FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 1 Round 1
Arithmetic: Percents

1)___________27.8%_________

2.) _______50___________

3.) _________68______________

1) During the first half of the baseball season, Slugger made a hit in 25% of his at-bats, and had a total of 60 hits. During the second half of the season, Slugger made a hit in 30% of his 300 at-bats. What percent of Slugger's at-bats for the entire season were hits? Round your answer to the nearest tenth of a percent.

   Slugger's at-bats for the first half of the season were 60/.25 = 240. Slugger's hits for the second half of the season were 0.3(300)= 90. Total hits divided by total at-bats is $\frac{150}{540} = \frac{5}{18}$ rounded to the nearest tenth of a percent is 27.8

2) 70% of 80% of 90% of Z is equal to (Z-10)% of 30% of 210. What is the value of Z?
   (0.7)(0.8)(0.9)Z = (0.01)(Z-10)(0.3)(210), so 0.504Z = 0.63Z - 6.3, so -0.126Z = -6.3, and Z=50

3) The value of stock PDQ increased by 50% in July, decreased by 40% in August, and increased by 30% in September. The value of stock WOW decreased by 50% in July, increased by 40% in August, and decreased by 30% in September. If the value of each stock was $57.33 at the end of September, what was the positive difference between the values of the two stocks at the beginning of July?
   Let x=value of stock PDQ. (1.5)(0.6)(1.3)x = 57.33, so x = 49.
   Let y = value of stock WOW. (0.5)(1.4)(0.7)y = 57.33, so y= 117
   117-49 = 68

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 1 Round 2
Algebra I: Equations

1.) ________-15__________

2.) ________-288__________

3.) _____2, $\frac{-5}{3}$ ____________

1.) Solve for x: $4-3(x+2(x-1))=1-2(x+3(x-4))$

4-3(x+(2x-2)) = 1 – 2(x+(3x-12)
4-3(3x-2) = 1 – 2(4x-12)
4-9x+6 = 1 – 8x + 24
10 – 9x = 25 – 8x
-15 = x

2) Solve for y: $\frac{1}{2}(y+4)(y-5)+\frac{1}{3}(y+6)(y-7)=(y+8)(y-9)-\frac{1}{6}y^2$

Multiply by 6 to get $3(y+4)(y-5)+2(y+6)(y-7)=6(y+8)(y-9)-y^2$
$3(y^2 - y - 20) + 2(y^2 - y - 42) = 6(y^2 - y - 72) - y^2$
$3y^2 - 3y - 60 + 2y^2 - 2y - 84 = 6y^2 - 6y - 432 - y^2$
-5y – 144 = -6y - 432
y = -288

3) Solve for z: $\frac{1}{2}-\frac{z-3}{4}=\frac{5}{z+\frac{14}{3}}$

Simplify $\frac{5}{z+\frac{14}{3}}$ $\frac{5}{z+\frac{14}{3}}$ to $\frac{15}{3z+14}$ $\frac{15}{3z+14}$. Multiply both sides by 4(3z+14) to get

2(3z+14) – (z-3)(3z+14) = 4*15
$6z + 28 - (3z^2 + 5z - 42) = 60$
$6z + 28 - 3z^2 - 5z + 42 = 60$
$-3z^2 + z + 10 = 0$
$3z^2 - z - 10 = 0$
(3z+5)(z-2)= 0   z = 2 or z = $\frac{-5}{3}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 1 Round 3
Geometry: Triangles
And Quadrilaterals

1.) __80 ________________

2.) _______36________________

3.) __ $\sqrt{130}+\sqrt{39}+13$________

1) A rhombus has side 10 cm. If the ratio of the lengths of the diagonals of the rhombus is 2:1, what is the area of the rhombus in square cm?

   The diagonals are perpendicular and bisect each other, so they form 4 right triangles. Let each right triangle have legs x and 2x. Then x^2 + $(2x)^2$ = 100, so $5x^2$ = 100, so x= 2√5 cm. One-half The product of the diagonals gives the area, so the area is (1/2) * 2(2√5)*(2(4√5) = 80 square cm.

2) Trapezoid ABCD has right angles at A and B. If a line is drawn from C parallel to AB and meets AD at point E, the ratio of the area of rectangle ABCE to triangle CDE is 8:1, what is the perimeter of the smallest possible trapezoid given that the lengths of all sides of the trapezoid are whole numbers?

Triangle CDE is a right triangle, and the smallest possible triangle with all whole number sides is a 3-4-5 triangle, and has area (1/2)(3)(4) = 6. The area of the rectangle must be 48, and one of the sides of the rectangle is either 3 or 4. Since the area of the rectangle must be 48, if one side is 3, the other side is 16. If one side is 4, the other side is 12. The perimeter of the trapezoid is either 16 + 3 + (16+4) + 5 or 12 + 4 + (12+3) + 5. The smallest of these is the second one, which has perimeter 36.

3) Right triangle ABC has AB as the hypotenuse. An altitude is drawn from C to AB and meets AB at D. If CD = $\sqrt{30}$ and AB = 13, find the perimeter of triangle ABC.

The altitude is the geometric mean between AD and BD, so AD*BD = $(\sqrt{30})^2$ = 30.
If AD*BD = 30 and AD+BD = 13, then if AD is the shorter side, AD=3 and BD = 10.
Find the length of AC by AC^2 = $(\sqrt{30})^2 + 3^2$, so AC =√39. Find the length of BC by BC^2 = $(\sqrt{30})^2 + 10^2$ = √130

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 1 Round 4
Algebra 2:
Simultaneous Equations

1.) _____a=11 b=4 ________

2.) ______20, -20_________

3.) _x= 1/3 y= 1/2 z= _1__

1.) Solve for a and b

$2a + 5b = 4a – 0.5b$

$a = 52 - (3a+2b)$

$2a = 5.5b$ becomes $2(2a)+2b=52$, so $2(5.5b)+2b=52$, so $13b=52$, and $b=4$.

$4a+2b=52$ Then $2a=22$, and $a=11$

2.) Solve for all real values of c:

$c^2 – d^2 = 375$

$cd = 100$

Solve by substitution: $d=100/c$, so $c^2 - (100/c)^2 = 375$

$$c^2 - \frac{10000}{c^2} = 375$$

$$c^4 - 10000 = 375c^2$$

$$c^4 - 375c^2 - 10000 = 0$$

$$(c^2 - 400)(c^2 + 25) = 0$$

$c^2+25=0$ has no solution, so $c^2-400=0$, so $c = \pm 20$

3.) Solve for x,y, and z:

$$\frac{2}{x} + \frac{3}{y} + \frac{1}{z-2} = 11$$

$$\frac{3}{x} - \frac{1}{y} + \frac{4}{z-2} = 3$$

$$\frac{4}{x} + \frac{2}{y} - \frac{6}{z-2} = 22$$

Let $u=1/x$, $v=1/y$, $w = 1/(z-2)$, so we have

$2u +3v + w = 11$ Eliminate v from the first 2: $2u +3v + w = 11$

$3u – v + 4w = 3$ $9u – 3v + 12w = 9$

$4u +2v – 6w = 22$ so $11u + 13w = 20$

Eliminate v from the second 2: $6u – 2v + 8w = 6$

$4u + 2v – 6w = 22$ so $10u + 2w = 28$

$11u+13w=20$ Eliminate w $22u + 26w = 40$

$10u + 2w = 28$ $-130u-26w = -364$, so $-108u = -324$, so $u = 3$ and $x=1/3$.

$66+26w=40$, so $26w=-26$, so $w=-1$, so $z=1$. $2u + 3v + w = 11$, so $6 +3v-1=11$, so $v=2$, and $y=1/2$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 1 Round 5
Trig: Right Triangles

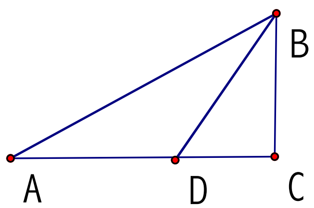
1.) ________ $\frac{\sqrt{5}}{3}$ ______________

2.) ________ $1+\sqrt{6}$ ________________

3.) _______ $30\sqrt{3}-30$ ________________

1)In right triangle ABC, the right angle is at C and 3 cos B = 2 tan A. What is sin A ?
Cos B = sin A, so 3 sin A = 2 tan A, and since tan A = sin A / cos A, we have 3 = 2 cos A, so cos A = $\frac{2}{3}$. Therefore $\sin^2 A = 1-(\frac{2}{3})^2 = \frac{5}{9}$, so sin A = $\frac{\sqrt{5}}{3}$

2.) In triangle ABC below, C is a right angle. $\sin(\angle BDC) = \frac{\sqrt{3}}{2}$ and $\sin(\angle BAC) = \frac{\sqrt{3}}{3}$. If AD=5, what is the length of DC?



Angle BDC must be 60 degrees. Let DC=x. Then BC = $x\sqrt{3}$, and BD = 2x. AB must be 3x since $\sin(\angle BAC) = \frac{\sqrt{3}}{3}$. By Pythagorean theorem, $(5+x)^2 + (x\sqrt{3})^2 = (3x)^2$, So $x^2 + 10x + 25 + 3x^2 = 9x^2$, so $5x^2 - 10x - 25 = 0$, so $x^2 - 2x - 5 = 0$, so $X = \frac{2+\sqrt{24}}{2} = \frac{2+2\sqrt{6}}{2} = 1+\sqrt{6}$

3.) A tree is located a certain horizontal distance from an observation tower. When viewed from the base of the tower, the angle of elevation to the top of the tree is 30 degrees. When viewed from the roof of the tower, the angle of depression to the top of the tree is 45 degrees. If the tower is 60 feet tall, how tall is the tree (in feet)? Remember to express answers with no radicals in the denominator.

Let y = horizontal distance from tower to tree. Let x = height of tree. Tan(30) = $\frac{x}{y}$, and tan(45) = 1 = $\frac{y}{60-x}$. So $\frac{x}{\tan(30)} = 60 - x$, $x\sqrt{3}$=60-x, so 60=x(1+$\sqrt{3}$), so

$$x=\frac{60}{\sqrt{3}+1} = \frac{60}{\sqrt{3}+1} * \frac{\sqrt{3}-1}{\sqrt{3}-1} = 30(\sqrt{3}-1) = 30\sqrt{3}-30$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 1 Round 6
Coordinate Geometry

1.) ________ $\frac{2}{3}$ ________________

2.) ______ $\frac{-4}{3}$ __________________

3.) ______ $y=\frac{4}{3}x-\frac{1}{2}$ _______and _____ $y=\frac{3}{4}x+\frac{3}{8}$ __________

1) A parallelogram has one vertex in each of the 4 quadrants. Three of the points are (-5,-2), (4, -1), and (-2,5). Find the slope of the line segment connecting (-5,-2) to the opposite vertex of the parallelogram.

The opposite vertex is in the first quadrant. Since the segment from (-5,-2) to (-2,5) goes over 3 and up 7, the segment from (4,-1) to the opposite vertex must do the same, so the opposite vertex is (7,6). The slope of the segment connecting (-5,-2) to (7,6) is $\frac{6-(-2)}{7-(-5)}=\frac{8}{12}=\frac{2}{3}$

2) Isosceles trapezoid ABCD has its two bases AD and BC parallel to the x-axis. The coordinates of A are (1,1) and the coordinates of B are (3, $1+2\sqrt{3}$). Line segments are drawn from B and C to the midpoint E of AD. If the area of the trapezoid is $16\sqrt{3}$, what is the product of the the slopes of BE and CE?

The segment from A to B goes over 2 and up $2\sqrt{3}$, so angle ABD measures 60 degrees. If you draw an altitude from B to AD meeting at C, the area of triangle ABC is = $2\sqrt{3}$, and since the trapezoid is isosceles, there is a corresponding triangle with area $2\sqrt{3}$ on the other side. That leaves $12\sqrt{3}$ for the part of the trapezoid that is a rectangle, and since we know the height is $2\sqrt{3}$, the base of that rectangle is 6. Therefore, AD must be 2+6+2 = 10, so D must have coordinates (11,1), and C must have coordinates (9, $1+2\sqrt{3}$). E must have coordinates (6,1). So multiply

$$\frac{(1+2\sqrt{3})-1}{3-6}*\frac{(1+2\sqrt{3})-1}{9-6}=\frac{12}{-9}=\frac{-4}{3}$$

3.) The line segment with endpoints (1,x) and (x,2) has length 5. Give both possible equations for its perpendicular bisector. Give your answers in the form y=mx+b.

Find the possibilities for x first by solving $1+2\sqrt{3}$ so x=5 or x=-2

$$\sqrt{(x-2)^2+(1-x)^2}=5$$

$$x^2-4x+4+1-2x+x^2=25$$

$$2x^2-6x-20=0$$

$$2(x-5)(x+2)=0$$

The midpoint of the segment from (1,5) to (5,2) is (3,3.5) and the segment has slope -3/4. The midpoint of the segment from (1,-2) to (-2,2) has midpoint (-0.5, 0) and has slope -4/3. The bisectors are y-3.5 = (4/3)(x-3), or y=(4/3)x-0.5 and y-0=(3/4)(x+0.5) y=(3/4)x+(3/8).

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014 Match 1 Team Round

1.) ________180____________ 4.) ___________4_________

2.) ______20, -39_________________ 5.) ________ $\frac{300}{7}$ _______

3.) ___________ $\frac{3}{5}$ ____________ 6.) _________ $\frac{-4\sqrt{3}}{3}$ ______________

1) (a+20)% of (b+30) is b% of (a+40). If b=2a, what is the sum (a+b)?

$$\frac{a+20}{100}(b+30)=\frac{b}{100}(a+40)$$
$$(a+20)(b+30)=ab+40b$$
$$(a+20)(2a+30)=2a^2+80a$$
$$2a^2+70a+600=2a^2+80a$$
$$600=10a$$
$$a=60$$
$$b=2*60=120$$
$$a+b=60+120=180$$

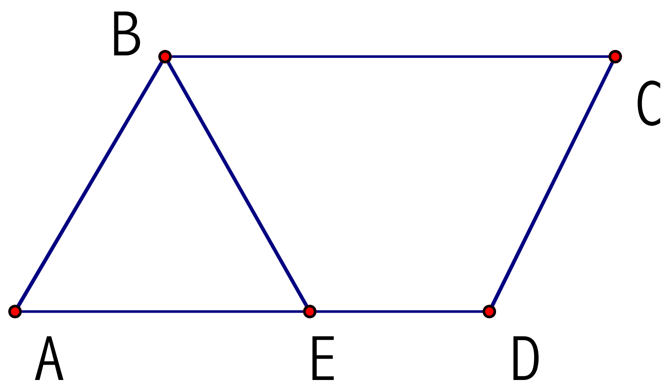
2.) Solve for all possible values of z: $\frac{1}{0.2m-3}-0.5=\frac{8}{0.5m+6}$

Multiply both sides by the LCD to get

$$0.5m+6-0.5(0.2m-3)(0.5m+6)=8(0.2m-3)$$
$$0.5m+6-0.5(0.1m^2-1.5m+1.2m-18)=1.6m-24$$
$$0.5m+6-0.05m^2+0.15m+9=1.6m-24$$
$$-0.05m^2-0.95m+39=0$$
$$0.05m^2+0.95m-39=0$$
$$m^2+19m-780=0$$
$$(m+39)(m-20)=0$$

so m=-39 or m=20

3) Parallelogram ABCD has angles of 60 degrees at points A and C. A line is drawn from B to AD at point E creating an equilateral triangle. The area of the triangle is one-third of the area of the parallelogram. Find the ratio of the perimeter of the triangle to the perimeter of the parallelogram. (Drawing not to scale)



Let the length of AB=x. The area of ΔABE is $\frac{x^2\sqrt{3}}{4}$. The height of the parallelogram must be $\frac{x\sqrt{3}}{2}$ since angle A is 60 degrees and side AB=x. If y = the length of AD, then The area of the parallelogram is $\frac{xy\sqrt{3}}{2}$, and we know

$$\frac{x^2\sqrt{3}}{4} = -2\sqrt{3} + \left(\frac{1}{3}\right)\frac{xy\sqrt{3}}{2} = \frac{xy\sqrt{3}}{6}.$$

Solve $\frac{x^2\sqrt{3}}{4} = \frac{xy\sqrt{3}}{6}$ for y to get y= $\frac{3x}{2}$, so the ratio $\frac{3x}{2x+2y} = \frac{3x}{3x+2x} = \frac{3}{5}$

4. For what value of k does the system

$$\begin{aligned} 2x + 3y + 5z &= 10 \\ x - 4y - 2z &= 8 \\ 7x - 6y + kz &= 44 \end{aligned}$$

have infinitely many solutions (x,y,z)?

Notice that for x terms, y terms, and the constants on the right hand side, 2*(first equation) + 3*(second equation) = third equation. So in order for the system to be dependent and have infinitely many solutions, we must have 2*5z+3(-2)z=kz, so k=4.

5. A string is tied to the top of a flagpole. A person on the ground holds the string some distance away from the flagpole. At a certain distance from the base of the flagpole as measured along the ground, the length of the string is √2 times the height of the flagpole. If the person walks back 100 more feet while unwinding more string, the angle of elevation to the top of the flagpole as measured from the ground is 16.7 degrees. Given tan $(16.7^{o}) = 0.3$, how tall is the flagpole in feet?
At the first distance, the angle must be 45 degrees in order for the length of the string to be twice the height of the flagpole, so if the height of the flagpole is x, the distance along the ground is x. The second reading gives $0.3 = \dfrac{x}{100+x}$, so

$$30 + 0.3x = x$$

$$30 = 0.7x$$

$$x = \frac{30}{0.7} = \frac{300}{7}$$

6. A rhombus with one of its vertices at the origin and one of its bases along the x-axis has sides of length 5 units each. The angle whose vertex is at the origin is 60 degrees, measured counterclockwise from the positive x-axis. What is the sum of the slopes of the 4 line segments formed by connecting the midpoints of the original rhombus?
The four points of the rhombus must be (0,0), (5,0), (2.5, 2.5√3), and (7.5, 2.5√3).
The midpoints of the segments are (2.5,0), (3.75,1.25√3),(5,2.5√3), and (6.25,1.25√3).
The slopes of two segments will be the same as the slopes of the opposite segments.
The two required slopes are $\dfrac{1.25\sqrt{3} - 0}{3.75 - 2.5}$ *and* $\dfrac{1.25\sqrt{3} - 2.5\sqrt{3}}{1.25 - 5} = -\sqrt{3}$ and $\dfrac{\sqrt{3}}{3}$, so the sum of the four slopes is $-2\sqrt{3} + \dfrac{2\sqrt{3}}{3} = \dfrac{-4\sqrt{3}}{3}$