CSAML 2017

April 6, 2017
Round 1: Arithmetic and Number Theory

1. (1 point) Find the positive difference, in seconds, between two and a half minutes and 10% of an hour.

2. (2 points) Suppose that A, B, C are positive integers such that $A+\cfrac{1}{B+\cfrac{1}{C}}=\frac{29}{13}$.

Compute $A+B+C$.

3. (3 points) Find the sum of all positive two digit numbers that have the property that the value of the number is the same as three times the product of the digits in the number.

1) ______________________________

2) ______________________________

3) ______________________________

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Round II: Algebra I, (Real numbers and no transcendental functions)

1.(1 point) Solve: $\left(\sqrt[3]{x-2}\right)^2 = 4$.

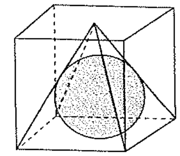
2. (2 points) Solve for x: $\dfrac{x^{-2}+1}{x^{-3}-x^{-4}} = \dfrac{-5x}{x^{-1}-1}$.

3. (3 points) Let S be the set of real numbers y such that $\sqrt{y} = 5\sqrt{x} - 2x$ for some real number x. Write S using interval notation.

1) ____________________________

2) ____________________________

3) ____________________________

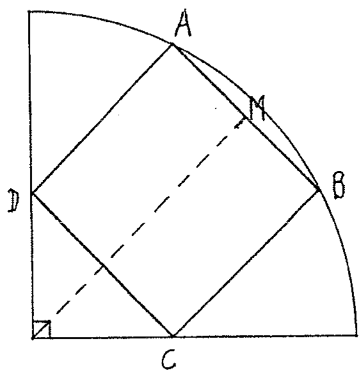


CSAML 2017

April 6, 2017
Round III: Geometry (figures are not to scale)

1. (1 point) Determine the number of sides of a regular polygon where the sum of two of its interior angles is 336°.

2. (2 points) The diagram shows square $ABCD$ inscribed in a 90° sector of a circle. Points A and B lie on the circle and points C and D lie on the perpendicular radii. M is the midpoint of AB. Each side of the square has length 6 meters, and the area of the region bounded by the entire circle is $k\pi$ square meters. Find the value of k.



3. (3 points) Let $\overline{AB}$ be a diameter of a circle and C be a point on $\overline{AB}$ with $2 \bullet AC = BC$. Let D and E be points on the circle such that $\overline{DC} \perp \overline{AB}$ and $\overline{DE}$ is a second diameter. What is the ratio of the area of triangle DCE to the area of triangle ABD?

1) ______________________________

2) ______________________________

3) ______________________________

April 6, 2017
Round IV: Algebra II

1. (1 point) Find the value of $\log_2\left(\sqrt{21}-\sqrt{5}\right)+\log_2\left(\sqrt{21}+\sqrt{5}\right)$.

2. (2 points) Find the largest x so that $|x|<\frac{1}{2}$ and $x^4+x^3-3x^2+x=0$.

3. (3 points) The parabola $y=ax^2+bx+c$ intersects the y-axis at (0, 8) and the x-axis in a single point (2, 0). How many pairs of integers (m, n) with -2000 < n < 2000 lie on the parabola?

1) ______________________________

2) ______________________________

3) ______________________________

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Round V: Analytic Geometry

1. (1 point) Let $f(x)=x^2+10x+5$. Express, as an ordered pair, the vertex of the graph of $y=f^{-1}(x)$.

2. (2 points) Q is the point of the circle $x^2-10x+y^2+6y+29=0$ which is furthest from the point P(-1, -6). Determine the distance PQ.

3. (3 points) The lines given by $y=x$ and $y=3x$ form an acute angle in the first quadrant. What is the slope of the line that bisects that angle?

1) ______________________________

2) ______________________________

3) ______________________________

April 6, 2017
Round VI: Trigonometry, Complex Numbers

1. (1 point) If $\sin t = \frac{4\sqrt{5}}{9}; 0^o < t < 90^o$, compute the value of $\log_3 \sin t + \log_3 \cot t$.

2. (2 points) Suppose that $\tan 2x = \frac{5}{12}$. Find all possible values of $\tan^2 x + \cot^2 x$.

3. (3 points) The equation $\left(2\sin x - \sqrt{3}\right)^2 + \left(2\cos x - 1\right)^2 + \left(\tan x - \sqrt{3}\right)^2 = 0$ has exactly 8 solutions in the range of $0^O < x \le \mu^O$. Find the smallest possible value of μ.

1) ______________________________

2) ______________________________

3) ______________________________

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April 6, 2017 TEAM ROUND

1) Let N be a 3-digit base ten positive integer whose middle digit is 0. N is a multiple of 11 and the quotient $\frac{N}{11}$ equals the sum of the squares of the digits of N. Determine N.

2) Determine a value for c.

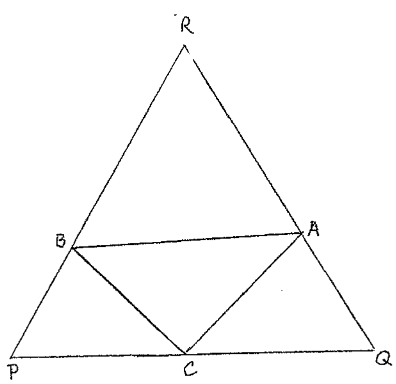
$$\begin{cases} \dfrac{ef-2d}{5}=18 \\ \dfrac{8cf+10d}{10f}=22 \\ \dfrac{9d-6cf}{15f}=54 \end{cases}$$

3) In $\Delta ABC, AB = AC = 65$ and the area of triangle ABC is 1848. Find the maximum possible length of the altitude from A to $\overline{BC}$.

4) Determine the sum of the y-coordinates of the three points of intersection of $y = x^2 - x - 5$ and $y = \frac{1}{x}$.

5) If the line $y = mx + 1$ intersects the conic $x^2 + 4y^2 = 1$ exactly once, find all possible values of m.

6) As shown in the diagram, right triangle ABC, with right angle C, is inscribed in equilateral triangle PQR. If PC = 3, BP = CQ = 2, compute AQ.



CSAML 2017
E.O.Smith H.S. (host)
April 6, 2017

Round I

1) 210

2) 9

3) 39

Round II

1) 10, -6

2) 2, -2

3) $\left[0, \frac{625}{64}\right]$

Round III

1) 30

2) 90

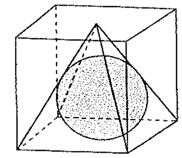
3) $1:3$ (or $\frac{1}{3}$)

Round IV

1) 4

2) $-1+\sqrt{2}$

3) 63



Round V

1) (-20, -5)

2) $4\sqrt{5}$

3) $\frac{1+\sqrt{5}}{2}$

Round VI

1) -2

2) $\frac{626}{25}$

3) 2580° or 2580

TEAM Round

1) 803

2) $\frac{-510}{11}$

3) 56

4) -5

5) $\pm\frac{\sqrt{3}}{2}$

6) $\frac{8}{5}$

Question Writing:
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