NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2019

Round 1: Arithmetic and Number Theory

1. ____________________

2. ____________________

3. ____________________

1. Simplify $\dfrac{\sqrt{0.0009}}{\sqrt{0.000036}}$.

2. Let $N=(3\_\_)\cdot(8\_)$, where $3\_\_$ and $8\_$ denote three-digit and two-digit base 10 integers, respectively. Compute the *largest* possible value of N that is divisible by 9, 12, 15, and 17.

3. If x, y, and z represent digits drawn from 1 through 9, where $x \neq y$, compute the number of distinct ordered triples (x, y, z) for which $0.\overline{xy} + 0.\overline{yx} = 0.\overline{z}$.

   Note: For any value of a, $0.\overline{a} = 0.aaa\ldots$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2019

Round 2: Algebra 1

1. ________ : ________

2. ____________________

3. ____________________

1. A whole ham costs \$2.00 per pound, and a half ham costs \$3.00 per pound. If the average price per pound of ham when one buys a particular whole ham and a particular half ham is \$2.25, what is the ratio of the weight of that whole ham to the weight of that half ham?

2. For $x > 0 > y$, compute the value of $\frac{x}{y}$, if $\frac{|x-y|}{x+2|y|} = \frac{3}{4}$.

3. Given: a set of five positive integers whose unique mode is 37, and whose median is 58. If A is the smallest possible average of the five integers, compute $\lceil A \rceil$.
   Note: $\lceil n \rceil$ denotes the smallest integer greater than or equal to n.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2019

Round 3: Geometry

1. ____________________ °

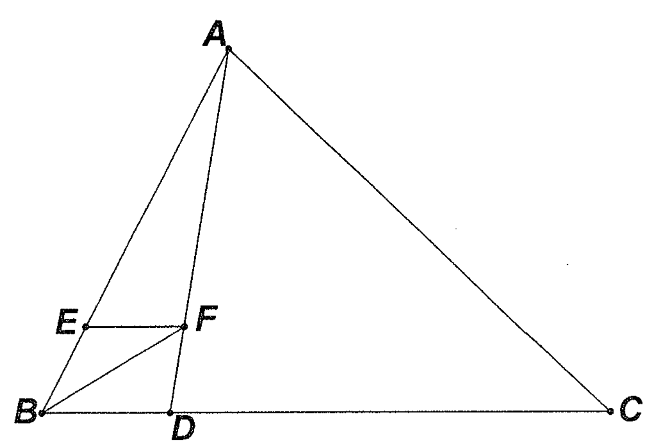
2. ______________________

3. ______________________

1. In regular hexagon $ABCDEF$ with reflective sides, a light ray from A hits $\overline{ED}$ at M. It is reflected, meeting $\overline{BC}$ at N, and again is reflected, meeting $\overline{AF}$ at P. Remember that the measure of the angle of incidence equals the measure of the angle of reflection. If $m\angle AME = 70°$, compute $m\angle NPA$ (in degrees).

2. A 4-by-8 rectangle lies wholly outside a circle whose radius is 12. When the diagonals of the rectangle are extended, each cuts off a 90° arc of the circle. Compute the distance from the center P of the rectangle to the point of intersection of a diagonal with the circle that is farther from P.

3. Given: ΔABC, $\overline{EF} \parallel \overline{BC}$, $AE = 3 \cdot EB$, $DC = 5 \cdot BD$, and the area of ΔEFB is 4. Compute the area of ΔABC.



NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2019

Round 4: Algebra 2

1. (________ , ________)

2. ____________________

3. ____________________

1. If $9^{x+\frac{5}{2}} = 2 + 27^{2y-8}$, where x and y are integers, compute the ordered pair (x,y).

2. Solve for x over the real numbers: $\sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}} = 3$.

3. If, for all real x, $f(x) - 2f(1-x) = x^2$, compute $f(2)$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2019

Round 5: Analytic Geometry

1. ______________________

2. ______________________

3. ______________________

1. Compute <u>all</u> values of a for which a line whose slope is $a+2$ is perpendicular to a line whose slope is $2a+1$.

2. Let point (a,b) be an ordered pair of positive integers. Compute <u>all</u> points (a,b) for which (a,b) is twice as far from $3x-y=6$ as it is from $3x+y=6$.

3. A circle with center at $P\left(0,\frac{25}{2}\right)$ is tangent to $y=x^2$ at two points. Compute the radius of the circle.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2019

Round 6: Trig and Complex Numbers

1. ______________________

2. (________ , ________)

3. ______________________

1. Compute the value of x for which $\sin\left(\text{Cos}^{-1} x\right) = x$.

2. A rectangular box has edges $AB = 8$, $AC = 15$, and $AD = 20$. If, in simplest form, $\cos\angle DCB = \frac{m}{n}$, where m and n are positive integers, determine the ordered pair (m, n).

3. If A, B, and C are the angles of a 45-45-90 right triangle, compute the value of $\tan\frac{A}{2}\cdot\tan\frac{B}{2}\cdot\tan\frac{C}{2}$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2019

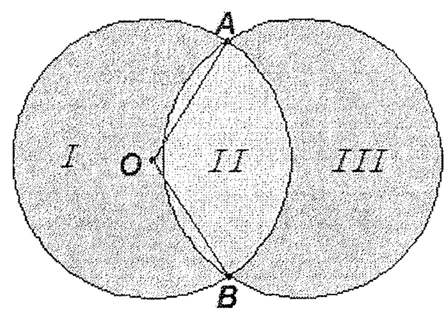
Team Round – Place all answers on the team round answer sheet.

1. Let N be the set of all distinct permutations of the letters of MASSASOIT. If a permutation is drawn at random from N, the probability (as a fraction in simplest form) that the three S's are *not* all adjacent in the permutation is $\frac{a}{b}$. Compute $a+b$.

2. Let a, b, and c be non-zero real numbers.
If $a+\frac{2}{b}=3$, $b+\frac{2}{c}=4$, and $c+\frac{2}{a}=5$, compute the value of $abc+\frac{8}{abc}$.

3. Compute the largest positive integer value of n such that $\frac{1^2+2^2+3^2+\ldots+(n-1)^2+n^2}{1^3+2^3+3^3+\ldots+(n-1)^3+n^3}>\frac{1}{15}$.

4. Two circles of radius 1 intersect in A and B as shown, such that the ratios of the areas of regions I, II, and III are 2 : 1 : 2. O is the center of the circle on the left, and the measure of $\angle AOB$ (in radians) is θ. In simplest form, $\sin\theta=\frac{k\theta-m\pi}{n}$. Compute the value of $k+m+n$, where k, m and n are unique positive integers.
Note: The lightly shaded region II is the intersection of the interiors of the circles.



5. In ΔABC, $m\angle A=30°$, $AB\in\{1,2,3\}$, and $AC\in\{1,2,3,\ldots,n\}$. Compute the value of n, such that the average area of all the triangles that can be formed using those sides is 20.
Note: Consider ΔABC formed with $AB=1$ and $AC=2$ to be distinct from the triangle formed with $AB=2$ and $AC=1$, since the named sides have different lengths.

6. The volume of a regular tetrahedron is 1. If h is the height, compute the value of h^3.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2019

Answer Sheet

Round 1

1. 5
2. 33,660
3. 32

Round 2

1. 3:1
2. -2
3. 51

Round 3

1. 70
2. $18\sqrt{2}$
3. 128

Round 4

1. $(-2,4)$
2. $9 \pm 4\sqrt{5}$
3. -2

Round 5

1. $-1, -\frac{3}{2}$
2. $(1, 1),\ (1, 9)$
3. $\frac{7}{2}$ (or 3.5)

Round 6

1. $\frac{\sqrt{2}}{2}$
2. $(9, 17)$
3. $3-2\sqrt{2}$

Team

1. 23
2. 36
3. 19
4. 7
5. 79
6. $\frac{8\sqrt{3}}{3}$