NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2011

Round 1: Arithmetic and Number Theory

1. ____(, ,)____

2. ______________

3. ______________

1. If $3(1.\overline{2})(.\overline{20}) = 0.\overline{ABC}$, find the ordered triple (A, B, C).

2. Find the largest integer less than 10,000 that has exactly 3 positive factors.

3. Given $N = .\overline{AB}$ and $M = .\overline{BA}$, find all numbers N such that $\frac{4}{7}N = M$. Express all answers as reduced fractions.

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Round 2: Algebra 1

1. _______________points

2. { _________________ }

3. _________________

1. At the end of the semester Bob built a backyard parabolic reflector and earned 6 points extra credit. If his average increased from 75% to 76%, how many total points were there for the semester?

2. Find the set of positive integers with the smallest sum in which the mode equals 1, the median equals 2, the mean is 3, and the largest element is as small as possible.

   Better wording: Call a set of positive integers a "Bozorgmir set" if it has the properties that it has a unique mode of 1, its median is 2, its mean is 3, and its largest element is minimized under these conditions. Find the Bozorgmir set with the smallest sum.

3. Find all real solutions to $\sqrt{1-\dfrac{2x-7}{(x-3)^2}}=\dfrac{1}{2}$.

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Round 3: Geometry

1. ____________________

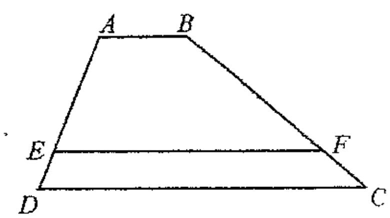
2. __________:__________

3. ____________________

1. To determine the area of an irregularly shaped plane figure A, Bob traced the figure on stiff plastic of uniform thickness and density and cut it out. He then traced a 4 cm by 4 cm square S on the same material and cut it out. When he attached the two shapes on opposite ends of a lever resting on a fulcrum, the lever balanced when shape A was 12 cm from the fulcrum and shape S was 10 cm from the fulcrum. How many cm^2 are in the area of plane figure A?

2. Diagonal $\overline{AC}$ of rhombus $ABCD$ is extended past C to E so that $CE : AC = 2{:}1$. Determine the ratio of the area of $ABCD$ to the area of $EDAB$.

3. $ABCD$ is a trapezoid with $\overline{EF} \parallel \overline{CD}$, $AB = 10$, $CD = 30$, and $AE : ED = 4 : 1$. The perimeter of $ABFE$ equals the perimeter of $EFCD$. How many linear units are in the perimeter of $ABCD$.



NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

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Round 4: Algebra 2

1. ____________________

2. ____________________

3. ____________________

1. If $a = \log 4^4$ and $b = \log 5^5$, find $\log 20^{20}$ in terms of a and b.

2. The integers from 1 to 20 are each written on a slip of paper and placed into a bag. Three slips are drawn out without replacement. What is the probability that the three numbers can be arranged in an increasing geometric sequence?

3. Determine the value of $1003^3 - 4 \cdot 1001^3 + 4 \cdot 999^3 - 997^3$.

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Round 5: Analytic Geometry

1. ____________________

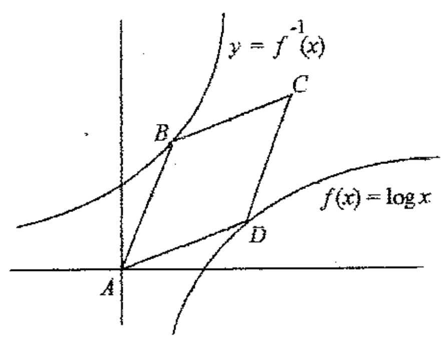
2. ____________________

3. ____________________

1. The line segment joining the intersection points of the graphs of $y = -3x^2 + 24x - 46$ and $y = -10$ is the minor axis of an ellipse. The vertex of the parabola is one end of the major axis. What is the number of units in the sum of the lengths of the major and minor axes?

2. Point $P(0,b)$ with $b > 1$, lies on the positive y-axis, $A = (0,0)$ and $B = (9,3)$. The ratio of the slopes of the lines drawn from P to the trisection points of $\overline{AB}$ is $\frac{1}{6}$. Find the value of b.

3. Shown are the graphs of $f(x) = \log x$ and its inverse. $ABCD$ is a rhombus with $A = (0,0)$ and $C = (4,4)$. Find the sum of the coordinates of D.



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Round 6: Trig and Complex Numbers

1. ______________

2. ______________

3. ______________

1. If $\text{Tan}^{-1}\left(-\frac{6}{5}\right) = A$, what is the value of $\csc 2A$?

2. In a polar coordinate system, the equation of a curve is $r = \dfrac{4}{2\sin\theta - 3\cos\theta}$.

   A rectangular coordinate system is superimposed on the polar coordinate system with the origin at the pole and the polar axis lying on the positive x-axis. If a point on the polar curve has an x-value of 5 in the rectangular system, find the y-value.

3. Find the area of the figure $ABCD$ in the complex plane whose vertices are $A = 0 + 0i$, $B = 1 + 4i$, $D = 4 + i$, and $C = B + D$.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2011

Team Round

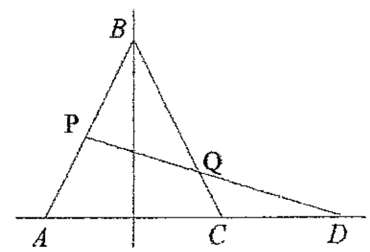
1. ________________ **4.** ________________

2. ________________ **5.** ________________

3. (___,___,___,___) **6.** ________________

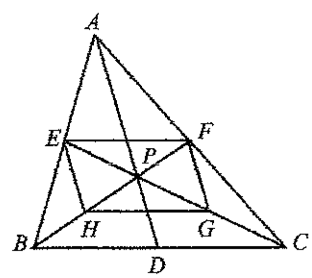
1. Each of the integers from 1 to 10^6 inclusive, is repeatedly replaced by the sum of its digits, until we obtain 10^6 one-digit numbers. Find the number of 1's.

2. Given points $A(-2,0)$, $B(0,6)$, $C(2,0)$, and $D(4,0)$, a line with slope m is drawn through D intersecting $\overline{BC}$ in Q and $\overline{AB}$ in P so that the area of ΔBPQ equals the area of quadrilateral $ACQP$. Find the value of m.



3. There are 3 gallons of paint, one red, one blue, and one yellow. To obtain one gallon of a color, various combinations of fifths of a gallon of the three main colors can be used. For example, one could have one gallon of red, a gallon consisting of 3/5 gallon of red and 2/5 gallon of blue, or a gallon consisting of 1/5 gallon of red, 2/5 gallon of blue, and 2/5 gallon of yellow. How many different colors of 1 gallon can one obtain?

4. Let $AB = 21$, $BC = 42$, and $AC = 27$. Points E, D, and F are midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively. Also, points H and G are midpoints of $\overline{BP}$ and $\overline{PC}$ respectively. Find the number of units in the perimeter of $EFGH$.



Note: the diagram is not drawn to scale.

5. On a test with 120 points the grade is determined by dividing the raw score N (an integer) by 120 to obtain a percentage that lies between 0 and 100. Percentages are then rounded to obtain a final grade: 58.3 is rounded to 58 and 58.5 is rounded to 59. Determine the number of raw scores N that exceed their final grade by exactly 5.

6. Find the maximum distance from the origin to a point on the curve $\left(x^2 + y^2\right)^3 = 9x^2y^2$.

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Answer Sheet

Round 1

1. $(7,4,0)$
2. 9409
3. $\frac{7}{33}, \frac{14}{33}, \frac{7}{11}, \frac{28}{33}$

Round 2

1. 600
2. $\{1, 1, 2, 5, 6\}$
3. $5, \frac{11}{3}$

Round 3

1. $\frac{40}{3}$
2. $\frac{1}{3}$
3. $\frac{220}{3}$

Round 4

1. $5a+4b$
2. $\frac{11}{1140}$
3. $-5{,}999{,}954$

Round 5

1. 28
2. $\frac{5}{2}$
3. 4

Round 6

1. $-\frac{61}{60}$
2. $\frac{19}{2}$
3. 15

Team

1. 111,112
2. $\frac{-4+\sqrt{7}}{3}$
3. 21
4. 50
5. 6
6. $\frac{3}{2}$