

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2018

Round 1: Arithmetic and Number Theory

1. _____
2. _____
3. _____

1. If $A = \frac{6^3}{3^2+4^2}$ and $B = \frac{5^3}{\sqrt{144}}$, compute the product AB . Write your answer in base 4.

2. If $(\overline{.72})^2 - (\overline{.27})^2$ is written as a rational number in simplest form, compute the sum of the numerator and denominator.

3. Given $n \in \{12, 13, 14, \dots, 29, 30\}$, for how many values of n is P a prime number where $P = 2 \cdot 3 \cdot 4 \cdot 5 \cdot 7 + n$?

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Round 2: Algebra 1

1. _____

2. _____

3. _____

1. If $3(x + 5) - 18 = 7 - 2x$ and $(y - 2)^2 = 16$, Compute the positive difference of the possible values of $x^{-3}y^2$.

2. Solve for x : $\frac{|x|}{|x|-1} = \frac{x+1}{x}$.

3. The solutions in x to $k(x+3) = \frac{1}{1+\frac{1}{x+3}}$ that lie between 16 and 26, inclusive, generate values of k satisfying $a \leq k \leq b$. What is the minimum possible value of $300(b-a)$?

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Round 3: Geometry

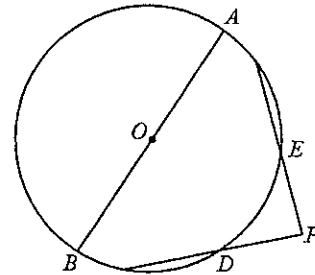
1. _____

2. _____

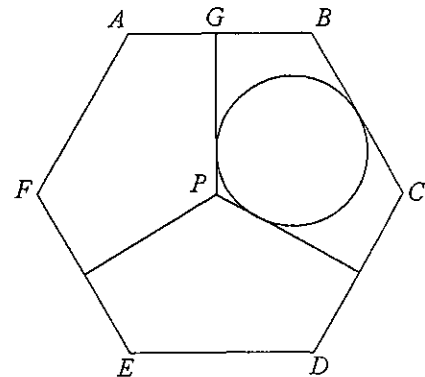
3. _____

1. A triangle is inscribed in a square of side 10 units with the base of the triangle coinciding with one side of the square and the vertex of the triangle on the opposite side of the square. Two circles are such that the diameter of one is the longest possible altitude of the triangle and the diameter of the other is a diagonal of the square. Compute the number of units in the positive difference of the areas of the circles.

2. If \overline{AOB} is a diameter of circle O and $m\widehat{DE} = 30^\circ$, compute the greatest lower bound of the degree measure of $\angle P$.



3. Regular hexagon $ABCDEF$ is partitioned into 3 congruent pentagons with $\overline{GP} \perp \overline{AB}$. If $AB = 6$ find the radius of a circle tangent to 3 sides of the pentagon as shown.



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Round 4: Algebra 2

1. _____
2. _____
3. _____

1. Tom can do a certain job in 15 hours. Ruth can do the same job in 9 hours. They work together for 3 hours, then are joined by Luis, who could have done the job alone in 6 hours. In how many more hours will the job be complete?

2. Find all real solutions to the following system:

$$x^3 - 3xy^2 = 125$$

$$3x^2y - y^3 = 0$$

3. Let $f(x)$ be an odd 5th degree polynomial with positive coefficients. If $f(1) = 7$, $f(2) = 9$, and $f(3) = 17$, what is the remainder when $f(6)$ is divided by 10?

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Round 5: Analytic Geometry

1. _____

2. _____

3. _____

1. The equation of a conic section is $4y^2 - 48y - 9x^2 - 36x = -72$. Compute the area of the triangle whose vertices are the center of the conic and its x -intercepts.
2. What are the coordinates of the reflection of $(1, 1)$ across $y = 2x$?
3. A line with negative slope passes through $(2, 1)$. The origin and the line's intercepts form a triangle whose area is 6. Compute the largest possible sum of the line's x - and y -intercepts.

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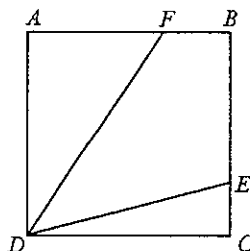
Round 6: Trig and Complex Numbers

1. _____
2. _____
3. _____

1. If $\cos 2\theta = \frac{7}{25}$ with $90^\circ < \theta < 180^\circ$, compute $\sin(\theta + 45^\circ)$.

2. Given $A(0, 0)$ and $B(3, 4)$, if A is fixed while segment \overline{AB} is rotated 30° in a counterclockwise direction, what is the new y -coordinate of B ?

3. $ABCD$ is a square of side 12. $FB = 4$ and $EC = 3$. Compute the tangent of $\angle FDE$.

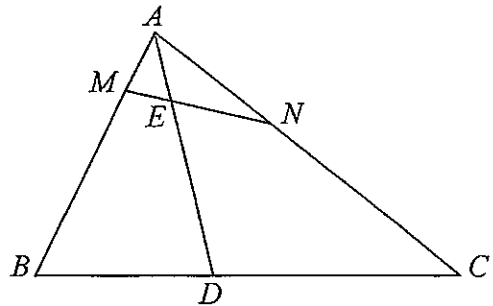


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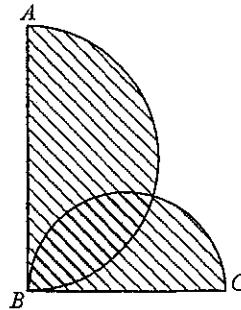
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Team Round - Place all answers on the team round answer sheet.

- Solve for x : $2\cos^{-1} x = \cos^{-1} \frac{x}{2}$.
- If $\frac{9}{x+y} = \frac{12}{3-y} = \frac{n}{x+3}$, what is the value of n ?
- In $\triangle ABC$, $AM = 1$, $MB = 3$, $AN = 2$, $NC = 4$, and \overline{AD} bisects $\angle BAC$. What is the ratio of the area of $\triangle AEN$ to the area of quadrilateral $MEDB$?



- In the diagram, $AB = 2\sqrt{3}$, $BC = 2$, and $\overline{AB} \perp \overline{BC}$. Semicircles are drawn on each of \overline{AB} and \overline{BC} . Compute the number of square units in the total area of all regions that are shaded?



- For $n \geq 1$, the average of a set of n consecutive integers starting with a_1 is k . If a_1 is removed, the average is doubled. Determine the largest possible value of the smallest number in the original set of n numbers.
- Let $\lfloor x \rfloor$ represent the greatest integer less than or equal to x . Thus, $\lfloor 3.7 \rfloor = 3$. For positive x and y , compute the area covered in the coordinate plane by the solution set to $\left\lfloor \frac{1}{x} \right\rfloor \cdot \left\lfloor \frac{1}{y} \right\rfloor = 8$. The answer can be written in simplest form as $\frac{a}{b}$. Compute $a + b$.

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Answer Sheet

Round 1

1. 1122 (base 4)
2. 16
3. 4

Round 2

1. 4
2. $-\frac{1}{2}$
3. 5

Round 3

1. $100^{25\pi}$
2. 75
3. $18 - 9\sqrt{3}$

Round 4

1. $\frac{42}{31}$
2. $(5, 0), \left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right), \left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$
3. 2

Round 5

1. $12\sqrt{3}$
2. $\left(\frac{1}{5}, \frac{7}{5}\right)$
3. $9 + \sqrt{3}$

Round 6

1. $\frac{\sqrt{2}}{10}$
2. $\frac{4\sqrt{3}+3}{2}$
3. $\frac{10}{11}$

Team

1. $\frac{1+\sqrt{33}}{8}$
2. 21
3. $\frac{10}{67}$
4. $\frac{7\pi}{6} + \sqrt{3}$
5. 0
6. 371