

Name \_\_\_\_\_ School \_\_\_\_\_ Grade \_\_\_\_\_

1) Suppose  $f(x) = x^4 - x^3 + ax + b$ ,  $f(1) = 4$  and  $f(2) = 6$ . What is the ordered pair  $(a, b)$ ?

2) Triangle  $ABC$  has a right angle at  $C$ , and  $\angle A = 20^\circ$ . If  $D$  lies on  $AC$ , and  $BD$  bisects angle  $B$ , how many degrees are in  $\angle BDA$ ?

3) The sequence 2, 5, 10, 50, 500, ... is formed so that each term after the second is the product of the two previous terms. The 15<sup>th</sup> term ends with exactly  $k$  zeros. What is the value of  $k$ ?

4) In the diagram, the rectangle is divided into nine smaller rectangles. The areas of five of these rectangles are given. Determine the area of the rectangle labeled  $R$ .

3	1	
	2	R
5		10

5) Determine all pairs  $(a, b)$  of positive integers for which  $a^2 + 2ab = 2013$ .

6) Let  $E$  be the event that in two flips of a fair coin one is Head and one is Tail. Let  $F$  be the event that in four flips of a fair coin two are Heads and two are Tails. Let  $G$  be the event that in five flips of a fair coin three are Heads and two are Tails. List these events ( $E$ ,  $F$ , and  $G$ ) in order of increasing probability (smallest to largest).

7) Suppose that, for some angles  $x$  and  $y$ ,  $\sin^2 x + \cos^2 y = \frac{3}{2}a$  and  $\cos^2 x + \sin^2 y = \frac{1}{2}a^2$ . Determine the possible value(s) of  $a$ .

8) The base of a triangle is 80, and one of the base angles is  $60^\circ$ . The sum of the lengths of the other two sides is 90. Find the length of the shortest side.

9) The distance between the centers of two circles is 15. One has radius 4 and the other has radius 5. What is the length of their common internal tangent?

10) Simplify  $\frac{3+i}{(1-i)^3}$ . Write you answer in  $a + bi$  form.

11) If  $\sin x + \cos x = 1.2$ , find  $\sin 2x$ .

12) The circle  $x^2 + y^2 = 17$  intersects line  $\ell : y = x + 3$  in two points  $(x_1, y_1)$  and  $(x_2, y_2)$ . Let  $P$  be the product  $\frac{1}{x_1} \cdot \frac{1}{x_2}$ , let  $J$  be the sum  $x_1 + x_2$ , and let  $K$  be the slope of a line perpendicular to  $\ell$ . Compute the **product**  $PJK$ .

<b>Answers</b>		
1)	2)	3)
4)	5)	6)
7)	8)	9)
10)	11)	12)

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13) In an arithmetic sequence, the sum of the first and third term is 6 and the sum of the second and fourth terms is 20. Determine the tenth term in the sequence.		
14) If $2a = 3b + 5$ , what is $\frac{4^a}{8^b}$ ?		
15) For how many positive real values of $x$ does $\log_4 x = 2 \sin x$ ?		
16) Square $ABCD$ has sides of length 3. Side $AB$ is extended through $B$ to $E$ with $BE = 1$ . Segment $DE$ intersects $BC$ at point $F$ . What is the area of triangle $CDF$ ?		
17) Determine all positive real numbers $x$ for which $\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$ .		
18) Let $P(x)$ be a polynomial of degree four such that $P(2) = P(-2) = P(-3) = -1$ and $P(1) = P(-1) = 1$ . Find $P(0)$ .		
19) Simplify: $\sqrt{19 + \sqrt{297}} - \sqrt{19 - \sqrt{297}}$ .		
20) In triangle $ABC$ point $F$ lies on $AC$ with the ratio $AF:FC = 2:3$ . Point $D$ is the midpoint of $BF$ , and $AD$ is extended to meet $BC$ at point $E$ . What is the ratio of $BE:EC$ ?		
21) If $\sin x + \cos x = \frac{1}{3}$ , what is the value of $\sin^3 x + \cos^3 x$ ?		
22) Define a function $f$ on the set of integers by $f(n) = \begin{cases} n-1 & \text{if } n \text{ is even} \\ n^2-1 & \text{if } n \text{ is odd.} \end{cases}$ List all values of $n$ for which $f(f(n)) = 8$ .		
23) Find the smallest positive integer which is divisible by all 1-digit primes but has no prime digits.		
24) EOSmith juniors and seniors were polled on the question "Do you believe there is life on Mars?" An equal number of juniors and seniors responded, and every respondent answered "yes" or "no". If 60% of those who said "yes" were seniors, and 80% of those who said "no" were juniors, then what percentage of the juniors polled said "yes"?		
<b>Answers</b>		
13)	14)	15)
16)	17)	18)
19)	20)	21)
22)	23)	24)