

FAIRFIELD COUNTY MATH LEAGUE 2018-2019

Match 3 Round 1
Arithmetic: Scientific &
Base Notation

1.) 5

2.) 4

3.) 126

1.) If $\frac{(3 \cdot 10^4)^3 (8 \cdot 10^{-3})}{(6 \cdot 10^5)^2} = a \cdot 10^b$, where $1 \leq a < 10$, find the value of $a + b$.

2.) How many permutations of 0's and 1's, up to three digits, have the property that the base 10 value of its base 3 representation is an integer multiple of the base 10 value of its base 2 representation? (For example, 010 would not, since 10_3 in base 10 is not an integer multiple of 10_2 in base 10.)

3.) If, for positive integers a and b , $123_a - 123_b = 260$ and $a + b = 24$, find the value of bb_a and express the value as a base 10 numeral. (Note: bb represents separate digits and not multiplication.)

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Match 3 Round 2
Algebra 1: Word Problems

1.) 2.4

2.) $\frac{28}{9}$

3.) 25

1.) If at a health food store cashews sell for \$12 per pound and walnuts sell for \$5 per pound, how many pounds of cashews are in a 5-pound mix that sells for \$41.80?

2.) Minnie Vann drives half way to Paytoo Park at an average speed of r miles per hour. She drives the second half at an average speed of $2r$ miles per hour. If it took 7 hours total for her to drive to Paytoo Park, how many hours would it have taken her to drive the entire way at an average speed of $3r$ miles per hour?

3.) Bucket A and Bucket B contain water. After five liters are poured from bucket A to Bucket B without spilling, Bucket B contains three times as much water as bucket A. If the original amount of water in bucket A is *then* poured from bucket B to bucket A without spilling, bucket B will contain 60% of the amount of water in bucket A. How many liters of water were originally in bucket B?

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Match 3 Round 3
Geometry: Polygons

1.) 172

2.) 9

3.) 7, 18

- 1.) If a regular octagon has exterior angles measuring d degrees, what is the measure of one interior angle of a regular d -gon in degrees?

- 2.) A particular n -gon has the property that multiplying the number of sides by one-third the number of sides gives the exact number of diagonals in the n -gon. Find the value of n .

- 3.) A particular regular k -gon has the property that ten times the total degree measure of all of its interior angles is exactly equal to the sum of twenty times the total degree measure of its exterior angles and the product of the number of its diagonals and the degree measure of one of its interior angles. Find all possible values of k .

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Match 3 Round 4
Algebra 2: Functions &
Inverses

1.) $y = \frac{9}{4}x - 15$

2.) $-\frac{2}{3}, \frac{\sqrt{10}}{2}, -\frac{\sqrt{10}}{2}$

3.) Domain: $[2, \infty)$

Range: $(0, \frac{1}{16}]$

Note: the inverse f^{-1} of a function f is not necessarily a function.

1.) If $f(x) = \frac{2}{3}x + 4$, find the equation for the graph of $f^{-1}(f^{-1}(x))$ in slope-intercept ($y = mx + b$) form.

2.) If $h(x) = \frac{3x^2 + 10x + 10}{(3x+5)(2x-1)}$ and (a, b) is a point on the graphs of both h and h^{-1} , find all possible values of a .

3.) If $f(3x + 1) = 4\sqrt{x - 5} + 2$ and $g(x) = \frac{1}{f^{-1}(x)}$, find the domain and range of $g(x)$. If you use inequalities to represent the domain and range, use x for the domain and y for the range.

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Match 3 Round 5
Precalculus: Exponents &
Logarithms

1.) 400

2.) $\frac{3}{8}$

3.) 125, $\sqrt[4]{5}$

1.) If $a^x = 4$ and $\log_a 5 = y$, find a^{2x+2y} .

2.) If $36^{2\log_6 b} = 27^{\log_9 a}$, find $\log_a(b)$.

3.) Solve for all real values of x : $\log_5(x^4) + \log_x\left(\frac{125}{x}\right) = 12$

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Match 3 Round 6
Miscellaneous: Matrices

1.) $\frac{21}{2}$

2.) -12

3.) $\frac{1 \pm \sqrt{43}}{2}, \frac{1 \pm \sqrt{47}}{2}$

1.) If $\begin{bmatrix} 3 & b \\ b & c \end{bmatrix} + 2 \begin{bmatrix} a & a \\ c & d \end{bmatrix} = \begin{bmatrix} 13 & 18 \\ 6 & 20 \end{bmatrix}$, find the value of d .

2.) If $B = \begin{bmatrix} 5 & 1 \\ 8 & 2 \end{bmatrix}$ and $AB = \begin{bmatrix} 7 & 1 \\ 3 & -3 \end{bmatrix}$, find the determinant of A .

3.) If C is a non-singular (invertible) matrix such that $\det(C) = \det(C^{-1})$, and

$C = \begin{bmatrix} k & 2 & k+3 \\ k & 3 & -1 \\ 2 & 1 & k \end{bmatrix}$, find all possible values of k .

Team Round

FAIRFIELD COUNTY MATH LEAGUE 2018-19 Match 3 Team Round

- Domain $[1, 10) \cup (10, \infty)$
- 1.) Range $(0, \frac{1}{6}) \cup (\frac{1}{6}, \frac{1}{3}]$
- 2.) 9
- 3.) $\frac{\pm\sqrt{47}}{2}, \pm\sqrt{5}$
- 4.) -2
- 5.) 17
- 6.) $(\frac{200}{11}, \frac{200}{9})$

- 1.) Find the domain and range of $f(x) = \frac{\sqrt{x-1}-3}{x-10}$. If you use inequalities, use x for the domain and y for the range.
- 2.) Find all possible values of x if $1001_{x+3} + 121_x = 725_{2x-2}$.
- 3.) A and B are matrices such that $AB = BA$. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 2p^2 + 3p & p^2 + q^2 \end{bmatrix}$, find all possible values of q .
- 4.) Find all values of t such that $1 + \log_9(t + 6) = \log_3(4 - t)$.
- 5.) The measure of one interior angle of regular a k -gon is the same as the measure of one exterior angle of a regular m -gon. Find the sum of all possible distinct values of $k + m$.
- 6.) For nonzero numbers x and y , twenty times the average (arithmetic mean) of x and y is equal to the product of x and y . If y is the same as x increased by $y\%$, find the ordered pair (x, y) .