

**FAIRFIELD COUNTY MATH LEAGUE 2017-2018**

Match 4 Round 1  
Arithmetic:  
Basic Statistics

1.) 3.286

2.) 819

3.) 20

1.) The scores of an AP test for a class are reported as follows: Six 5's, ten 4's, nine 3's, eight 2's, and 2 1's. Give the mean score rounded to three decimal places.

2.) The upper quartile of a set of data is the median of the upper half of the data and the lower quartile is the median of the lower half of the data. The inter-quartile range is the upper quartile minus the lower quartile. Give the interquartile range of the set of the twelve smallest perfect cubes of natural numbers.

3.)\_ The Math Team bus costs \$200 for one match. X students are planning to come to the match. If 4 of those students decide not to go, the average (mean) cost per student increases by \$2.50. Find X.

## FAIRFIELD COUNTY MATH LEAGUE 2017-2018

Match 4 Round 2  
Algebra 1:  
Quadratic  
Equations

1.) 3, 5, -3, -5

2.)  $96x^2 - 44x - 35 = 0$

3.)  $2, \frac{5}{m}$

1.)\_ Find all integer values of  $k$  such that  $(x+k)^2 = k^2 + kx - 2.25$  has rational solutions.

2.) Find a quadratic equation whose solutions are  $-\frac{5}{12}$  and  $\frac{7}{8}$ .

Express your answer as  $ax^2 + bx + c = 0$ , where  $a > 0$  and  $a, b, c$  are relatively prime integers.

3. Give two solutions to the quadratic equation  $mx^2 - 2mx + 4 = 5x - 6$ , one involving  $m$  and one not involving  $m$ .

**FAIRFIELD COUNTY MATH LEAGUE 2017-2018**

Match 4 Round 3  
 Geometry:  
 Similarity

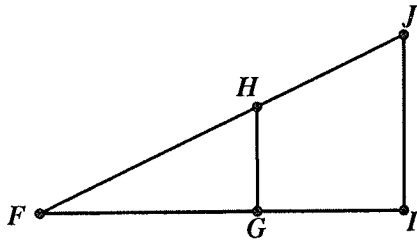
Note: Diagrams are not  
 Necessarily drawn to scale

1.)           4          

2.)            $\frac{16\sqrt{10}}{3}$            cm

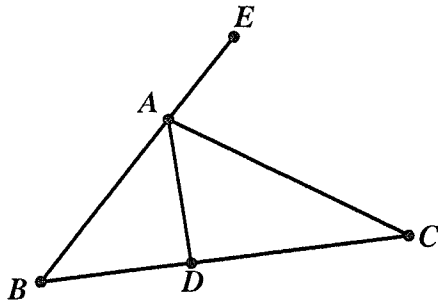
3.)           18           cm

1.)  $\overline{GH}$  is parallel to  $\overline{JI}$ .  $FI=5x+4$ ,  $GI=3x-2$ ,  $IJ=5x-8$ , and  $GH=2x-1$ .  
 Find  $x$ .



2.) The area of a regular octagon with side  $x$  is  $(1 + \sqrt{2})x^2$ . The ratio of the sides of two regular octagons is 3:2. The area of the larger octagon is  $10 + 10\sqrt{2}$  cm<sup>2</sup>. What is the perimeter of the smaller octagon?

3.) In the diagram below,  $\angle EAC = \angle CAD$ .  $AB=10$  cm,  $BD=12$  cm,  $AD=6$  cm. Find  $DC$ . (Hint: Draw a line through  $D$  parallel to  $\overline{AC}$ .)



## FAIRFIELD COUNTY MATH LEAGUE 2017-2018

Match 4 Round 4  
Algebra 2:  
Variation

1.)  $\frac{4.41}{\quad}$

2.)  $\frac{-22}{27}$

3.)  $k = \frac{1}{4}$   $n = \frac{1}{2}$

1.)  $p$  varies directly with the square root of  $q$ , and  $q$  varies inversely with  $r$ . If  $p=7$  when  $r=9$ , what is  $r$  when  $p=10$ ?

2.)  $(z+2)$  varies inversely with the cube of  $w$ . If  $z=2$  when  $w=4$ , what is the value of  $z$  when  $w=6$ ?

3.) The ordered pair  $(64,2)$  belongs to the function  $y = kx^n$  and the ordered pair  $(64,8)$  belongs to the function  $y = k^3x^{n+1}$ . If  $k$  is positive, find the values of  $k$  and  $n$ .

## FAIRFIELD COUNTY MATH LEAGUE 2017-2018

Match 4 Round 5  
Trig Expressions  
and DeMoivre's  
Theorem

1.)  $8i$  ( $0+8i$  acceptable)

2.)  $\sec 2\theta$  ( $1 \cdot \sec 2\theta$  acceptable)

3.)  $128$

1.) Express the complex number  $(2(\cos(\frac{5\pi}{6}) + i\sin(\frac{5\pi}{6})))^3$  in the form  $a+bi$ .

2.) Express the following in the form  $a[f(x)]^n$  where  $a$  and  $n$  are constants,  $n > 0$ , and  $f(x)$  is a trig function of  $x$ .

$$\frac{(1 + \csc \phi)(1 - \csc \phi)(1 - \sec \phi)(1 + \sec \phi)}{(1 + \sin \phi)(1 - \sin \phi)}$$

3. If  $\cos(8\theta)$  is expressed in terms of  $\cos(\theta)$ , what is the coefficient of  $\cos^8(\theta)$ ?

**FAIRFIELD COUNTY MATH LEAGUE 2017-2018**

Match 4 Round 6  
Conics

1.)  $(3, \frac{6}{5})$

2.)  $(3, 4)$

3.)  $\frac{2\sqrt{273}}{3}$

1.)\_ A parabola has vertex (3,1), opens upward, and passes through the point (5,6). Give the location of the focus of the parabola as an ordered pair.

2.) A circle and a hyperbola are centered at the origin. The hyperbola has equation  $\frac{x^2}{9} - \frac{y^2}{16} = 1$ . The circle passes through the two foci of the hyperbola. What are the coordinates of the point in the first quadrant where the circle intersects one of the asymptotes of the hyperbola?

3.) An ellipse is centered at (0,0) and passes through the points (3,4) and (5,2). If the foci are on one of the axes, give the sum of the distances from a point on the ellipse to each of the foci.

**FAIRFIELD COUNTY MATH LEAGUE 2017-2018 Match 4 Tm Round**

1.) 79

4.) 
$$\frac{-30\sqrt{6} + 73}{77}$$

2.) 
$$\frac{54 + 18\sqrt{3}}{18 + 6\sqrt{3}}$$

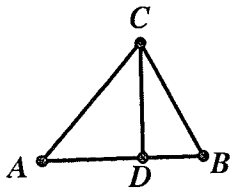
5.) 
$$\frac{40}{3}$$

3.) 
$$(-3, 2), (15, 8), (15, -4)$$

6.) 
$$\frac{41}{400}$$

1.)\_ There is a set of 5 consecutive prime numbers, all of which are less than 100, for which the mean of the numbers is equal to the median of the numbers. Give the median of these numbers.

2.)\_ In the diagram below, not necessarily drawn to scale,  $\triangle ABC$  is a right triangle. The altitude from C intersects  $\overline{AB}$  at D.  $AD=4x-9$ ,  $BD=9$ , and  $CD= x\sqrt{3}$ . Give all possible values for the perimeter of  $\triangle ABC$ .



3.) A parabola has focus at  $(-2.5, 2)$  and its directrix is the line  $x = -3.5$ . Find all points of intersection with the parabola and the circle with equation  $x^2 - 14x + y^2 - 4y = 47$ .

4.) A is an angle in the first quadrant and B is an angle in the second quadrant.

$\sec(A) = \frac{7}{5}$  and  $\csc(B) = \frac{11}{5}$ . Find  $\cos(A + B) - \sin(A - B)$ .

5.) The acceleration due to gravity on the surface of a spherical planet is directly proportional to its mass and inversely proportional to the square of its radius. The density of an object is its mass divided by its volume. The volume of a sphere is directly proportional to the cube of its radius. Suppose acceleration due to gravity of earth is 10 meters/second<sup>2</sup>. What is the acceleration due to gravity in meters/second<sup>2</sup> on the surface of a spherical planet that has twice the radius of the earth and its density is two-thirds of the density of the earth?

6.) An ellipse with equation  $Px^2 + Qy^2 = 1$  has foci at  $(3,0)$  and  $(-3,0)$  and passes through the point  $(\frac{5\sqrt{5}}{3}, \frac{8}{3})$ . Find the sum  $P+Q$ .