FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 6 Round 1
Geometry: Lines and Angles

1.) ___________15_________degrees

2.) __________196_________degrees

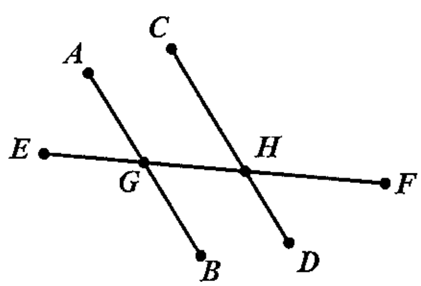
Note: Figures not necessarily
Drawn to scale

3.) ___________159________degrees

1.)_ What is the degree measure of the acute angle formed by the intersection of the lines $2x-2y=1$ and $4x-(4\sqrt{3})y=2$?

The two lines intersect at the point (0.5, 0). The first line has slope 1, so is at a 45 degree angle with the horizontal. The second line has slope $\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ so it is at a 30 degree angle with the horizontal. The acute angle formed is 45-30 = 15 degrees.

2.)_ $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$. The lines are cut by transversal $\overleftrightarrow{EF}$, which intersects line $\overleftrightarrow{AB}$ at G and $\overleftrightarrow{CD}$ at H. If $m(\angle AGH)=(11+6x)°$ and $m(\angle FHD)=(12x-92)°$, find $m(\angle CHF)+m(\angle EGB)$.



$m(\angle FHD)=m(\angle CHG)$ by vertical angles.

$m(\angle AGH)+m(\angle CHG)=180°$ since they are same-side interior angles.

$(11+6x)+(12x-92)=180$

$18x-81=180$

$18x=261$

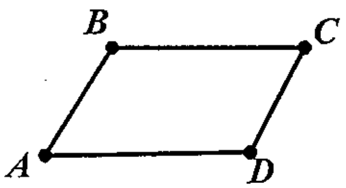
$x=14.5$

$\angle CHF$ and $\angle EGB$ are both congruent to $\angle AGH$, which has measure 11+6*14.5=98 degrees. The sum is 2*98 = 196 degrees.

3.) In parallelogram ABCD,

$\angle BAD = (8x-3)°, \angle BCD = (3x^2-2x)°$.

Find the degree measure of $\angle ADC$.



Opposite angles of a parallelogram are congruent, so the measures of angle A and angle C are equal.

$$8x+3=3x^2-2x$$

$$3x^2-10x+3=0$$

$$(3x-1)(x-3)=0$$

$$x=3, x=\frac{1}{3}$$

If $x=\frac{1}{3}$, then $\angle BCD$ has a negative degree measure, so x=3.

$\angle BAD$=8*3 -3 = 21 degrees; and this must be 180 – m($\angle ADC$), so $\angle ADC$ measures 159 degrees.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2016-2017

Match 6 Round 2 Algebra: Literal Equations

1.) ______ $z = 4x + 3y - 2$______________

2.) ________ $b = a^2 - 3$______________

3.) ______ k = __$\frac{m+1}{2}, \frac{m}{2}$______________

1.) Solve for z in terms of x and y: $8x + 6y - 2z = 4$

$$8x + 6y - 2z = 4$$
$$-2z = 4 - 8x - 6y$$
$$z = \frac{4 - 8x - 6y}{-2}$$
$$z = 4x + 3y - 2$$

2.)_ If a≠6, solve for b in terms of a:
$ab + 6a^2 + 3a = a^3 + 6b + 18$

$$ab + 6a^2 + 3a = a^3 + 6b + 18$$
$$-a^3 + 6a^2 + 3a - 18 = 6b - ab$$
$$-(a^3 - 6a^2 - 3a + 18) = b(6 - a)$$
$$-(a^2(a - 6) - 3(a - 6)) = b(6 - a)$$
$$-(a^2 - 3)(a - 6) = b(6 - a)$$
$$b = a^2 - 3$$

3. Solve for k in terms of m:

$$4k^2+m=4mk+2k-m^2$$

$$4k^2-(4m+2)k+m^2+m=0$$

$$k=\frac{(4m+2)\pm\sqrt{(4m+2)^2-4*4(m^2+m)}}{8}$$

$$k=\frac{(4m+2)\pm\sqrt{(16m^2+16m+4-16m^2-16m}}{8}$$

$$k=\frac{(4m+2)\pm\sqrt{4}}{8}=\frac{4m+4}{8}\_or\_\frac{4m}{8}=$$

$$\frac{m+1}{2},\frac{m}{2}$$

FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 6 Round 3
Geometry:
Solids and
Volumes

1.)_ $25\pi+40\pi\sqrt{2}$__ in^2_

2.)_______ $108\pi\sqrt{3}-216$_ cm^3

3.) _____ $18\sqrt{7}$ _______ cm^2

1) The volume of a cylinder with height 8 inches is 100π in^3. What is its total surface area, including the bases ?

$$V=\pi hr^2$$

$$100\pi=\pi*8*r^2$$

$$r^2=\frac{100}{8}=\frac{25}{2}, r=\frac{5\sqrt{2}}{2}$$

$$A=2\pi r^2+h(2\pi r)=$$

$$2\pi(\frac{25}{2})+8*2*\pi*\frac{5\sqrt{2}}{2}$$

$$=25\pi+40\pi\sqrt{2}$$

2. A cube of side 6 cm is inscribed in a sphere. What is the volume that is inside the sphere but outside the cube?

The radius of the sphere must be half the diagonal of the cube. The diagonal of the cube is $\sqrt{6^2+6^2+6^2}=\sqrt{108}=6\sqrt{3}$. The radius of the sphere is $3\sqrt{3}$. The volume of the sphere is $\frac{4}{3}\pi(3\sqrt{3})^3=108\pi\sqrt{3}$. The volume of the cube is $6*6*6=216$. The desired volume is $108\pi\sqrt{3}-216$ cm^3

3. The lateral area of a pyramid is the surface area, not including the area of the base. A triangular pyramid has an equilateral triangle of side 6 cm for its base and isosceles triangles for its sides. The pyramid has a height of 5 cm. Find the lateral area of this pyramid.

Draw the altitude for one of the triangles on the side. It is the hypotenuse for a triangle with one of the sides being the distance from the midpoint of one of the sides of the base to the center of the base, and the other side being 5 cm. The segment from the midpoint of the base to the center of the equilateral triangle is the apothem of the equilateral triangle. The area of a triangle is half the product of the perimeter and the apothem. The area of the triangle is $\frac{6^2\sqrt{3}}{4} = 9\sqrt{3}$ cm^2 The perimeter is 18 cm, so the apothem is $2 * \frac{9\sqrt{3}}{18} = \sqrt{3}$ cm. The altitude first drawn then has length $\sqrt{5^2 + (\sqrt{3})^2} = \sqrt{28} = 2\sqrt{7}$. The lateral area consists of 3 triangles with base 6 and height $2\sqrt{7}$, so the total area is

$3 * \frac{1}{2} * 2\sqrt{7} * 6 = 18\sqrt{7}$ cm^2.

FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 6 Round 4 Radical Expressions and Equations

1.) ____________ $\frac{208\sqrt{10}}{15}$ ________

2.) ______________________ $-\frac{2}{3}$ ________

3.) ____________________ 3____________

1.)_ Express as a single reduced fraction in simplest radical form:

$$\sqrt{160}-\frac{4}{\sqrt{90}}+5\sqrt{40}$$

$$\sqrt{160}-\frac{4}{\sqrt{90}}+5\sqrt{40}$$

$$=4\sqrt{10}-\frac{4}{3\sqrt{10}}+5*2\sqrt{10}$$

$$=4\sqrt{10}-\frac{4\sqrt{10}}{30}+10\sqrt{10}$$

$$=\frac{120\sqrt{10}-4\sqrt{10}+300\sqrt{10}}{30}=$$

$$\frac{416\sqrt{10}}{30}=\frac{208\sqrt{10}}{15}$$

2) Solve for all real values of x:

$$\sqrt{x^2} = 2x+2$$

$$|x| = 2x+2$$
$$x = 2x+2\_or\_x = -2x-2$$
$$-x = 2\_or\_3x = -2$$
$$x = -2\_or\_x = \frac{-2}{3}$$

But x=-2 is extraneous so $x = -\frac{2}{3}$

3._Solve for all real values of x:

$$\sqrt{3x+7} - \sqrt{x-2} = \sqrt{6x-9}$$
$$\sqrt{3x+7} - \sqrt{x-2} = \sqrt{6x-9}$$
$$3x+7+x-2-2\sqrt{(3x+7)(x-2)} = 6x-9$$
$$-2\sqrt{(3x+7)(x-2)} = 2x-14$$
$$-\sqrt{(3x+7)(x-2)} = x-7$$
$$(3x+7)(x-2) = (x-7)^2$$
$$3x^2+x-14 = x^2-14x+49$$
$$2x^2+15x-63 = 0$$
$$(2x+21)(x-3) = 0$$
$$x = \frac{-21}{2}, x = 3$$

But $\frac{-21}{2}$ is extraneous, so x=3.

FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 6 Round 5 Polynomials and Advanced Factoring

1.) ________5________________

2.) _____________4___ _______

3.) __ $(5x^2-6y^2+2xy)(5x^2-6y^2-2xy)$__

1.) When $x^{16}-1$ is completely factored into binomials with integer coefficients, how many binomials are there?

$$x^{16}-1=(x^8+1)(x^8-1)$$
$$=(x^8+1)(x^4+1)(x^4-1)$$
$$=(x^8+1)(x^4+1)(x^2+1)(x^2-1)$$
$$=(x^8+1)(x^4+1)(x^2+1)(x+1)(x-1)$$

so there are 5 binomials.

2.) 1+3i is a zero of the polynomial $x^3 + ax^2 + bx + 40$ for some integer values of a and b. Find a+b.

$1+3i$ _and_ $1-3i$ are both zeros of the polynomial since it has integer coefficients. The quadratic with these as zeros must have $-\frac{b}{a}$=2, and $\frac{c}{a}$=10. a must be 1, so b=-2, c=10, and the quadratic is x^2 -2x + 10. Since the last term is 40 and the first term is x^3, the other factor must be x+4. (x^2 -2x + 10)(x+4) = x^3 +2x^2 + 2x + 40., so 2+2 = 4.

3) Factor into two trinomials: $25x^4-64x^2y^2+36y^4$

$$
\begin{aligned}
&25x^4-64x^2y^2+36y^4\\
&=25x^4-60x^2y^2+36y^4-4x^2y^2\\
&=(5x^2-6y^2)^2-(2xy)^2\\
&=(5x^2-6y^2+2xy)(5x^2-6y^2-2xy)
\end{aligned}
$$

FAIRFIELD COUNTY MATH LEAGUE 2016-17

Match 6 Round 6
Counting and
Probability

1.) ________48________

2.) ________$\frac{3}{8}$________

3.) ________48, 54, 66________

1.)_ For how many four-digit whole numbers in which no digits are repeated is the sum of the digits equal to 28?

What possibilities of 4 distinct digits add to 28? The only possibilities are 9, 8, 6, 5 or 9, 8, 7, 4. So we have 9865 and all of its permutations, which is 24 numbers, plus 9874 and all of its permutations, which is another 24 numbers, so there are 48.

2.) A men's tennis match is complete when one player wins three sets. Suppose two players are evenly matched so that the probability of either player winning any one set is 0.5. What is the probability that the match will last for five sets?

If the two players are A and B, there are 8 equally likely possibilities for the first 3 sets, AAA, AAB, ABA, BAA, AAB, ABA, ABB, and BBB. Two of those 8 represent matches that will last only 3 sets, so there is $\frac{6}{8} = \frac{3}{4}$ probability that the match goes more than three sets. After the 4^{th} set, the equally likely outcomes are AABA, AABB, ABAA, ABAB, … ABBA, ABBB. Six of those twelve possibilities have one player winning three sets, so there is a probability of $\frac{1}{2}$ that if the match goes for more than 3 sets, it will end after the fourth set, so there is a $\frac{3}{4} * \frac{1}{2} = \frac{3}{8}$ probability that the match goes exactly 4 sets, so a probability of $\frac{1}{4}+\frac{3}{8}=\frac{5}{8}$ that the match goes 4 sets or less. That leaves a remaining probability of $\frac{3}{8}$ that the match lasts five sets.

3.) A container of N balls contains R blue balls. The probability that if you draw one ball from the container it is blue is p. If 6 blue balls are removed from the container, the probability that if you draw one ball from the container it is blue is p-0.1. Find the three possible values of N+R

$$\frac{R}{N}-\frac{R-6}{N-6}=\frac{1}{10}$$

$$10R(N-6)-10N(R-6)=N(N-6)$$

$$10RN-60R-10RN+60N=N(N-6)$$

$$N^2-6N-60N+60R=0$$

$$R=\frac{66N-N^2}{60}=\frac{(66-N)N}{60}$$

N can not be greater than or equal to 66.
Between N and 66-N, the two numbers must contain 2 factors of 2, 1 factor of 3, and 1 factor of 5 in order to be evenly divisible by 60.
Consider the numbers N for which 66-N and N contain factors of 5:
Consider N=5,10,15,20,25,...65. For these numbers 66-N contains a factor of 3 only if N also contains a factor of 3. N=15, 30, 45, 60 contain both factors of 3 and 5, but if N=15 or 45, 66-N does not contain any factors of 2. N can be 30 or 60 – if N=60, the expression is certainly divisible by 60. If N=30, 66-N does have an additional factor of 2. Consider 66-N=5,10,15,20,...65. 66-N=60 means N=6 and (66-N)N is divisible by 60. 66-N=30 means N =3 6gives N an additional factor of 2, so (66-N)N would be divisible by 60. The candidates for N are 6, 30, 36, and 60. If N=6, the original equation gives a zero in the denominator. If N=30, R=18. If N=36, R=18. If N=60, R=6. N+R could be 48, 54, or 66

FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 6 Team Round		
	1.) D= 1440-19A	4.) 40
	2.) 48	5.) 41, -43
	3.) $5\sqrt[3]{2}$	6.) $\frac{1}{4}$

1) Angles A, B, C and D are the four interior angles of a convex quadrilateral. Twenty less than six times the measure of the complement of $\angle A$ is equal to the measure of the supplement of $\angle B$. Two times the measure of $\angle B$ is sixty degrees more than the measure of $\angle C$. Find $\angle D$ in terms of the measure of $\angle A$ if all measurements are in degrees. Use D for the measure of $\angle D$ and A for the measure of $\angle A$.

A+B+C + D=360
6(90-A)-20=180-B, so B=20-6(90-A)+180
2B=C+60, so C=2B-60
A+(20-6(90-A)+180)+(2(20-6(90-A)+180)-60+D=360
A+ 20-540+6A+180 + 40- 1080+12A+360-60+D=360
19A-1080+D=360
D=1440-19A

2.) How many triangles that are not equilateral can be made by connecting 3 of the 8 vertices of a cube?

There are ${}_8C_3$ ways of choosing ${}_8C_3 = 56$ ways of choosing 3 vertices out of the 8. How many are equilateral? Choose any vertex P. Three vertices are adjacent to this vertex by an edge and could not be used to create an equilateral triangle. One of the remaining 4 vertices is on the opposite corner of P and would create the main diagonal of the cube if you connected P to that vertex, so choose any of the other three vertices and draw one segment of the triangle. Now there are two remaining vertices for which neither of the original two vertices create a main diagonal, and choose one of those vertices to complete the triangle. So we have 8*2*3 = 48 equilateral triangles, but we could have chosen the three points in 3! orders, so divide 48 by 6 to get 8. Therefore, there are 56-8 = 48 triangles that are not equilateral.

3). The vertex of a right circular cone which extends above the x-y plane is at the origin and the cone has base radius 6 inches and height 10 inches. The plane z=k cuts the cone so that the volume below z=k is one-fourth of the volume of the entire cone. Find the value of k.

The base of the cone has radius 6, and its height is 10, so its volume is

$$\frac{1}{3}\pi * 10 * 6^2 = 120\pi$$

At a height of k inches, the cone must be have a volume of 30π. To find the radius of this circle, use similar triangles, $\frac{r}{k} = \frac{6}{10}, r = \frac{3}{5}k$. So

$$30\pi = \frac{1}{3}\pi * k(\frac{3}{5}k)^2$$

$$30 = \frac{3}{25}k^3$$

$$k^3 = 250, k = \sqrt[3]{250} = 5\sqrt[3]{2}$$

4.) The following can be expressed as $a + b\sqrt[3]{c} + d\sqrt[3]{e}$ where a,b,c,d and e are natural numbers and all radicals are in simplest form:

$$\frac{\sqrt[3]{9}+\sqrt[3]{4}}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}} + \frac{\sqrt[3]{3}+\sqrt[3]{2}}{\sqrt[3]{3}-\sqrt[3]{2}}$$

Give the sum a+b+c+d+e.

Use the factoring pattern $a^3 - b^3 = (a^2 + ab + b^2)(a-b)$ with $a = \sqrt[3]{3}$, $b = \sqrt[3]{2}$ to get a common denominator to add the fractions.

$$\frac{\sqrt[3]{9}+\sqrt[3]{4}}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}}+\frac{\sqrt[3]{3}+\sqrt[3]{2}}{\sqrt[3]{3}-\sqrt[3]{2}}$$

$$\frac{(\sqrt[3]{9}+\sqrt[3]{4})(\sqrt[3]{3}-\sqrt[3]{2})}{(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4})(\sqrt[3]{3}-\sqrt[3]{2})}+\frac{(\sqrt[3]{3}+\sqrt[3]{2})(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4})}{(\sqrt[3]{3}-\sqrt[3]{2})(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4})}=$$

$$\frac{\sqrt[3]{27}+\sqrt[3]{12}-\sqrt[3]{18}-\sqrt[3]{8}+(\sqrt[3]{27}+2\sqrt[3]{18}+2\sqrt[3]{12}+\sqrt[3]{8})}{(\sqrt[3]{27}+\sqrt[3]{18}+\sqrt[3]{12}-\sqrt[3]{18}-\sqrt[3]{12}-\sqrt[3]{8})}=$$

$$\frac{2\sqrt[3]{27}+\sqrt[3]{18}+3\sqrt[3]{12}}{(\sqrt[3]{27}+\sqrt[3]{18}+\sqrt[3]{12}-\sqrt[3]{18}-\sqrt[3]{12}-\sqrt[3]{8})}=\frac{6+\sqrt[3]{18}+3\sqrt[3]{12}}{1}$$

$$=6+\sqrt[3]{18}+3\sqrt[3]{12}$$

6+1+18+3+12 = 40

5.) $x^3-12x^2+cx-30$ has three distinct integer zeros. Find all possible values of c.

If the zeros are p, q, and r, we need pqr=30 and p+q+r=12. 30 is small enough that we can check by trial and error that there are two solutions, the integers {5,6,1} and {15,-2,-1}. c= (pq+pr+qr), so c can be (5*6+5*1+6*1) = 41, or (15*(-2)+15*(-1)+(-2)*(-1)) = -43

6) A standard deck of 52 cards has 13 spades. Your friend removes two cards at random. You draw a card from the deck of 50 cards. What is the probability that you draw a spade?

There are three possibilities for the remaining 50 cards of the deck. The deck could have all 13 spades. Then your friend would have removed two non-spades, so the probability is

$$\frac{{}_{39}C_2 * {}_{13}C_0}{{}_{52}C_2}=\frac{39*38}{52*51}=\frac{3*38}{4*51}$$

or your friend could have removed exactly one spade with probability

$$\frac{{}_{39}C_1 * {}_{13}C_1}{{}_{52}C_2}=\frac{2*39*13}{52*51}=\frac{3*13}{2*51}$$

or your friend could have removed two spades, with probability

$$\frac{{}_{39}C_0 * {}_{13}C_1}{{}_{52}C_2} = \frac{13*12}{52*51} = \frac{12}{4*51}$$

The probability that you draw a spade from the deck with13 spades is $\frac{13}{50}$, so the total probability is $\frac{3*38}{4*51}*\frac{13}{50}$. The probability you draw a spade from the deck with 12 spades is $\frac{12}{50}$, so the total probability is $\frac{3*13}{2*51}*\frac{12}{50}$. The probability you draw a spade from the deck with 11 spades is $\frac{12}{4*51}*\frac{11}{50}$

Multiplying and reducing these fractions gives $\frac{3*19}{4*51}*\frac{13}{25}=\frac{741}{5100}$, $\frac{3*13}{2*51}*\frac{12}{50}=\frac{468}{5100}$, and $\frac{12}{4*51}*\frac{11}{50}=\frac{6*11}{2*51*50}=\frac{66}{5100}$, adding them together gives $\frac{1275}{5100}=\frac{1}{4}$.