}$$

$$\frac{5}{c-4} - \frac{6}{d+3} = \frac{11}{2}$$

$Multiply\_by\_5\_and\_2$

$10u + 20v = -21$

$10u - 12v = 11$

$Subtract$

$32v = -32$

$v = -1, so$

$$-1 = \frac{1}{d+3}, d = -4$$

$$10u-12(-1)=11,$$
$$10u+12=11,$$
$$10u=-1, u=\frac{-1}{10}$$
$$\frac{-1}{10}=\frac{1}{c-4}$$
$$c=-6$$

$$10u-12(-1)=11,$$
$$10u+12=11,$$
$$10u=-1, u=\frac{-1}{10}$$
$$\frac{-1}{10}=\frac{1}{c-4}$$
$$c=-6$$

3) Find all values of k such that the system below has infinitely many solutions (x,y,z):

$$-2kx+3y-z=6$$
$$3x+ky+2z=13$$
$$12x+25y-11z=-34$$
$$E\lim inate\_z$$
$$-4kx+6y-2z=12$$
$$3x+ky+2z=13$$
$$(-4k+3)x+(6+k)y=25$$
$$and$$
$$33x+11ky+22z=143$$
$$24x+50y-22z=-68$$
$$57x+(11k+50)y=75$$
$$Since\_3*25=75, We\_need$$
$$3(6+k)=(11k+50), 3(-4k+3)=57$$
$$18+3k=11k+50, -4k+3=19$$
$$-8k=32, -4k=16, so$$
$$k=-4$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

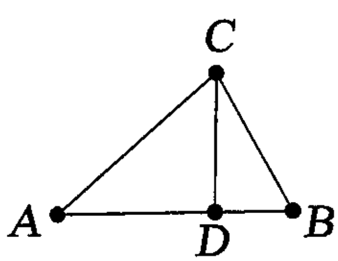
Match 1 Round 5
Trig: Right Triangles

1.) ______ $12+4\sqrt{3}+4\sqrt{2}$

2.) __________ $\dfrac{2\sqrt{39}}{13}$ __________

3.) _____ $\dfrac{24\sqrt{5}-15\sqrt{2}}{10}$ _____

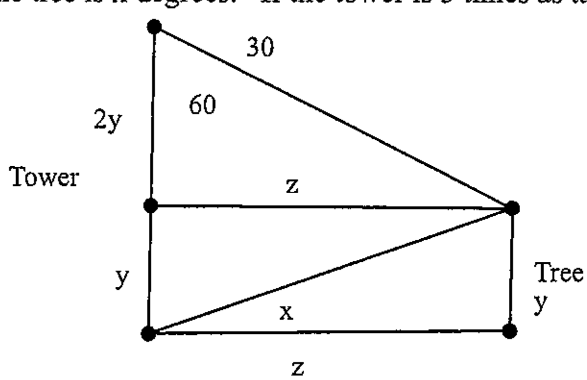
1.), For ΔABC, an altitude is drawn from C and intersects $\overline{AB}$ at point D. If $\tan(\angle CBA)=1$ and $\sin(\angle CAB)=0.5$ and $CD=4$, what is the perimeter of ΔABC?



$\tan(\angle CBA)=1$, and CD=4, so BD=4 and BC=4√2.

$\sin(\angle CAB)=0.5$, so since CD=4, AC=8, and AD=$4\sqrt{3}$. The sum of the sides of ΔABC is

2) A tree is located a certain horizontal distance from an observation tower. When viewed from the top of the tower, the angle of depression to the top of the tree is 30 degrees. When viewed from the bottom of the tower, the angle of elevation to the top of the tree is x degrees. If the tower is 3 times as tall as the tree, what is $\cos(x)$?



$\tan(60) = \dfrac{z}{2y}$

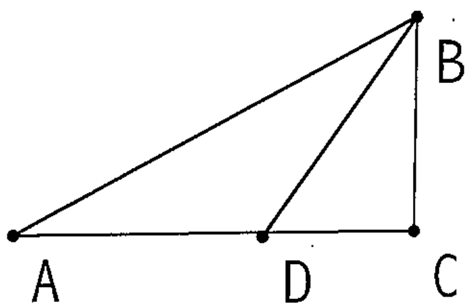
$z = y(2\sqrt{3})$

The length of the line from the bottom of the tower to the top of the tree is

$\sqrt{y^2 + (2y\sqrt{3})^2} = y\sqrt{13}$

$\cos(x) = \dfrac{2y\sqrt{3}}{y\sqrt{13}} = \dfrac{2\sqrt{39}}{13}$

3.) In triangle ABC below, $\angle C$ is a right angle, $\cos(\angle BAC) = \dfrac{2}{3}$, $\cos(\angle BDC) = \dfrac{1}{3}$, and $BC = 6$. What is the length of AD?



$\cos(\angle BDC) = \dfrac{1}{3}, \therefore \sin(\angle BDC) = \dfrac{2\sqrt{2}}{3}, \tan(\angle BDC) = 2\sqrt{2}$, so

$2\sqrt{2} = \dfrac{BC}{CD} = \dfrac{6}{CD} \therefore CD = \dfrac{6}{2\sqrt{2}} = \dfrac{3\sqrt{2}}{2}$

$\cos(\angle BAC) = \dfrac{2}{3}, \sin(\angle BAC) = \dfrac{\sqrt{5}}{3}, \tan(\angle BAC) = \dfrac{\sqrt{5}}{2}$

$\cos(\angle BAC) = \dfrac{2}{3}, \sin(\angle BAC) = \dfrac{\sqrt{5}}{3}, \tan(\angle BAC) = \dfrac{\sqrt{5}}{2}$

$\dfrac{\sqrt{5}}{2} = \dfrac{BC}{AC} = \dfrac{6}{AC} \therefore AC = \dfrac{12}{\sqrt{5}} = \dfrac{12\sqrt{5}}{5}$

$AD = \dfrac{12\sqrt{5}}{5} - \dfrac{3\sqrt{2}}{2} = \dfrac{24\sqrt{5}}{10} - \dfrac{15\sqrt{2}}{10} = \dfrac{24\sqrt{5} - 15\sqrt{2}}{10}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-15 Match 1 Team Round

1.) $\frac{15\sqrt{10}-20\sqrt{5}}{2}$

4.) $\frac{7}{3}$, $\frac{9\pm\sqrt{13}}{2}$

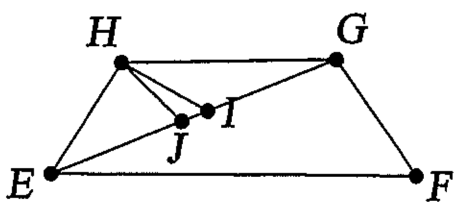
2.) $\frac{3\sqrt{5}}{5}$

5.) $25, -12$

3.) $x=200, y=10, z=50$

6.) 20

1)



1. Isosceles trapezoid EFGH has EH = $5\sqrt{2}$ and angle HEF measures 45 degrees. The area of the trapezoid is 75 cm^2. Diagonal $\overline{EG}$ measures $5\sqrt{10}$ cm. The median of ΔEGH from angle H is drawn to point I on $\overline{EG}$. The angle bisector of angle EHG intersects $\overline{EG}$ at point J. What is the length of segment $\overline{IJ}$?

Since angle HEF is 45 degrees and EH = $5\sqrt{2}$, each of the triangles on either side of the trapezoid must be isosceles right triangles of side 5, so they each have area 12.5 cm^2, so a total of 25 cm^2. That means GH must be 10, since the height of the trapezoid is 5, and the middle rectangle of the trapezoid must give the other 50 cm^2. The distance EI is $\frac{5\sqrt{10}}{2}$ by definition of median. The angle bisector of angle EHG splits GE into the same ratio as GH to HE, so let x = EJ.

$$\frac{5\sqrt{2}}{10}=\frac{x}{5\sqrt{10}-x} \qquad 10x=25\sqrt{20}-5x\sqrt{2}$$

$$10x=25\sqrt{20}-5x\sqrt{2}$$

Then , so , and

$$x=\frac{50\sqrt{5}}{10+5\sqrt{2}}$$

Subtract this from (1/2) the length of the diagonal $\frac{5\sqrt{10}}{2}$

$$\frac{5\sqrt{10}}{2}-\frac{50\sqrt{5}}{10+5\sqrt{2}}=$$

$$\frac{5\sqrt{10}}{2}-\frac{50\sqrt{5}(10-5\sqrt{2})}{(10+5\sqrt{2})(10-5\sqrt{2})}=$$

$$\frac{5\sqrt{10}}{2}-\frac{500\sqrt{5}-250\sqrt{10}}{50}$$

$$\frac{125\sqrt{10}}{50}-\frac{500\sqrt{5}-250\sqrt{10}}{50}=$$

$$\frac{375\sqrt{10}-500\sqrt{5}}{50}$$

$$=\frac{15\sqrt{10}-20\sqrt{5}}{2}$$

2.) ΔABC has vertices A(2,6), B(1,4), and C(3,5). Find the length of the altitude from point A to $\overline{BC}$.

The altitude from (2,6) to the segment joining (1,4) and (3,5) must have slope -2, so $\frac{y-6}{x-2}=-2$ and $y=\frac{1}{2}x+3.5$, so (0.5x+3.5)-6=-2(x-2), so 0.5x-2.5=-2x+4,

So 2.5x=6.5, so x=2.6, and y=(0.5)(2.6)+3.5 = 4.8. The altitude has length

$$\sqrt{(2.6-2)^2+(4.8-6)^2}=\sqrt{0.36+1.44}=\sqrt{1.8}=\sqrt{\frac{9}{5}}$$

$$=\frac{3\sqrt{5}}{5}$$

3.) X% of Y = Z-30. Y% of Z =0.025X. Z% of X = 10Y. Find x, y, and z.

$\frac{x}{100}*y=z-30$, $\frac{y}{100}*z=\frac{x}{40}$, $\frac{z}{100}*x=10y$

xy=100z-3000, 40yz=100x, xz=1000y, so $y=\frac{xz}{1000}$, substitute this for y into the two other equations.

$$\frac{x}{100}*\frac{xz}{1000}=z-30$$

$$\frac{xz}{10000}*z=\frac{x}{40}$$

The x's cancel from the second equation to get $40z^2=10000$, so $z^2=2500$, so z=50.

Then $\frac{50x^2}{10000}=50-30$, so $50x^2=2000000$, $x^2=400000$, and x=200. Then y= $\frac{2000*50}{10000}$ =10.

4.) In ΔKLM, K has coordinates (1,3), L has coordinates (x,5), and M has coordinates (2,x). Find all possible real values of x such that ΔKLM is a right triangle.

If there is a right angle at K, then $\frac{5-x}{x-2}=-\frac{1}{\frac{5-3}{x-1}}$, or, so -2x+6=x-1, so 3x=7, and x= $\frac{7}{3}$. If there is a right angle at L, then $\frac{5-x}{x-2}=-\frac{1}{\frac{5-3}{x-1}}$, so $\frac{x-5}{x-2}=\frac{x-1}{2}$, and $x^2-3x+2=2x-10$, so $x^2-5x+12=0$. No real solutions since the discriminant is negative. If there is a right angle at M, then $\frac{5-x}{x-2}=-\frac{1}{\frac{x-3}{2-1}}$ or $\frac{5-x}{x-2}=\frac{-1}{x-3}$, or $\frac{x-5}{x-2}=\frac{1}{x-3}$, so $x^2-8x+15=x-2$, so $x^2-9x+17=0$, so x= $\frac{9\pm\sqrt{13}}{2}$

5.) Find all values of d such that $\frac{d+1}{2}-\frac{d+3}{4}-\frac{d+5}{6}=\frac{25}{d}$.

$$\frac{d+1}{2}-\frac{d+3}{4}-\frac{d+5}{6}=\frac{25}{d}$$

$$Mult.by\_12d$$

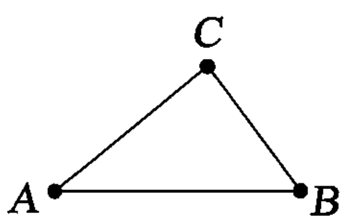
$$6d(d+1)-3d(d+3)-2d(d+5)=300$$

$$6d^2+6d-3d^2-9d-2d^2-10d=300$$

$$d^2-13d-300=0$$

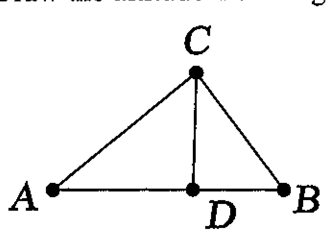
$$(d-25)(d+12)=0$$

d=25 or d=-12



6.) Right triangle ABC above has a right angle at C. If $\tan(\angle ABC)$=2 and AB=10, what is the area of the triangle?

Draw the altitude from angle C to segment AB to meet at D.



$$\tan(\angle ABC)=2,$$

$$\sin(\angle ABC)=2\cos(\angle ABC)=2\sin(\angle BAC)$$

$$\sin(\angle ABC)=\frac{CD}{BC},\sin(\angle BAC)=\frac{CD}{AC}$$

$$BC\sin(\angle ABC)=AC\sin(\angle BAC)$$

$$BC*2\sin(\angle BAC)=AC\sin(\angle BAC)$$

$$AC=2*BC$$

$$\sqrt{(BC)^2+(2*BC)^2}=10$$

$$5*(BC)^2=100, BC=\sqrt{20}, AC=2\sqrt{20}.$$

If $\tan(\angle ABC)$=2, then $\sin(\angle ABC) = \frac{2}{\sqrt{5}}$. DC=BC$\sin(\angle ABC) = \sqrt{20} * \frac{2}{\sqrt{5}} = 4$,
So the area is (1/2)*10*4 = 20.