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**FAIRFIELD COUNTY MATH LEAGUE 2017-2018**

Match 5 Round 1  
 Algebra I:  
 Fractions and  
 Exponents

- 1.) 1.15
- 2.)  $\frac{a}{54}$
- 3.)  $\frac{7}{3,000,000}$

1. Simplify as much as possible and express your answer as a decimal:  $\frac{1}{2} + \left(\frac{3}{4}\right)\left(\frac{5}{6}\right) - \frac{7}{8} + \frac{9}{10}$

2) Simplify as much as possible. Express your answer with no negative exponents. Simplify all numerical coefficients.

$$\frac{(2ab^2)^{-5}(3a^2b^3)^{-3}(a^2b)^4}{(4a^5b^3)^{-2}\left(\frac{a^2}{b^3}\right)^3}$$

3.)\_ If  $m+2n = -1$ , express the following as a single fraction in simplest form with no exponents.

$$\frac{3^m 9^n 10^{6m} 5^{12n} \left(\frac{1}{7}\right)^m}{8^{-4n} 49^n}$$

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Match 5 Round 2  
Algebra I:  
Fractional  
Expressions and  
Equations

1.)  $\frac{11x+4}{2x+1}$

2.)  $5, \frac{25}{12}$

3.)  $\frac{x^2+6}{6x}$

Assume no values of  $x$  make any denominator equal to zero

1). Express as the quotient of two linear binomials:

$$4 + \frac{3}{2 + \frac{1}{x}}$$

2). Solve for all possible values of  $x$ :

$$\frac{5}{2x-5} - 3 = \frac{3x-25}{x}$$

3.) Simplify as much as possible:

$$\frac{(x-1)(x-2)(x-3) - (9-2x)(x+2)}{(4x-3)(x-5) + (2x+5)(x-3)}$$

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Match 5 Round 3  
 Geometry:  
 Circles

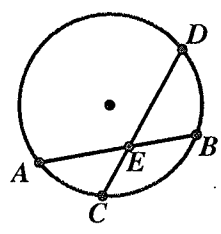
1.) 30

2.) 62 degrees

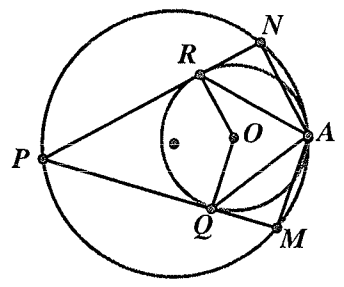
3.)  $\frac{65}{8}$

Note: Diagrams not necessarily to scale

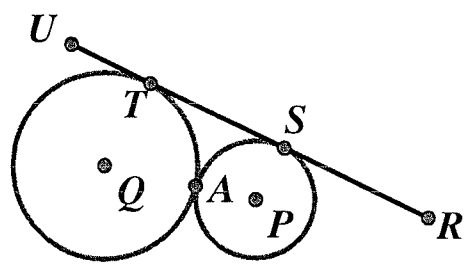
1.) In the picture below,  $\overline{AB}$  and  $\overline{CD}$  are chords of a circle that intersect at E.  $AE=x+5$ ,  $BE=x+3$ ,  $CE=x+1$ , and  $DE=4x$ . Find the sum  $AB + CD$ .



2.) Two circles are internally tangent at A as shown below. O is the center of the smaller circle. Chord  $\overline{PN}$  of the larger circle is tangent to the smaller circle at R and chord  $\overline{PM}$  of the larger circle is tangent to the smaller circle at Q. The measure of angle  $\angle MAN$  is  $124^\circ$ . Find the measure of  $\angle RAQ$ .



3.) Two circles with centers P and Q are externally tangent as shown at A.  $\overline{RU}$  is tangent to the circle with center P at point S and tangent to the circle with center Q at point T.  $AP=4$  and  $TS=\sqrt{130}$ . What is the radius of the circle with center Q?



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Match 5 Round 4  
Quadratic  
Equations and  
Complex  
Numbers

1.)  $\frac{-1}{2} + i$

2.)  $-2, -8.5$

3.)  $-\frac{1}{10} + \frac{1}{5}i, -\frac{2}{5} + \frac{1}{5}i$

1) Express in the form  $a+bi$ :  $\frac{3+4i}{2-4i}$

2) If  $(4+ai)(2+bi) = 23-14i$  where  $a$  and  $b$  are real, give all possible values of  $a+b$

3) Solve for all complex  $z$ :  $(10i)z^2 + (4+5i)z + 1 = 0$ . Express your answers in  $a+bi$  form.

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Match 5 Round 5  
Solving Trig  
Equations

1.) 20, 80, 140, 200, 260, 320 degrees

2.) 0,  $\frac{\pi}{6}$ ,  $\frac{5\pi}{6}$

3.)  $\frac{\sqrt{3} \pm 2\sqrt{2}}{6}$

1) Solve for all  $x$   $0^\circ \leq x < 360^\circ$ :  $\tan(3x) = \sqrt{3}$

2) Solve for all  $x$   $0 \leq x < 2\pi$  if  $\sin(2x) - 2\sin(x) - \cos(x) = -1$

3.) If  $\sin(x + \frac{\pi}{3}) = \frac{1}{3}$ , what are all possible values for  $\cos(x)$ ?

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Match 5 Round 6  
Sequences and  
Series

1.) 5

2.) 4, -2

3.)  $-2 + 2\sqrt{2}$

1.)\_ What is the smallest value of n such that  $\sum_{k=1}^n \frac{1}{k^2} > 1.45$

2.) The first term of an arithmetic sequence is 2. The square of the third term is four more than the seventh term. Give all possible values for the fifth term of the sequence.

3. An infinite geometric series converges to 16. The second term of the original geometric sequence is -4. Give all possible values for the third term of the sequence.

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Match 5 Team Round

1.) 45.5

4.)  $\frac{-6 \pm 6\sqrt{6}}{5}$

2.)  $2-7i, 2+i$

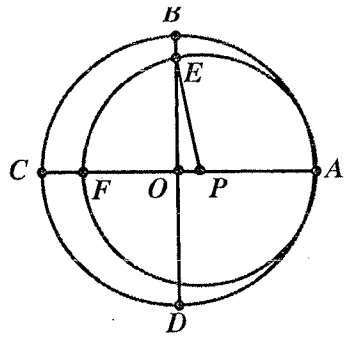
5.)  $\frac{64}{x^2y^{15}}$

Note: Diagrams not necessarily drawn to scale.

3.)  $\frac{3}{5}, 1$

6.) 12

1.) The circles below with centers O and P are internally tangent at A. B, C, and D are on the larger circle. Both centers lie on AC and AC is perpendicular to BD and both are diameters of the larger circle. F lies on AC and on the smaller circle. A radius is drawn from P to BD and intersects BD at E. FC=9, OP=4.5, and BE=5. Find the sum of the radii of the two circles.



2.) A geometric sequence of complex numbers has second term  $3 - 4i$  and fourth term  $-7 - 24i$ . Find all possible values for the first term.

3.) If  $\cos(x) + 2\sin(x) = 1$ , find all possible values of  $\cos(x)$ .

4.) Solve for x:  $\frac{x^2}{x^2 - 5x + 6} - \frac{2x^2}{4 - x^2} = 3$

5.) If  $x \neq 0$  and  $y \neq 0$ , express in simplest form with no negative exponents:

$$\frac{(3x)^5 (2y^{-5}) (9x^3y)^{-4}}{(4y^6)^2 (24xy^2)^{-3}}$$

6.)  $\{a_n\}$  is an arithmetic sequence of numbers with  $a_3 = 3.2$  and  $a_{10} = -7.3$ . How many terms of  $\{a_n\}$  must be added to get a sum of  $-24.6$ ?