

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 5 Round 1

Algebra I:  
Fractions and  
Exponents

1.) 3.14159

2.) 1296

3.)  $96k^7p$

1) Express the following as a decimal, correct to 5 decimal places:

$$3 + \frac{1}{7 + \frac{1}{16}}$$

2.) Express as an integer or a reduced fraction:

$$(36)^5 (18)^{-4} (12)^3 (6)^{-2} (3)^1 (2)^{-6}$$

3.) If  $k \neq 0$  and  $p \neq 0$ , simplify the following as much as possible. Do not leave any negative exponents in your answer:

$$\frac{(4k^2)^{2p+1} (3p)^{2-3k}}{(2k)^{4p-3} (27p^3)^{-k+1}} (3kp)^2$$

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 5 Round 2  
Algebra I:  
Fractional  
Expressions and  
Equations

1.)  $\frac{2(2x+3)}{3(x+3)}$  accept  $\frac{4x+6}{3x+9}$

2.)  $-2, 1$

3.)  $\frac{y^4 + 3xy^2 + x^2}{y^3 + 2xy}$

1). Simplify the product as much as possible if no values of x make any denominators equal to zero:  $\frac{x^2 + 7x - 30}{3x^2 - 27} * \frac{8x^2 + 24x + 18}{2x^2 + 23x + 30}$

2). Solve for all possible values of x:

$$4 - \frac{9}{x+5} = \frac{3x+7}{x+3}$$

3). Simplify as much as possible given that  $x > 0$  and  $y > 0$ . Express your answer as a single fraction.

$$y + \frac{x}{y + \frac{x}{y + \frac{x}{y}}}$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 5 Round 3  
 Geometry:  
 Circles

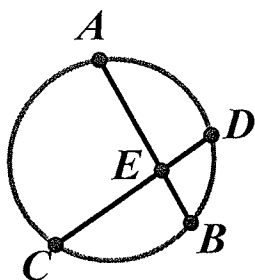
Note: Diagrams not necessarily to scale.

1.) 11

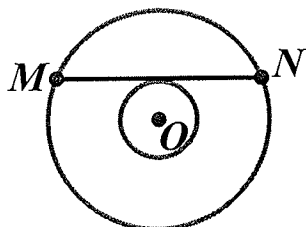
2.)  $225\pi$   $\text{cm}^2$

3.)  $4\sqrt{6}$   $\text{cm}$

1). Points A, B, C, and D lie on a circle.  $\overline{AB}$  intersects  $\overline{CD}$  at point E.  $AE=6$ ,  $EB=4$ ,  $CE=x$ ,  $DE=x-5$ . Find the length of  $\overline{CD}$ .

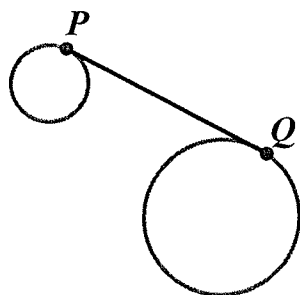


2). Two circles are concentric. Points M and N lie on the larger circle and  $\overline{MN}$  is tangent to the smaller circle. If  $MN = 30$  cm, find the area between the two circles.



3.) A segment is tangent to two circles as shown at points P and Q. The radius of the smaller circle is 2 cm and the radius of the larger circle is 4 cm.

The distance between the two centers is 10 cm. Find the length of  $\overline{PQ}$



# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 5 Round 4  
Quadratic  
Equations and  
Complex  
Numbers

1.)         -64        

2.)         169        

3.)          $\frac{-i}{4}, \frac{1}{3}$         

1) Simplify:  $\frac{(i - 3i^3)^3}{i^{33}}$

2)  $a + bi = \frac{(5 + 12i)}{(3 + 2i)}(3 - 2i)^3$ . If  $a + bi$  is plotted on the complex plane, how far is it from the origin?

3) Solve for all complex  $z$ :  $12iz^2 - (3 + 4i)z + 1 = 0$

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 5 Round 5  
Solving Trig  
Equations

1.)  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

2.)  $\frac{5}{9}$

3.)  $-\frac{\pi}{6}, \frac{\pi}{6}$

1) Solve for all  $x$  if  $0 \leq x < 2\pi$ :  $\cos(4x) = -1$

2) If  $\cos(x) + \sin(x) = \frac{2}{3}$ , give the value of  $\sin(2x)$ .

3) Solve for  $x$  if  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ :  $3\sec(x)\tan^2(x) - \sec(x) + \frac{3}{\cos^2(x)} = 4$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 5 Round 6  
Sequences and  
Series

1.) 204

2.)  $\frac{3}{8}$

3.) 3, -59

1.) Evaluate  $\sum_{n=1}^8 n^2$ .

2.) For a geometric sequence  $\{a_n\}$ ,  $\sum_{n=1}^{\infty} a_n = 4$ . If  $a_2 = -3$ , what are all possible values for the fifth term of the sequence?

3.) In an arithmetic sequence, the twelfth term is 30 less than the square of the second term. If the third term is 19, find all possible values for the first term of the sequence.

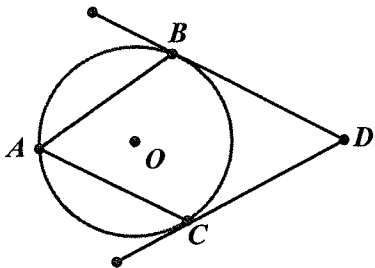
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Match 5 Team Round

- 1.)  $16\sqrt{3}$  4.)  $13-9i, -13+9i$   
 2.)  $16a^3b - 16ab^3$  5.)  $\frac{-4 \pm 3\sqrt{3}}{10}$   
 3.)  $3, -\frac{3}{4}$  6.)  $101, -89$

Note: Diagrams not necessarily drawn to scale.

- 1) In the figure below, circle O has radius 4 cm.  $\angle BAC$  is inscribed in the circle with center O. Arc BAC measures 240 degrees.  $\overline{BD}$  and  $\overline{CD}$  are tangent to circle O.  $AB=AC$ . Find the perimeter of quadrilateral ABDC.



- 2.) Simplify as much as possible: 
$$\frac{\frac{a^2 - b^2}{ab^3} - \frac{b^2 - a^2}{a^3b}}{\frac{a^2 + b^2}{(2ab)^4}}$$
- 3.) Solve this equation for all possible values of x: 
$$2 + \frac{3}{2 + \frac{3}{x}} = \frac{3}{2 - \frac{3}{x}}$$
- 4.) A geometric sequence  $\{a_n\}$  of complex numbers has  $a_1 = 1+i$  and  $a_3 = 7-i$ . Find all possible values for  $a_4$ .
- 5.) Find all values of  $\sin(A)$  such that  $\cos\left(A + \frac{\pi}{6}\right) = \frac{4}{5}$
- 6.) An arithmetic sequence of real numbers has as its first 3 terms:  $a, 2a-1, a^2 - 90$ . Find all possible values of for the 10<sup>th</sup> term of the sequence.