FAIRFIELD COUNTY MATH LEAGUE 2018-2019

Match 2 Round 1
Arithmetic: Factors
And Multiples

1)________15______________

2.) ____0, 5, 8, 9________

3.) _______1830____________

1.)_ How many natural numbers M where 1≤M≤100 can be factored as p^2q where p and q are primes and p≠q ?

M can be 2^2*3, 2^2*5, 2^2*7, 2^2*11, 2^2*13, 2^2*17, 2^2*19, 2^2*23
3^2*2, 3^2*5, 3^2*7, 3^2*11
5^2*2, 5^2*3
7^2*2
8+4+2+1 = 15

2.) N is a whole number 0≤N≤9. For what values of N is the expression $2^N + 3$ not a prime number?

2^0+3 = 4 = 2*2
2^1+3 = 5 prime
2^2+3 = 7 prime
2^3+ 3 = 11 prime
2^4+3 = 19 prime
2^5+3 = 35 =5x7

2^6 + 3 = 67 prime
2^7+3 = 131 prime (check all primes through 11 to see if they divide)
2^8+3 = 259 = 7 x 37
2^9+3 = 515 = 5 x 103

3.) A and B are positive integers. The greatest common factor of A and B is 30. The least common multiple of A and B is 27000. What is the smallest possible value of A+B ?

$30 = 2*3*5$ $27000 = 27x1000 = 3^3*1000= 2^3*3^3*5^3$.
One possibility is A = 30, B = 27000. A+B = 27030.
It could also be
$A = 2^3*3*5$, $B= 2*3^3*5^3$ A=120, B=6750, A+B = 6870
$A = 2*3^3*5$, $B= 2^3*3*5^3$ A = 270, B=3000, A+B = 3270
$A = 2*3*5^3$, $B= 2^3*3^3*5$ A= 750, B=1080, A+B = 1830.

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Match 2 Round 2
Algebra: Polynomials
And Factoring

1.)____231____

2.) ___ $2(x^4+6x^2+1)$_

3.) _ $(2a+3b-2)(2a-3b)$_

1.)_ $(x+2)(3x+4)(5x+6)=ax^3+bx^2+cx+d$. Find a+b+c+d.

$(x+2)(3x+4)=3x^2+10x+8$

$(3x^2+10x+8)(5x+6)=15x^3+18x^2+50x^2+60x+40x+48$

$=15x^3+68x^2+100x+48$

$15+68+100+48=231$

2.) Factor completely over the integers: $(x+1)^4+(x-1)^4$

$(x+1)^4=x^4+4x^3+6x^2+4x+1$

$(x-1)^4=x^4-4x^3+6x^2-4x+1$

$(x+1)^4+(x-1)^4=2x^4+12x^2+2$

$=2(x^4+6x^2+1)$

3. Express as the product of a trinomial and a binomial:

$4a^2-9b^2+6b-4a$

$$4a^2 - 9b^2 + 6b - 4a =$$

$$(2a + 3b)(2a - 3b) + 2(3b - 2a)$$

$$= (2a + 3b)(2a - 3b) - 2(2a - 3b)$$

$$= (2a + 3b - 2)(2a - 3b)$$

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Match 2 Round 3
Geometry:
Area and Perimeter

1.______60_________ cm^2_

2. __$12p - 18\sqrt{3}$__ cm^2

3.______ 23 _________cm

1.) A right triangle has sides of length x cm, (x+7) cm, and (x+9) cm. Find the area of the triangle in cm^2.

$$x^2+(x+7)^2=(x+9)^2$$

$$x^2+x^2+14x+49=x^2+18x+81$$

$$2x^2+14x+49=x^2+18x+81$$

$$x^2-4x-32=0$$

$$(x-8)(x+4)=0$$

$$x=8$$

$$x+7=15$$

$$\frac{1}{2}*8*15=60$$

2.)_ A circle is circumscribed about a regular hexagon which has perimeter $12\sqrt{3}$ cm. Find the area that is inside the circle but outside the hexagon.

Each side of the hexagon has length $2\sqrt{3}$ cm. The hexagon consists of 6 equilateral triangles of side $2\sqrt{3}$ cm, so each triangle has area $\frac{(2\sqrt{3})^2\sqrt{3}}{4} = \frac{12\sqrt{3}}{4} = 3\sqrt{3}$. The total area of the hexagon is $18\sqrt{3}$ cm^2. The radius of the circle is also $2\sqrt{3}$ cm, so its area is $p(2\sqrt{3})^2 = 12p$, so the total area is $12p - 18\sqrt{3}$ cm^2.

3.)_ An isosceles trapezoid has area 30 cm^2. The height of the trapezoid is 6 cm and one base is 5 cm longer than the other base. Find the perimeter of the trapezoid.

$$\frac{1}{2}(b_1 + b_2) * 6 = 30$$
$$b_1 + b_2 = 10$$
$$b_1 - b_2 = 5$$
$$so\_b_1 = 7.5, b_2 = 2.5$$

Since the trapezoid is isosceles, the longer base is broken up into 3 intervals of 2.5 cm each. Since the height is 6 cm, the trapezoid consists of one rectangle of area (2.5)(6) = 15 cm^2 and two right triangles with legs 2.5 cm and 6 cm. The hypotenuse of each triangle is $\sqrt{(2.5)^2 + 6^2} = 6.5$ cm. The total perimeter is 2.5 + 7.5 + 6.5 + 6.5 = 23 cm.

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Match 2 Round 4
Algebra 2: Inequalities
And Absolute value

1)___ $\frac{-9}{5} < x < 3$______

Remember to use AND or OR or
the shorthand notation for a conjunction

if you answer with <. >, ≤. or ≥ .

2.) _____ $\frac{-5}{4}, \frac{3}{2}$______

You may use union and intersection symbols if you answer using interval notation.

3.) __x<-1 or 0<x<1 or x>5____

1.) Solve for x: $\frac{|3-5x|}{2} + 7 < 13$

$$\frac{|3-5x|}{2} + 7 < 13$$

$$\frac{|3-5x|}{2} < 6$$

$$|3-5x| < 12$$

$$-12 < 3 - 5x < 12$$

$$-15 < -5x < 9$$

$$3 > x > -\frac{9}{5}$$

$$\frac{-9}{5} < x < 3$$

2. Solve for x: |5x-2|=|x-4|+3

If x>4, 5x-2=x-4+3, so 4x=1, x=0.25, not in the domain.
If 0.4<x<4, then 5x-2=4-x+3, 6x=9, so x=1.5
If x<0.4, then 2-5x = 4-x+3 so -2=4x+3, so 4x=-5, so x=-1.25

3.) Solve for x: $\frac{3}{x+1} < 1 - \frac{2}{x-1}$

$$Keep\_same\_sense\_if\_x > 1\_or\_x < -1$$

$$3(x-1) < (x^2-1) - 2(x+1)$$

$$3x - 3 < x^2 - 2x - 3$$

$$x^2 - 5x > 0$$

$$x(x-5) > 0$$

$$x < 0\_or\_x > 5\_if\_x > 1\_or\_x < -1, so$$

$$x < -1\_or\_x > 5.$$

$$Change\_sense\_if \ -1 < x < 1$$

$$\frac{3}{x+1} > 1 - \frac{2}{x-1}, so$$

$$x^2 - 5x < 0$$

$$0 < x < 5\_if\_-1 < x < 1, so$$

$$0 < x < 1$$

$$Final\_answer:$$

$$x < -1\_or\_0 < x < 1\_or\_x > 5$$

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Match 2 Round 5
Trigonometry:
Laws of Sine and Cosine

Note: Drawings not necessarily drawn to scale. _

1.) __________ $2\sqrt{6}$ ___________

2.) ______________ $\dfrac{\sqrt{2}}{3}$ _______

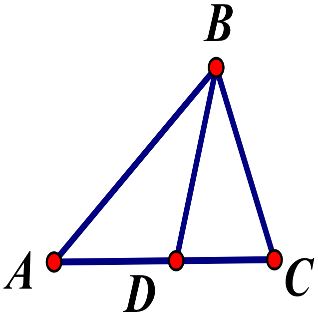
3.)________ $700+400\sqrt{3}$ ____

1.) In ΔXYZ, YZ=4, , $\angle X = 45°, \angle Y = 15°$. Find XY. $\angle Z = 120°$

$$\frac{\sin(120)}{XY} = \frac{\sin(45)}{4}$$
$$XY * \frac{\sqrt{2}}{2} = 4 * \frac{\sqrt{3}}{2} = 2\sqrt{3}$$
$$XY = 2\sqrt{3} * \frac{2}{\sqrt{2}} = 2\sqrt{3}\sqrt{2} = 2\sqrt{6}$$

.

2.) $\overline{BD}$ bisects $\angle ABC$ in ΔABC. If AD=4 CD=6, and $\angle BAD = 45°$, find the sine of $\angle BCD$



.

$$\frac{\sin\angle ABD}{4}=\frac{\sin(45°)}{BD}$$

$$BD*\sin\angle ABD=4*\frac{\sqrt{2}}{2}=2\sqrt{2}$$

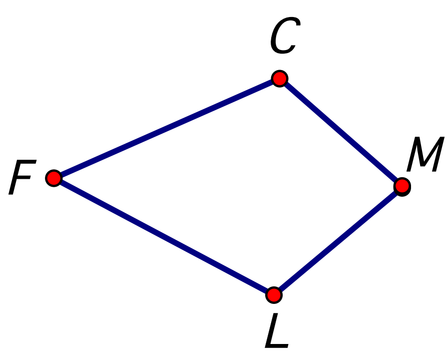
$$\frac{\sin\angle CBD}{6}=\frac{\sin\angle BCD}{BD}$$

$BD*\sin\angle CBD=6*\sin\angle BCD$, but $\sin\angle CBD$=$\sin\angle ABD$,so

$$2\sqrt{2}=6*\sin\angle BCD$$

$$\sin\angle BCD=\frac{2\sqrt{2}}{6}=\frac{\sqrt{2}}{3}$$

3.) In kite FCML, CM=ML, FC=FL, $\angle F=30°$ and $\angle M=150°$. and CM=10. Find $(FC)^2$



Find CL first by law of cosines.

$$CL^2=10^2+10^2-2*10*10*\cos(150°)$$

$$=200-200(-\frac{\sqrt{3}}{2})=200+100\sqrt{3}$$

Then

$$FC = FL, so$$

$$CL^2 = FC^2 + FC^2 - 2*(FC)(FC)\cos(30°)$$

$$200 + 100\sqrt{3} = 2(FC)^2 - 2(FC)^2\frac{\sqrt{3}}{2} = (FC)^2(2 - \sqrt{3})$$

$$FC^2 = \frac{200 + 100\sqrt{3}}{2 - \sqrt{3}} = \frac{(200 + 100\sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{400 + 400\sqrt{3} + 300}{1} =$$

$$700 + 400\sqrt{3}$$

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Match 2 Round 6
Equations of Lines

1.) ___ $y = -\frac{2}{11}x + \frac{58}{11}$ _______

2.) ____ $y = -\frac{3}{4}x + \frac{25}{2}$ __________

3.) _____ $y = \frac{1}{2}x - \frac{11}{2}$ ______

1.) ΔABC has vertices A(7,4), B(5,2), and C(-2,8). Give the equation of the line that contains the median of the triangle from point A. Express your answer as y=mx+b

Midpoint of BC is (1.5, 5) Slope connecting (7,4) and (1.5, 5) is.

$$\frac{5-4}{1.5-7} = \frac{1}{-5.5} = -\frac{2}{11}$$

$$y - 4 = -\frac{2}{11}(x - 7)$$

$$y - 4 = -\frac{2}{11}x + \frac{14}{11}$$

$$y = -\frac{2}{11}x + \frac{58}{11}$$

y

2.) Give the equation of the line tangent to the circle $(x-3)^2+(y-4)^2=25$ passing through (6,8). Express your answer as y=mx+b

Slope of radius connecting center (3,4) to (6,8) is $\frac{8-4}{6-3}=\frac{4}{3}$.

Tangent is perpendicular to radius, to it has slope $-\frac{3}{4}$, so

$$y-8=\frac{-3}{4}(x-6)$$

$$y-\frac{16}{2}=-\frac{3}{4}x+\frac{9}{2}$$

$$y=-\frac{3}{4}x+\frac{25}{2}$$

3.)_ A line intersects the parabola $x=y^2-4$ at the points (k+16, n) and (k, n-8). Find the equation of the line. Express your answer as y=mx+b.

$k+16=n^2-4$, and $k=(n-8)^2 - 4$, so
$(n-8)^2 - 4 + 16 = n^2 - 4$
$n^2 - 16n + 64 - 4 + 16 = n^2 -4$
-16n+76=-4, -16n=-80, n=5.

$k+16=5^2 - 4$, so k=5.
Points of intersection are (5+16,5) and (5,5-8), or (21,5) and (5,-3). Slope of line is $\frac{-3-5}{5-21}=\frac{-8}{-16}=\frac{1}{2}$.

$$y - (-3) = \frac{1}{2}(x - 5)$$

$$y + 3 = \frac{1}{2}x - \frac{5}{2}$$

$$y = \frac{1}{2}x - \frac{11}{2}$$

Team Round

FAIRFIELD COUNTY MATH LEAGUE 2018-19 Match 2 Team Round

1.) ____78, 390________ 4.)______ $5, -5, \sqrt{7}, -\sqrt{7}$ ____

2.) _ $x(x+2)(x^2+x-1)$ _ 5.) _________ $\frac{2}{3}$ ______

3.) _ $x < -2$ _or_ $2 < x < 6$ _ 6.) ___ $y = \frac{11}{2}x + 40$ __________

1.)_The greatest common factor of N and 702 is 78 . The least common multiple of N and 1755 is 3510. Find all possible values of N.

78 = 13*3*2, so N has at least 13*3*2. 702 = 13*3^3*2, so N must have a factor of 13 and exactly one factor of 3 and exactly one factor of 2. 1755 = 13*3^3*5, and 3510 is 13*3^3*2*5, so N could have the 5, or the 5 could come from the 1755. N could be 13*3*2=78. N could be 13*3*2*5=390.

Answers are 78, 390.

2.) Factor as the product of a monomial, binomial, and trinomial with integer coefficients:

$$x^4+3x^3+x^2-2x$$

$$x^4+3x^3+x^2-2x=$$

$$x(x^3+3x^2+x-2)=$$

$$x(x^3+2x^2+x^2+x-2)$$

$$=x((x^2(x+2)+(x+2)(x-1))$$

$$=x(x+2)(x^2+x-1)$$

$x(x^3+3x^2+x-2)=$

$x(x^3+2x^2+x^2+x-2)$

$=x((x^2(x+2)+(x+2)(x-1))$

$=x(x+2)(x^2+x-1)$

3.)_ Solve for all values of x: $2x^3-12x^2-8x+53<5$

$$2x^3-12x^2-8x+53<5$$

$$2x^3-12x^2-8x+48<0$$

$$2(x^3-6x^2-4x+24)<0$$

$$2(x^2(x-6)-4(x-6))<0$$

$$2(x^2-4)(x-6)<0$$

$$2(x+2)(x-2)(x-6)<0$$

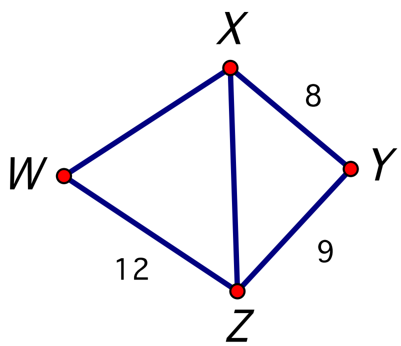
If x>6, all three linear factors are positive, so the expression is not less than zero.
If 2<x<6, then x-6 is negative while the others are positive, so the expression is less than 0.
If -2<x<2, two of the linear factors are negative, so the expression is not less than 0.
If x<-2, all three linear factors are negative, so the expression is less than 0.
Solution: x<-2 or 2<x<6

4.) Two adjacent sides of a rectangle have lengths |x+4| cm and |x-4| cm. The area of the rectangle is 9 cm^2. Find all possible values for x.

If x>4, (x-4)(x+4)=9, so x^2-16=9, x=5. If x<-4, (-x-4)(-x+4)=9, so x^2-16=9, and x=-5. If -4<x<4, then (x+4)(4-x)=9, 16-x^2 = 9, -x^2 = -7, so x^2=7, x= $\sqrt{7}, -\sqrt{7}$

. Solution: $5, -5, \sqrt{7}, -\sqrt{7}$

5.) In quadrilateral WXYZ, XY=8, YZ=9, ZW=12, $\angle Y = 60°$. The total area of the quadrilateral is $18\sqrt{3} + 4\sqrt{73}$. Diagonal $\overline{XZ}$ is drawn. Find the sine of $\angle XZW$.



Use the area formula $Area = \frac{1}{2}ab * \sin(C)$ for a triangle from the unit on Law of Sines and Cosines. The area of ΔXYZ =

$\frac{1}{2} * 8 * 9 * \sin(60) = \frac{1}{2} * 8 * 9 * \frac{\sqrt{3}}{2} = 18\sqrt{3}$.

Find XZ by $XZ^2 = 9^2 + 8^2 - 2*8*9*\cos(60) = 81+64-72$, so $XZ = \sqrt{73}$. Use the same formula on ΔWXZ with area =

$\frac{1}{2} * 12 * \sqrt{73} * \sin(\angle XZW) = \sin(\angle XZW) = \frac{2}{3} * \sin(\angle XZW)$.

Equate the total area $18\sqrt{3} + 4\sqrt{73}$ =

$18\sqrt{3} + 6\sqrt{73} * \sin(\angle XZW)$, so

$\sin(\angle XZW) = \frac{2}{3}$.

6. In ΔABC, Point A lies in the second quadrant and point C lies in the third quadrant. $\overline{AB}$ is contained in the line $3x+4y=10$ and $\overline{BC}$ is contained in the line $4x-3y=5$. $\overline{BC}$ has twice the length of $\overline{AB}$ and the area of ΔABC is 100. Find the equation of the line that contains. Express your answer as y=mx+b.

Solve the system 3x+4y=10 and 4x-3y=5 to get point B (2,1). Since the slopes of the two lines are $\frac{-3}{4}$ and $\frac{4}{3}$, $\overline{AB}$ is perpendicular to $\overline{BC}$ and the triangle is a right triangle with legs $\overline{AB}$ and $\overline{BC}$, so its area $100=\frac{1}{2}(AB)(BC)$ but $BC=2(AB)$, so 100=$(AB)^2$ and AB = 10. Combine the fact that the slope of the line 3x + 4y =10 has slope $\frac{-3}{4}$, and the distance from A to B is 10 = so go back 8 and up 6 from (2,1) since $\sqrt{(-8)^2+6^2}$ =10 and A has coordinates (-6, 7). To find point C, combine the fact that the slope of 4x-3y=5 is $\frac{4}{3}$ and the distance to C is $20=\sqrt{(-12)^2+(-16)^2}$ to go back 12 and down 16 from (2,1) to get to the coordinates of point C (-10, -15). AC goes through (-6,7) and (-10,-15), so it has slope

$$\frac{7-(-15)}{(-6)-(-10)}=\frac{22}{4}=\frac{11}{2}$$

Then use point slope form with A or C.

$$y + 15 = \frac{11}{2}(x + 10)$$

$$y + 15 = \frac{11}{2}x + 55$$

$$y = \frac{11}{2}x + 40$$