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FAIRFIELD COUNTY MATH LEAGUE 20~~17~~-20~~18~~

Match 5 Round 1
Algebra I:
Fractions and
Exponents

1.) ________1.15_____

2.) ________ $\frac{a}{54}$ ___________

3.) _____ $\frac{7}{3000000}$ _____

1. Simplify as much as possible and express your answer as a decimal: $\frac{1}{2}+(\frac{3}{4})(\frac{5}{6})-\frac{7}{8}+\frac{9}{10}$

$$\frac{1}{2}+(\frac{3}{4})(\frac{5}{6})-\frac{7}{8}+\frac{9}{10}=$$

$$\frac{1}{2}+\frac{15}{24}-\frac{7}{8}+\frac{9}{10}=$$

$$\frac{60}{120}+\frac{75}{120}-\frac{105}{120}+\frac{108}{120}=$$

$$\frac{135}{120}+\frac{3}{120}=$$

$$\frac{138}{120}=\frac{69}{60}=\frac{23}{20}=1.15$$

2) Simplify as much as possible. Express your answer with no negative exponents. Simplify all numerical coefficients.

$$\frac{(2ab^2)^{-5}(3a^2b^3)^{-3}(a^2b)^4}{(4a^5b^3)^{-2}(\frac{a^2}{b^3})^3} =$$

$$\frac{(4a^5b^3)^2(a^2b)^4(b^3)^3}{(2ab^2)^5(3a^2b^3)^3(a^2)^3} =$$

$$\frac{(16a^{10}b^6)(a^8b^4)(b^9)}{(32a^5b^{10})(27a^6b^9)(a^6)} =$$

$$\frac{16a^{18}b^{19}}{32*27a^{17}b^{19}} = \frac{a}{54}$$

3.)_ If m+2n = -1, express the following as a single fraction in simplified form with no exponents:

$$\frac{3^m9^n10^{6m}5^{12n}(\frac{1}{7})^m}{8^{-4n}49^n}$$

$$\frac{3^m 9^n 10^{6m} 5^{12n} (\frac{1}{7})^m}{8^{-4n} 49^n} =$$

$$\frac{3^m 3^{2n} 2^{6m} 5^{6m} 5^{12n} (2^3)^{4n}}{7^m 7^{2n}} =$$

$$3^{m+2n} 2^{6(m+2n)} 5^{6(m+2n)}$$

$$3^{m+2n} 2^{6(m+2n)} 5^{6(m+2n)}$$

$$\frac{3^{m+2n} 2^{6(m+2n)} 5^{6(m+2n)}}{7^{m+2n}}$$

$$= \frac{3^{m+2n} 10^{6(m+2n)}}{7^{m+2n}}$$

$$= \frac{7^1}{1000000 * 3} = \frac{7}{3000000}$$

$=$

FAIRFIELD COUNTY MATH LEAGUE 20~~17~~18-20~~18~~19

Match 5 Round 2
Algebra I:
Fractional
Expressions and
Equations

1.) $\dfrac{11x+4}{2x+1}$

2.) $5, \dfrac{25}{12}$

3.) $\dfrac{x^2+6}{6x}$

Assume no values of x
make any denominator equal to zero.

1). Express as the quotient of two linear binomials:

$$4+\frac{3}{2+\frac{1}{x}}$$

$$4+\frac{3}{2+\frac{1}{x}}=4+\frac{3}{\frac{2x+1}{x}}=4+\frac{3x}{2x+1}=\frac{4(2x+1)+3x}{2x+1}=\frac{11x+4}{2x+1}$$

2). Solve for all possible values of x:

$$\frac{5}{2x-5}-3=\frac{3x-25}{x}$$

$$\frac{5}{2x-5}-3=\frac{3x-25}{x}$$

$$(2x-5)(x)[\frac{5}{2x-5}-3=\frac{3x-25}{x}]$$

$$5(x)-3(2x-5)(x)=(3x-25)(2x-5)$$

$$5x-6x^2+15x=6x^2-65x+125$$

$$-6x^2+20x=6x^2-65x+125$$

$$12x^2-85x+125=0$$

$$(x-5)(12x-25)=0$$

$$x=5, x=\frac{25}{12}$$

3.) Simplify as much as possible:

$$\frac{(x-1)(x-2)(x-3)-(9-2x)(x+2)}{(4x-3)(x-5)+(2x+5)(x-3)}$$

$$\frac{(x-1)(x-2)(x-3)-(9-2x)(x+2)}{(4x-3)(x-5)+(2x+5)(x-3)}$$

$$\frac{(x^2-3x+2)(x-3)-(-2x^2+5x+18)}{(4x^2-23x+15)+(2x^2-x-15)}$$

$$\frac{(x^3-6x^2+11x-6)+(2x^2-5x-18)}{6x^2+24x} =$$

$$\frac{(x^3-4x^2+6x-24)}{6x^2-24x} = \frac{x^2(x-4)+6(x-4)}{6x(x-4)} = \frac{(x^2+6)(x-4)}{6x(x-4)} = \frac{x^2+6}{6x}$$

$=$

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FAIRFIELD COUNTY MATH LEAGUE (FCML) 201~~7~~-201~~8~~

Match 5 Round 3
Geometry:
Circles

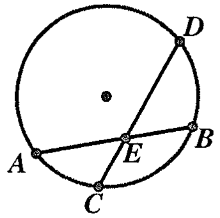
1.) ________30____________

2.) __________62______degrees

Note: Diagrams not necessarily to scale

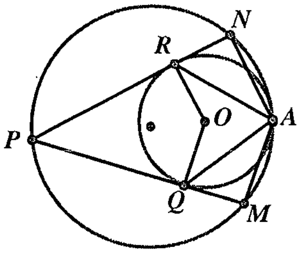
3.) ________ $\frac{65}{8}$ ______________

1.)_ In the picture below, $\overline{AB}$ and $\overline{CD}$ are chords of a circle that intersect at E. AE=x+5, BE=x+3, CE=x+1, and DE=4x. Find the sum AB + CD.



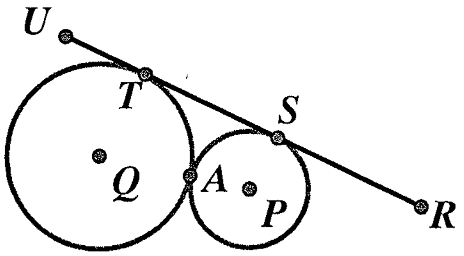
AE*BE=CE*DE. (x+3)(x+5)=4x(x+1), .$x^2 + 8x + 15 = 4x^2 + 4x$, $3x^2 + 4x - 15 = 0$, ((x-3)(3x+5)=0. x=3 since x=$\frac{-5}{3}$ makes (x+1) negative, so AE=6, BE=8 so AB=14. CE=12, DE=4, so CD=16. 14+16 = 30.

2.)_ Two circles are internally tangent at A as shown below. O is the center of the smaller circle. Chord $\overline{PN}$ of the larger circle is tangent to the smaller circle at R and chord $\overline{PM}$ of the larger circle is tangent to the smaller circle at Q. The measure of angle $\angle MAN$ is $124°$. Find the measure of $\angle RAQ$.

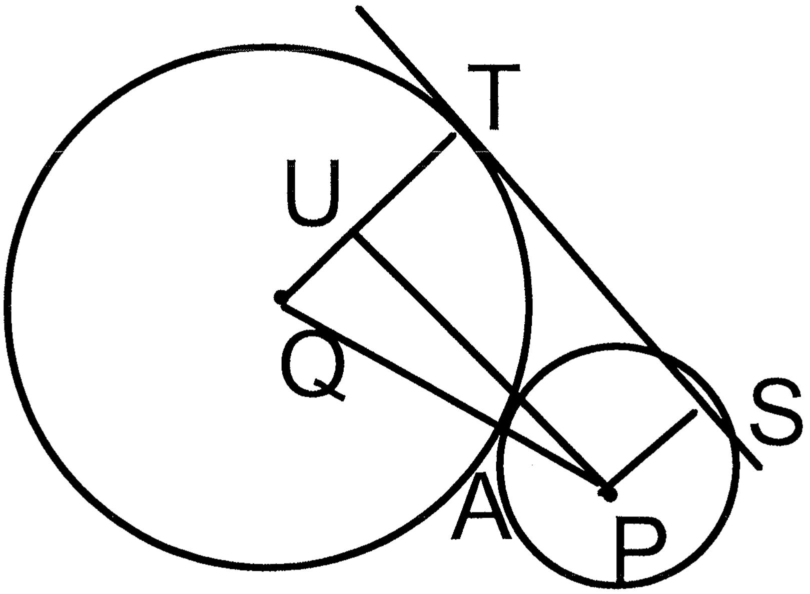


Since PNAM is a quadrilateral whose vertices all lie on the same circle, opposite angles are equal so the measure of angle NPM = 180-124 = 56 degrees. Since $\overline{OR}$ and $\overline{OQ}$ are tangents and quadrilateral OPRQ has angles that add to 360 degrees. Since $\angle ROQ$ has measure $360 - 56 - 90 - 90 = 124$ degrees. That is a central angle for the smaller circle, so arc QR also measures 124 degrees, and $\angle RAQ$ as an inscribed angle has measure of half of the intercepted arc, so it has measure 62 degrees.

3. Two circles with centers P and Q are externally tangent as shown at A. $\overline{RU}$ is tangent to the circle with center P at point S and tangent to the circle with center Q at point T. AP=4 and TS=$\sqrt{130}$. What is the radius of the circle with center Q?



Say the radius of circle Q is x. Draw the line from P perpendicular to $\overline{QT}$. Say it intersects $\overline{QT}$ at U. TUPS must be a rectangle since it has 4 right angles. PU = ST = $\sqrt{130}$. ΔPQU is a right angle with hypotenuse $\overline{QP}$. QU=x-4, QP=x+4, so $(x-4)^2 + (\sqrt{130})^2 = (x+4)^2$, so $x^2 - 8x + 16 + 130 = x^2 + 8x + 16$, so 16x=130, and x=$\frac{130}{16} = \frac{65}{8}$



FAIRFIELD COUNTY MATH LEAGUE 20~~17~~18-20~~18~~19

Match 5 Round 4
Quadratic
Equations and
Complex
Numbers

1.) ______ $-\frac{1}{2}+i$ _________

2.) ______-2, -8.5_________

3.) __ $-\frac{1}{10}+\frac{1}{5}i, \frac{-2}{5}+\frac{1}{5}i$ __

1) Express in the form a+bi: $\frac{3+4i}{2-4i}$

$$\frac{3+4i}{2-4i}=$$

$$\frac{(3+4i)(1+2i)}{2(1-2i)(1+2i)}=$$

$$\frac{-5+10i}{2*5}=\frac{-5+10i}{10}=-\frac{1}{2}+i$$

2) If $(4+ai)(2+bi)=23-14i$ where a and b are real,, give all possible values of $a+b$

$(4+ai)(2+bi)=23-14i$

$(4+ai)(2+bi)=8+(2a+4b)i+abi^2=8+(2a+4b)i-ab$

$8-ab=23\_and\_2a+4b=-14$

$ab=-15\_and\_2a+4b=-14$

$b=\frac{-15}{a},so\_2a+4\frac{-15}{a}=-14$

$2a^2-60=-14a$

$2a^2+14a-60=0$

$a^2+7a-30=0$

$(a+10)(a-3)=0$

$a=-10\_or\_a=3$

$If\_a=-10,-20+4b=-14,b=1.5,a+b=-8.5$

$If\_a=3,6+4b=-14,b=-5,a+b=-2$

3) Solve for all complex z: $(10i)z^2+(4+5i)z+1=0$. Express your answers in a+bi form.

$$(10i)z^2 + (4+5i)z + 1 = 0$$

$$z = \frac{-4-5i \pm \sqrt{(4+5i)^2 - 4*10i*1}}{20i} =$$

$$\frac{-4-5i \pm \sqrt{16+40i-25-40i}}{20i} =$$

$$\frac{-4-5i \pm \sqrt{-9}}{20i} = \frac{-4-5i \pm 3i}{20i}$$

$$= \frac{-4-2i}{20i} \_or \frac{-4-8i}{20i}$$

$$= \frac{-2-i}{10i} \_or \frac{-2-4i}{10i}$$

$$= \frac{(-2-i)(-i)}{(10i)(-i)} \_or \frac{(-2-4i)(-i)}{(10i)(-i)} =$$

$$\frac{(2i-1)}{10} \_or \frac{(2i-4)}{10}$$

$$= -\frac{1}{10} + \frac{1}{5}i, \frac{-2}{5} + \frac{1}{5}i$$

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FAIRFIELD COUNTY MATH LEAGUE 20~~17~~-20~~18~~

Match 5 Round 5
Solving Trig
Equations

1.) _20,~~60~~ 80,140,200,260,320 _degrees

2.) __________ $0, \frac{\pi}{6}, \frac{5\pi}{6}$ __________

3.) __________ $\frac{\sqrt{3} \pm 2\sqrt{2}}{6}$ __________

1) Solve for all x $0° \le x < 360°$: $\tan(3x) = \sqrt{3}$

$$3x = 60, 240, 420, 600, 780, 960$$
$$x = 20, 60, 140, 200, 260, 320$$

2) Solve for all x $0 \le x < 2\pi$ if $\sin(2x) - 2\sin(x) - \cos(x) = -1$

$$2\sin(x)\cos(x) - 2\sin(x) - \cos(x) + 1 = 0$$
$$2\sin(x)(\cos(x) - 1) - 1(\cos(x) - 1) = 0$$
$$(\cos(x) - 1)(2\sin(x) - 1) = 0$$
$$\cos(x) = 1\_or\_\sin(x) = \frac{1}{2}$$
$$x = 0, \frac{\pi}{6}, \frac{5\pi}{6}$$

3.) If $\sin(x+\frac{\pi}{3})=\frac{1}{3}$, what are all possible values for cos(x)?

$$\sin(x+\frac{\pi}{3})=\frac{1}{3}$$

$$\sin(x)\cos\frac{\pi}{3}+\cos(x)\sin\frac{\pi}{3}=\frac{1}{3}$$

$$\sin(x)(\frac{1}{2})+\cos(x)\frac{\sqrt{3}}{2}=\frac{1}{3}$$

$$\frac{1}{2}\sqrt{1-\cos^2 x}=\frac{1}{3}-\frac{\sqrt{3}}{2}\cos(x)$$

$$\frac{1}{4}(1-\cos^2 x)=\frac{1}{9}-\frac{\sqrt{3}}{3}\cos(x)+\frac{3}{4}\cos^2(x)$$

$$\cos^2 x-\frac{\sqrt{3}}{3}\cos(x)+\frac{1}{9}-\frac{1}{4}=0$$

$$\cos^2 x-\frac{\sqrt{3}}{3}\cos(x)-\frac{5}{36}=0$$

$$\cos(x)=\frac{\frac{\sqrt{3}}{3}\pm\sqrt{\frac{1}{3}-4*\frac{-5}{36}}}{2}$$

$$\cos(x)=\frac{\frac{\sqrt{3}}{3}\pm\sqrt{\frac{12}{36}+\frac{20}{36}}}{2}$$

$$\cos(x)=\frac{\frac{\sqrt{3}}{3}\pm\sqrt{\frac{8}{9}}}{2}$$

$$\cos(x)=\frac{\frac{\sqrt{3}}{3}\pm\frac{2\sqrt{2}}{3}}{2}=\frac{\sqrt{3}\pm2\sqrt{2}}{6}$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 20~~17~~18-20~~18~~19

Match 5 Round 6
Sequences and
Series

1.)________5 _______________

2.)_____4, -2 _______________

3.) ____ $-2+2\sqrt{2}$ ________

1.) What is the smallest value of n such that $\sum_{k=1}^{n} \frac{1}{k^2} > 1.45$

$$1+\frac{1}{4}=1.25, 1+\frac{1}{4}+\frac{1}{9}=1.25+.111...=1.36111...,$$

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}=1.36111...+0.625=1.4261111...$$

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}=1.42611..+0.04$$

$$which\_is\_>1.45$$

$$n=5$$

2.) The first term of an arithmetic sequence is 2. The square of the third term is four more than the seventh term. Give all possible values for the fifth term of the sequence.

$$(2+2d)^2=(2+6d)+4$$

$$4d^2+8d+4=6d+6$$

$$4d^2+2d-2=0$$

$$2d^2+d-1=0$$

$$(2d-1)(d+1)=0$$

$$d=\frac{1}{2}\_or\_d=-1$$

$$a_5=4\_or\_-2$$

3. An infinite geometric series converges to 16. The second term of the original geometric sequence is -4. Give all possible values for the third term of the sequence.

$$a_1r=-4,\frac{a_1}{1-r}=16$$

$$\frac{-4}{r(1-r)}=16$$

$$-16r^2+16r+4=0$$

$$4r^2-4r-1=0$$

$$r=\frac{4\pm\sqrt{32}}{8}=\frac{4\pm4\sqrt{2}}{8}=\frac{1\pm\sqrt{2}}{2}$$

If r = $\frac{1+\sqrt{2}}{2}$, $\frac{1+\sqrt{2}}{2}>1$ so the series won't converge.

$$r = \frac{1-\sqrt{2}}{2}$$

If

$$a_3 = -4 * (\frac{1-\sqrt{2}}{2}) = -2 + 2\sqrt{2}$$

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FAIRFIELD COUNTY MATH LEAGUE 201~~7~~-201~~8~~

Match 5 Team Round

1.) 45.5

2.) $2-i,-2+i$

3.) $\frac{-3}{5},1$

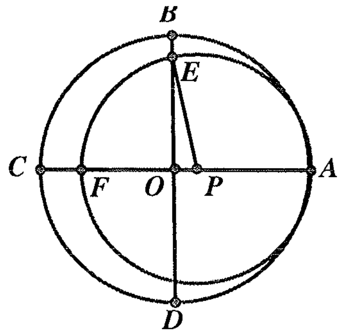
4.) $\frac{-6\pm 6\sqrt{6}}{5}$

5.) $\frac{64}{x^4y^{15}}$

6.) 12

Note: Diagrams not necessarily drawn to scale

1. The circles below with centers O and P are internally tangent at A. B, C, and D are on the larger circle. Both centers lie on $\overline{AC}$ and $\overline{AC}$ is perpendicular to $\overline{BD}$ and both are diameters of the larger circle. A radius is drawn from P to $\overline{BD}$ and intersects $\overline{BD}$ at E. FC=9, OP=4.5, and BE=5. Find the sum of the radii of the two circles.



Find OC + OB. OF = OC-9. Let OE=x. If r is the radius of the smaller circle and R is the radius of the larger circle, then R=OE+5=4.5+r, so $r = OE + \frac{1}{2}$, and $r = \sqrt{(OE)^2 + 4.5^2}$, so

$$(OE + \frac{1}{2})^2 = (\sqrt{(OE)^2 + 4.5^2})^2$$

$$OE^2 + OE + \frac{1}{4} = OE^2 + 20.25$$

$$OE = 20$$

R=5+OE=5+20=25.

$$r = OE + \frac{1}{2}, r = 20 + \frac{1}{2} = 20.5$$

Sum is 25+20.5 = 45.5.

2.) A geometric sequence of complex numbers has second term $3-4i$ and fourth term $-7-24i$. Find all possible values for the first term.

$$-7-24i=(3-4i)r^2$$

$$r^2=\frac{-7-24i}{3-4i}=\frac{(-7-24i)(3+4i)}{(3-4i)(3+4i)}=\frac{75-100i}{25}=3-4i$$

$$\sqrt{3-4i}=(a+bi)$$

$$3-4i=a^2-b^2+2abi$$

$$a^2-b^2=3,2ab=-4$$

$$a=2,b=-1\_or\_a=-2,b=1$$

$$r=2-i\_or\_-2+i$$

3.) If $\cos(x)+2\sin(x)=1$, find all possible values of cos(x).

$$2\sin(x)=1-\cos(x)$$

$$4\sin^2(x)=1-2\cos(x)+\cos^2(x)$$

$$4(1-\cos^2(x))=1-2\cos(x)+\cos^2(x)$$

$$4-4\cos^2(x)=1-2\cos(x)+\cos^2(x)$$

$$5\cos^2(x)-2\cos(x)-3=0$$

$$(5\cos(x)+3)(\cos(x)-1)=0$$

$$\cos(x)=\frac{-3}{5},1$$

4.) Solve for x: $\dfrac{x^2}{x^2-5x+6}-\dfrac{2x^2}{4-x^2}=3$

$$\frac{x^2}{x^2-5x+6}-\frac{2x^2}{4-x^2}=3$$

$$\frac{x^2}{(x-3)(x-2)}-\frac{2x^2}{(2-x)(x+2)}=3$$

$$\frac{x^2}{(x-3)(x-2)}+\frac{2x^2}{(x-2)(x+2)}=3$$

$$[\frac{x^2}{(x-3)(x-2)}+\frac{2x^2}{(x-2)(x+2)}=3](x-3)(x-2)(x+2)$$

$$x^2(x+2)+2x^2(x-3)=3(x-3)(x-2)(x+2)$$

$$x^3+2x^2+2x^3-6x^2=3(x-3)(x^2-4)$$

$$3x^3+2x^2-6x^2=3x^3-9x^2-12x+36$$

$$-4x^2=-9x^2-12x+36$$

$$5x^2+12x-36=0$$

$$x=\frac{-12\pm\sqrt{12^2-4*5*(-36)}}{10}=\frac{-12\pm\sqrt{864}}{10}=\frac{-12\pm12\sqrt{6}}{10}=\frac{-6\pm6\sqrt{6}}{5}$$

5.) If x≠0 and y≠0, express in simplest form with no negative exponents:

$$\frac{(3x)^5(2y^{-5})(9x^3y)^{-4}}{(4y^6)^2(24xy^2)^{-3}}$$

$$\frac{(3x)^5(2y^{-5})(9x^3y)^{-4}}{(4y^6)^2(24xy^2)^{-3}}=$$

$$\frac{3^5x^5 2*2^9*3^3x^3y^6}{y^5 3^8x^{12}y^4 2^4y^{12}}=$$

$$\frac{2^{10}x^8y^6}{2^4x^{12}y^{21}}=\frac{64}{x^4y^{15}}$$

- 6.)_ $\{a_n\}$ is an arithmetic sequence of numbers with $a_3 = 3.2$ and $a_{10} = -7.3$. How many terms of $\{a_n\}$ must be added to get a sum of -24.6 ?

$a_3 = 3.2$

$a_{11} = -7.3$

$a_{10} = a_3 + 7d$

$-7.3 = 3.2 + 7d$

$-10.5 = 7d$

$d = -1.5$

$a_1 = a_3 - 2d = 3.2 - (2)(-1.5) = 6.2$

$$S_n = \frac{n(2*6.2 + (n-1)(-1.5))}{2}$$

$$-24.6 = \frac{n(12.4 + (n-1)(-1.5))}{2}$$

$-49.2 = n(12.4 - 1.5n + 1.5)$

$-37.2 = -1.5n^2 + 13.9n$

$1.5n^2 - 13.9n - 49.2 = 0$

$15n^2 - 139n - 492 = 0$

$(n-12)(15n+41) = 0$

$n = 12$