2014 CT-ARML runoff **PLEASE PRINT ALL INFO** **Name** ____________________

Round 1 (1 hour) **NO CALCULATORS** **School** ______________

1) What is the sum of the digits of $(1010101)^2$?

2) A student takes three exams. The second has twice as many questions as the first, and the third has three times as many questions as the first. The student answers exactly 75% of the question correctly on the first exam, exactly 81% on the second exam, and exactly 85% on the third exam. Out of all the questions on the three exams, what percent did he answer correctly?

3). List all positive solutions of $x^3 + x^2 - 2x - 2 = 0$.

4) Find all possible ordered pairs (A, B) of digits for which the decimal number $\underline{7}\,\underline{A}\,\underline{8}\,\underline{B}$ is divisible by 45.

5) A sequence satisfies $a_1 = 3$, $a_2 = 5$ and $a_{n+2} = a_{n+1} - a_n$ for $n \geq 1$. What is the value of a_{2013}?

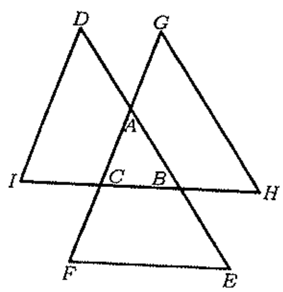
6) Find the ordered triple (a, b, c) of positive integers which satisfies both $(a + b)(a + c) = 77$ and $(a + b)(b + c) = 42$.

7) How many positive integers less than 1200 have no repeating digits; i.e. no digit occurs more than once.

8) The sum of the first ten terms of a nonzero geometric series is 244 times the sum of the first five terms. What is the common ratio?

9) Five circles of equal radius are placed inside a square of side length 1 in such a way that no two intersect in more than one point. What is the largest possible radius for these circles?

10) In the diagram, DA = AB = BE, GA = AC = CF, and IC = CB = BH. If EF = 5, DI = 6, and GH = 7, what is the area of triangle ABC?



11) Assume that for every person the probability that they have exactly one child is ¼, the probability that they have exactly 2 children is ½, and the probability that they have exactly three children is ¼. What is the probability that a person will have exactly four grandchildren?

12) If $x = 2 + \sqrt{3}$, find an integer or fraction equal to $x^4 + \frac{1}{x^4}$.

Answers		
1)	2)	3)
4)	5)	6)
7)	8)	9)
10)	11)	12)

2014 CT-ARML runoff
Round 2 (1 hour)

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NO CALCULATORS **School** ______________

13)) Let S = {3, 4, 5, 6, 8, 9, 10, 11, 12}. What is the sum of the elements in S which are divisors of 7021500420?

14) What is the largest base-10 number whose base-3 and base-4 expansions have the same number of digits?

15) Find the area of triangle ABC satisfying the following properties. AC = 13 and CM = $4\sqrt{10}$ where M is the midpoint of AB. The altitude from C does not meet AB itself, but it meets the extension of AB, and this altitude has length 12.

16) The bases of a trapezoid have lengths 12 and 15. Find the length of the segment parallel to the bases which passes through the point of intersection of the diagonals and extends from one side to the other.

17)) How many ordered pairs of integers (x, y) satisfy
$x + y - xy = 49$?

18) Compute the number of ordered 4-tuples (a, b, c, d) of positive integers such that
$a + b + c + d = 14$.

19)) List all numbers which can be written as $x + y$ where x and y are positive integers satisfying $x^4 + y^4 = 15266$.

20)) Out of all positive integers n which have exactly 2107 positive divisors, what is the largest number of positive divisors that n^2 could have?

21) A rectangle is placed inside an isosceles right triangle in such a way that two vertices of the rectangle are on the hypotenuse, and the other two vertices lie, one on each leg. The area of the original triangle is 2(units)^2, and the area of the rectangle is one quarter of that. In the figure thus created, find the area of the triangle which does not touch the hypotenuse of the larger, isosceles triangle.
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22) In triangle ABC, C is a right angle and M is on $\overline{AC}$. A circle with radius r is centered at M, is tangent to $\overline{AB}$, and is tangent to $\overline{BC}$ at C. If AC = 5 and BC = 12, compute r.

23) The product of the first five terms of a geometric progression is 32. The fourth term is 17, compute the second term.

24) Solve for θ, $0° \le \theta < 360°$: $\tan^2\frac{\theta}{2}(1+\cos\theta)+\cos 2\theta = 0$.

Answers		
13)	14) .	15)
16)	17)	18)
19)	20)	21)
22)	23)	24)