NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2014

Round 1: Arithmetic and Number Theory

1. ____________________

2. ____________________

3. ____________________

1. Compute: $1+\cfrac{1}{.\overline{1}+\cfrac{.\overline{8}}{.\overline{2}+\cfrac{.\overline{7}}{.\overline{3}+\cfrac{.\overline{6}}{.\overline{4}+.\overline{5}}}}}$

2. A store clerk needs to give a customer \$0.23 in change. The change is to be made with k coins, a collection made up exclusively of pennies, nickels and/or dimes. How many possible values of k?

3. A combination lock for a bicycle consists of four cylinders, each of which has all the integers from 1 to 6. Alvin forgot his combination so he tried each integer in order, starting with 1111, then 1112, followed by 1113, and so on until the lock finally opened at 2563. Compute the number of unsuccessful integers that he tried.

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Round 2: Algebra 1

1. ____________________

2. ____________________

3. ____________________

1. If $x = 3.000001$, approximate $\dfrac{x^2+5x+2}{x^2-9}$ to the nearest hundred thousand.

2. Compute the <u>largest</u> possible product xy given that (x, y) is a solution of the system:

   $x^2 + y = 8$ and $y - x = -4$.

3. Compute all values of k for which there are real x and y such that $\dfrac{k}{x-y} = \dfrac{1}{x} - \dfrac{1}{y}$.

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Round 3: Geometry

1. ____________________

2. ____________________

3. ____________________

1. A sphere is inscribed in a cube of edge 6. Determine the exact number of cubic units in the volume of the space interior to the cube but exterior to the sphere.

2. The lengths of three segments are $x^2 + 3x$, $x^2 + x$, and 16. Find all values of x such that when the three segments are joined at their endpoints, a triangle is formed.

3. The lengths of the sides of a triangle are 3, 9, and 10. By what non-zero factor should the lengths be multiplied so that the numerical value of the perimeter of the triangle equals the numerical value of its area?

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Round 4: Algebra 2

1. ______________________

2. ______________________

3. ______________________

1. Let $f^2(x) = f(f(x))$, $f^3(x) = f\left(f(f(x))\right)$, etc.

   For $f(x) = \frac{x-1}{x+1}$, find the value of $f^{2014}(-2)$.

2. If $4x^2 - kx + 15$ is factorable over the integers, compute the number of possible integer values of k.

3. Find the least positive integer value of n such that

   $\log_{10}\frac{1}{2} + \log_{10}\frac{1}{3} + \log_{10}\frac{1}{4} + \ldots + \log_{10}\frac{1}{n} < -4$

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Round 5: Analytic Geometry

1. ____________________

2. ____________________

3. ____________________

1. A line with negative slope passes through (4, 5). If its x-intercept is four times its y-intercept, and if point (x, 18) lies on the line compute the value of x.

2. A diameter of a circle is the transverse axis of a hyperbola whose equation is

$$4(x-5)^2 - 9(y-3)^2 = 36$$

Also, the circle is tangent to the hyperbola at both of its vertices. Compute the length of the vertical chord of the circle which passes through $(4, 0)$.

3. Compute the largest possible value of the y-intercept of the tangents to $(x-5)^2 + (y-5)^2 = 25$ that have a slope of $-\dfrac{4}{3}$.

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Round 6: Trig and Complex Numbers

1. ____________________

2. ____________________

3. ____________________

1. Let α and β be angles of a triangle. For positive a and b, if $\tan\alpha = \frac{a}{b}$ and $\tan\beta = \frac{b+a}{b-a}$, compute $\tan(\alpha - \beta)$.

2. In ΔABC, $A = (0,0)$, $B = (\sin a, \cos a)$, and $C = (\cos b, \sin b)$ where a and b are in radians with $0 < a < b < \frac{\pi}{4}$. If the area of ΔABC is 1/10, compute the tangent of $\angle BAC$.

3. Let a and b be unequal positive integers, $0 < b < a < \frac{\pi}{4}$, and let $M(a, 83°)$ and $N(b, 23°)$ be points in the polar coordinate plane. Compute the least possible sum of a and b in which the distance MN is an integer.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2014

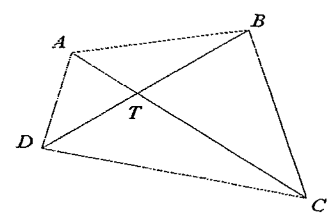
Team Round - Place all answers on the team round answer sheet.

1. If m and n are relatively prime positive integers that sum to 143, compute the number of ordered pairs (m, n).

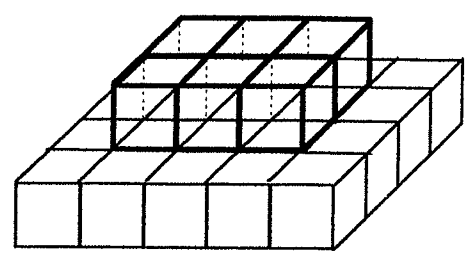
2. Let $\lfloor x \rfloor =$ the greatest integer less than or equal to x. For $2 \le a \le 4$, compute the least possible value of $\lfloor \log_a(8a) + \log_{8a}(64a) \rfloor$.

3. Let S be the set of positive 4-digit numbers none of whose digits are 0. A number is chosen at random from S. Compute the probability that at least two adjacent digits are the same.

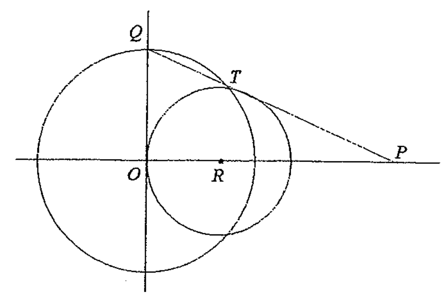
4. The area of quadrilateral $ABCD$ is 120. The diagonals of $ABCD$ meet at T. If the area of ATB is 20 and the area of ATD is 28, compute the number of square units in the area of DTC.



5. Using sticky toothpicks, a student builds a structure consisting of a base of 20 cubes in a 5 by 4 arrangement and a second layer of 6 cubes in a 2 by 3 arrangement. The top layer has edges in common with the bottom as indicated in the diagram. Whenever edges are in common, only 1 toothpick was used. Compute the number of toothpicks that were used. Note: The diagram doesn't show all the toothpicks.



6. Circle R with center $\left(\frac{25}{2}, 0\right)$ is tangent to the y-axis. Q is the positive y-intercept of circle O whose center is the origin. T is the intersection of circles O and R while P is the x-intercept of $\overleftrightarrow{QT}$. If $7 \le OQ \le 15$, let the smallest possible value of OP be a and the largest be b. Compute the ordered pair (a, b).



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Answer Sheet

Round 1

1. 2
2. 9
3. 392

Round 2

1. 4,300,000
2. 32
3. $k \leq 0$ or $k \geq 4$

Round 3

1. $216 - 36\pi$ or $36(6 - \pi)$
2. $(-8, -4) \cup (2,8)$ or $-8 < x < -4$ or $2 < x < 8$
3. $\frac{\sqrt{11}}{2}$

Round 4

1. $\frac{1}{2}$
2. 12
3. 8

Round 5

1. −48
2. $4\sqrt{2}$
3. 20

Round 6

1. −1
2. $\frac{\sqrt{6}}{12}$
3. 11

Team

1. 120
2. 4
3. $\frac{217}{729}$
4. 42
5. 157
6. (45,49)