FAIRFIELD COUNTY MATH LEAGUE 2026–2027
Match 2
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Factors and Multiples

1-1 Find the average (the arithmetic mean) of all multiples of 5 between 5 and 555, inclusive.
[Answer: 280]

Since pairs $(5,555)$, $(10,550)$, $(15,545)$, etc. have the same arithmetic mean, it follows that the arithmetic mean of the set of all pairs will be the same, which is $\frac{5+555}{2} = 280$.

1-2 Determine the largest possible value of $m+n$ such that $2^m \cdot 3^n$ is a factor of $100!$
[Answer: 145]

The value of m comes from the number of times 2 is a factor of all integers up to 100. This is most easily found by counting occurrences of multiples of powers of 2 starting with lower powers: $\frac{100}{2} = 50$ multiples of 2, $\frac{100}{4} = 25$ multiples of 4, $floor\left(\frac{100}{8}\right) = 12$ multiples of 8, $floor\left(\frac{100}{16}\right) = 6$ multiples of 16, $floor\left(\frac{100}{32}\right) = 3$ multiples of 32, and finally $floor\left(\frac{100}{64}\right) = 1$ multiple of 64, so $m = 50 + 25 + 12 + 6 + 3 + 1 = 97$. Similarly, we can count multiples of powers of 3: $(3,33), (9,11), (27,3), (81,1)$ making $n = 33 + 11 + 3 + 1 = 48$. This makes the desired quantity $97 + 48 = 145$.

1-3 A set A contains the numbers 1, 2, and 3, and has the following property: if n is an element in A, the smallest whole number that is both divisible by n and also has n factors including 1 and itself is also in A. Find the sum of all perfect square elements of A.
[Answer: 46]

We know $A = \{1,2,3\}$. The inclusion of 1 means A contains 1, and the inclusion of 2 means A contains 2. However, the inclusion of 3 means A contains a number with 3 factors, including 3. This means A contains 3^2, or 9. Now A contains the smallest number with 9 factors, including 3^2. This has to be $2^2 * 3^2 = 36$. Since this number is even, the next included value will have an even number of factors and therefore will not be a perfect square. Additionally, all new numbers will have even numbers as factors and therefore be even. This means that the only perfect square factors of A must be $1, 9$, and 36, making the desired quantity $1 + 9 + 36 = 46$.

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Round 2: Polynomials and Factoring

2-1 Find the sum of all (not necessarily distinct) zeros for the polynomial function $y = (2x - 5)^2(3x^3 - 12x^2 - 12x + 48)$.
[Answer: 9]

The first factor has two zeros of $\frac{5}{2}$, and the second factor has zeros that must add to $-\left(-\frac{12}{3}\right) = 4$, making the total sum of the zeros $2\left(\frac{5}{2}\right) + 4 = 9$. Alternatively, the student can factor the cubic and find zeros of $-2, 2$, and 4.

2-2 The polynomial expression $64x^a y^b - 1$ is factorable for integers a and b with $0 < b < a < 7$. Let all ordered pairs of a and b that satisfy the information above be in the form (a, b). For all such ordered pairs, if A is the sum of all non-distinct a-values and B is the sum of all non-distinct b-values, Find $A \div B$.
[Answer: 2]

Depending on the values of a and b (whether both are multiples of 2 or 3), the expression will be factorable as a difference of squares or cubes. Pairs in the restrictions with both values even are $(6,4)$, $(6,2)$, and $(4,2)$. The only pair with both values as multiples of three is $(6,3)$. Therefore $A = 6 + 6 + 4 + 6 = 22$, and $B = 4 + 2 + 2 + 3 = 11$, making the desired quantity $\frac{22}{11} = 2$.

2-3 The polynomial $f(x) = x^3 - 14x^2 + ax + b$, where a and b are integer constants, has only positive integer zeros, each with a multiplicity of 1. Find the positive difference between the lowest and highest possible values of b.
[Answer: 68]

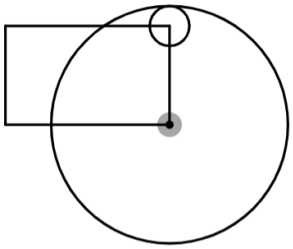
From the quadratic coefficient, we know the zeros must add to 14. We also know they must be positive distinct integers. Since b comes from the product of the zeros, we need the greatest and least possible products. The greatest possible product will occur when the zeros are close in value: $\{3,4,7\}$ and $\{3,5,6\}$ have products of 84 and 90, making 90 the largest product. The least product will occur when the zeros are far in value: $\{1,2,11\}$ will produce the smallest product of 22. Therefore the desired quantity is $90 - 22 = 68$.

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Round 3: Area and Perimeter

3-1 Charlie is out walking his dog on a leash and he stops at his neighbor Lucy's house. While talking in the yard, Charlie ties the end of the leash to the corner of Lucy's shed, which is a 20 ft by 10 ft rectangle. If the leash is 12 feet long and there are no other obstacles, then the area of the space in which Charlie's dog can run in square feet can be expressed in the form $a\pi$. Find a.
[Answer: 109]



See the diagram. If the dog's leash is connected to the highlighted point, then the area of the dog's run is $\frac{3}{4}$ of a circle with radius 12 ft and $\frac{1}{4}$ of a circle with radius $12 - 10 = 2$ feet long, making a total area in square feet of $\frac{3}{4}\pi(12^2) + \frac{1}{4}\pi(2^2) = 108\pi + \pi = 109\pi$, so $a = 109$.

3-2 The diagonal of a rectangle is 8 units longer than one of the sides. If the perimeter of the rectangle is 82, find the area.
[Answer: 420]

Let one of the sides have length x. The other side will then have length $41 - x$. Note that this means that the area of the rectangle is $x(41 - x)$. Then we have the equation $(x + 8)^2 = x^2 + (41 - x)^2$. Expanding yields $x^2 + 16x + 64 = x^2 + 1681 - 82x + x^2$, so $x^2 - 98x + 1617 = 0$, which factors into $(x - 21)(x - 77) = 0$, but since $x = 77$ is extraneous, we have that one side of the rectangle has length 21 and the other must have length $41 - 21 = 20$, making the area $(21)(20) = 420$.

3-3 The circumference of a circle with area 100π is unraveled and used to make two squares, one of which has integer side lengths, whose perimeters are as close as possible. The square with non-integer side lengths has area $a\pi^2 + b\pi + c$, where $a, b,$ and c are integers. Find $a + b + c$.
[Answer: 9]

Since the radius of the circle is 10, the circumference is $20\pi \approx 62.82$. The multiple of four closest to half of this quantity is 32, leaving $20\pi - 32$ as the perimeter of the second square. One side of the square is therefore $5\pi - 8$, and therefore the area is $(5\pi - 8)^2 = 25\pi^2 - 80\pi + 64$, making the desired quantity $25 - 80 + 64 = 9$.

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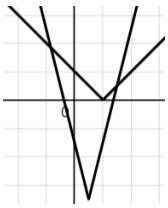
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Round 4: Absolute Value & Inequalities

4-1 The solutions to $|4x - a| = 8$ are plotted on a number line. If a is a real number, what is the distance between the two points?
[Answer: 4]

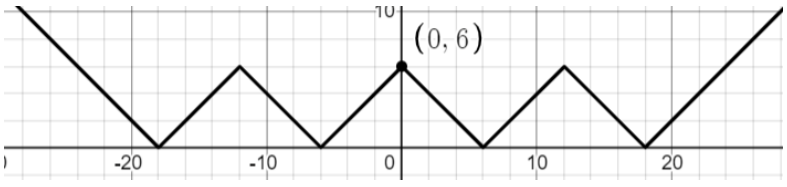
Dividing the equation by 4 yields $\left|x - \frac{a}{4}\right| = 2$, and therefore the solutions are each 2 units away from the value $\frac{a}{4}$, so regardless of the value of a, the distance between them is $2 + 2 = 4$.

4-2 What is the sum of all integer solutions to $|x - 2| \geq 4|x - 1| - 7$?
[Answer: 5]



Creating a sketch like the one shown in the diagram can simplify the work and avoid a multi-step algebraic solution. Substituting $x = -2$ yields $4 \geq 5$, and then substituting $x = -1$ yields $3 \geq 1$. Therefore the smallest integer solution is $x = -1$. Substituting $x = 3$ yields $1 \geq 1$, making $x = 3$ the largest integer solution. The sum of all integer solutions is therefore $-1 + 0 + 1 + 2 + 3 = 5$.

4-3 The set $(a, b) \cup (c, d)$ is the set of all values k such that the equation $\left|\left||x| - 12\right| - 6\right| = k^2 - 5k$ has 8 solutions for x. Find $a + b + c + d$.
[Answer: 10]



It helps to start by drawing the graph of the absolute value expression on the left. An absolute value graph centered at the origin is shifted down 12 units, then taking the absolute value yields a graph with points $(12,0)$, $(0,12)$, and $(-12,0)$. This is shifted down again six units yielding points of $(18,0)$, $(-18,0)$, $(12,-6)$, $(-12,-6)$, and $(0,6)$. When we take the absolute value one last time, we get a graph like the one in the diagram. Note that the greatest number of solutions, 8, is found when $0 < y < 6$. Therefore $0 < k^2 - 5k < 6$, or $k^2 - 5k > 0$ AND $k^2 - 5k - 6 < 0$. The first inequality is solved by $k \in (-\infty, 0) \cup (5, \infty)$ and the seccond is solved by $k \in (-1,6)$, and therefore the interesection of these solutions is $k \in (-1,0) \cup (5,6)$, making the desired quantity $-1 + 0 + 5 + 6 = 10$.

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Round 5: Law of Sines and Cosines

5-1 In triangle ABC, $AB = 8$ and $BC = 15$. The area of the triangle is $30\sqrt{3}$, and $\angle B$ is acute. Find $(AC)^2$.
[Answer: 169]

Since $30\sqrt{3} = \frac{1}{2}(8)(15)\sin(\angle B)$, we have $\sin(\angle B) = \frac{\sqrt{3}}{2}$, and therefore $\cos(\angle B) = \frac{1}{2}$ (positive because the angle is acute). Then by the law of cosines, $(AC)^2 = 8^2 + 15^2 - 2(8)(15)\left(\frac{1}{2}\right) = 289 - 120 = 169$,

5-2 In triangle ABC, $\cos(A) = \frac{3}{4}$, $AB = 10$, and $BC = 8$. Find the sum of all possible perimeters of ABC.
[Answer: 51]

We do not know AC, but we can still set up the law of cosines as $8^2 = 10^2 + x^2 - 2(10)(x)\left(\frac{3}{4}\right)$, or $x^2 - 15x + 36 = 0$. This yields values of $AC = 12$ or $AC = 3$, and since both work in the triangle, the desired quantity is $(10 + 8 + 3) + (10 + 8 + 12) = 51$.

5-3 An acute triangle has side lengths that are three consecutive integers. If the sine of the smallest angle is $\frac{\sqrt{7}}{4}$, find the perimeter of the triangle.
[Answer: 15]

Let the smallest side be n. If the sine of its opposite angle is $\frac{\sqrt{7}}{4}$, then the cosine is $\frac{3}{4}$. Therefore we have $n^2 = (n+1)^2 + (n+2)^2 - 2(n+1)(n+2)\left(\frac{3}{4}\right)$, or $n^2 = 2n^2 + 6n + 5 - \frac{3}{2}(n^2 + 3n + 2)$, or $\frac{1}{2}n^2 - \frac{3}{2}n - 2 = 0$, or $n^2 - 3n - 4 = 0$. This gives $n = 4$ or $n = -1$, which is extraneous. Therefore the sides of the triangle must be 4,5,and 6, and therefore the perimeter is $4 + 5 + 6 = 15$.

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Round 6: Equations of Lines

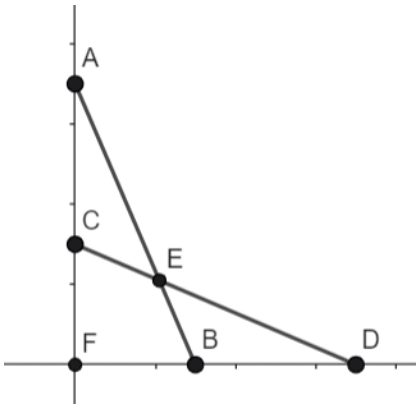
6-1 The points $(4, a)$, $(a, 7)$, and $(8, 13)$ are collinear. Find the sum of all possible values of a.
[Answer: 21]

Setting slopes from two pairs of points equal yields the equation $\frac{7-a}{a-4} = \frac{13-7}{8-a}$, or $(7-a)(8-a) = 6(a-4)$, which yields the equation $a^2 - 21a + 80 = 0$. From here the quadratic can be factored to yield values of $a = 16$ and $a = 5$ or one can notice that the sum of the solutions must be the opposite of the linear coefficient, or 21.

6-2 The line $3x + 4y = k$ is tangent to Circle O, which has a center of $(2,3)$ and a radius of 1. What is the minimum value of k?
[Answer: 13]

Since the line described has slope $-\frac{3}{4}$, it will be tangent to O at a point whose radius makes a slope of $\frac{4}{3}$ with the center. With a radius of one, this means that the point will be shifted $\frac{3}{5}$ horizontally and $\frac{4}{5}$ vertically from the center, either right and up or left and down. Since we want to minimize k, we shift left and down from the center: $\left(2 - \frac{3}{5}, 3 - \frac{4}{5}\right)$, or $\left(\frac{7}{5}, \frac{11}{5}\right)$. This is the point of tangency, and therefore $k = 3\left(\frac{7}{5}\right) + 4\left(\frac{11}{5}\right) = \frac{21}{5} + \frac{44}{5} = \frac{65}{5} = 13$.

6-3 See the diagram (not drawn to scale). Point F is the origin, points A and C lie on the y-axis, and points B and D lie on the x-axis. Line l contains points A and B, and line m contains the points C and D. If the areas of triangles AEC and DEB and quadrilateral $BFCE$ are all equal and point E has coordinates $(10,20)$, find the y-coordinate of point A.
[Answer: 80]



Let the distance from A to C be a, the distance from C to $(0,20)$ be b, the distance from $(10,0)$ to B be c, and the distance from B to D be d. From similarity, we have $\frac{a+b}{10} = \frac{20}{c}$ and $\frac{b}{10} = \frac{20}{c+d}$. This means $ac + bc = 200$ and $bc + bd = 200$, or $ab + bc = bc + bd$, giving $ac = bd$. Next, setting areas equal: $5a = 5(b+20) + 10(10+c) = 10d$, or simplified, $a = b + 2c + 40 = 2d$. Substituting $a = 2d$ into $ac = bd$ yields $b = 2c$. Substituting this into $a = b + 2c + 40$ yields $a = 2b + 40$. Finally, putting these substitutions into $\frac{a+b}{10} = \frac{20}{c}$ yields $\frac{3b+40}{10} = \frac{20}{\frac{1}{2}b}$, which after cross multiplication yields a quadratic in b: $3b^2 + 40b - 400 = 0$. This is factorable into $(b+20)(3b-20) = 0$, and since the

negative value of b is extraneous, we have $b = \frac{20}{3}$, and so the desired quantity is $a + b + 20$ or $3b + 60$, or $3\left(\frac{20}{3}\right) + 60 = 80$.

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Team Round

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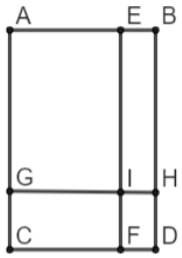
1. If N is a positive integer that has a remainder of 5 when divided by 9 and a remainder of 3 when divided by 7, what is the largest possible value of N less than 1000?
[Answer: 941]

   Our number N must be in the format $9a + 5$ and $7b + 3$, and so we have $9a + 5 = 7b + 3$, or $9a - 7b = -2$. By inspection we see $(6,8)$ works, corresponding to a value of $N = 59$. Subsequent values must larger than this value by a multiple of $lcm(9,7) = 63$. Therefore we have $59 + 63t < 1000$. The largest integer value of t that satisfies this inequality is $t = 14$, and so the desired quantity is $59 + 63(14) = 941$.

2. The polynomial $f(x) = x^4 - 2026x^3 + ax^2 + bx + 2025$, where a and b are integer constants, has a graph that bounces off the x −axis at $x = 3$. Find $f(-1)$.
[Answer: 35936]

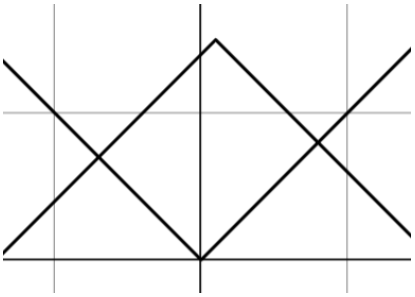
   The fastest way to solve this is to note that f must have a factor of $x^2 - 6x + 9$ since it must have $x = 3$ as a zero with a multiplicity of 2. This means there exist numbers c and d such that $x^4 - 2026x^3 + ax^2 + bx + 2025 = (x^2 - cx + d)(x^2 - 6x + 9)$. Through either distribution or applying Viete's formulas we see that $c + 6 = 2026$ and $9d = 2025$, and therefore $c = 2020$ and $d = 225$. Thus $f(x) = (x^2 - 2020x + 225)(x^2 - 6x + 9)$, and therefore $f(-1) = ((-1)^2 - 2020(-1) + 225)((-1)^2 - 6(-1) + 9) = (2246)(16) = 35936$

3. Consider rectangle $ABDC$ with area 300 and two line segments that subdivide it into four smaller rectangles, as shown in the diagram (not drawn to scale). If the area of $IHDF$ is an integer and the areas of $EBHI$ and $GIFC$ are equal (and non-zero), find the sum of all possible integer values of $\frac{area\ AEIG}{area\ IHDF}$.
[Answer: 98]



   Let the areas of each rectangle $EBHI$ and $GIFC$ be a and the area of $IHDF$ be b. We will call the desired ratio k, and so the area of $AEIG$ is kb. Because the areas of any two rectangles in a row or column are proportional, we have $(kb)(b) = (a)(a)$, and so $kb^2 = a^2$. We also know $kb + b + a + a = 300$, or $b(k + 1) + 2a = 300$. Thies means $k + 1 + \frac{2a}{b} = \frac{300}{b}$, or $k + 2\sqrt{k} + 1 = \frac{300}{b}$, or $\left(\sqrt{k} + 1\right)^2 = \frac{300}{b}$. Because b and k are both integers, it follows that k must be a perfect square and b must be a factor of 300. Only three ordered pairs (k, b) work: $(1,75), (16,12), (81,3)$, making the desired quantity $1 + 16 + 81 = 98$.

4. The graphs of the functions $f(x) = |x|$ and $g(x) = -|x - 1| + k$, where $k > 1$, enclose a quadrilateral with area 112. Find the value of k.
[Answer: 15]



See the diagram. The sides of the quadrilateral all have slope 1 or -1, so the quadrilateral is a rectangle. Let the x −value (and consequently y −value) of the point of intersection of the two functions in Quadrant I be a. Then the side in Quadrant I with a positive slope has length $a\sqrt{2}$ and the side in Quadrant I with a negative slope has length $(a-1)\sqrt{2}$. The area of the figure is then $2a(a-1)$, so $2a(a-1) = 112$, or $a^2 - a - 56 = 0$. Therefore $a = 8$, and $k = a + a - 1 = 15$.

5. In acute triangle ABC, $\frac{BC}{AC} = \frac{3}{2}$, $\frac{\cos(A)}{\cos(B)} = \frac{1}{3}$, and $\sin^2(B) = \frac{p}{q}$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.
[Answer: 109]

We have $\frac{BC}{AC} = \frac{\sin(A)}{\sin(B)} = \frac{3}{2}$ by the law of sines, and so $2\sin(A) = 3\sin(B)$, or $4\sin^2(A) = 9\sin^2(B)$. We also have $9\cos^2(A) = \cos^2(B)$, or $9(1 - \sin^2(A)) = 1 - \sin^2(B)$. Substituting $\sin^2(A) = \frac{9}{4}\sin^2(B)$ gives us $9\left(1 - \frac{9}{4}\sin^2(B)\right) = 1 - \sin^2(B)$, or $8 = \frac{77}{4}\sin^2(B)$, so $\sin^2(B) = \frac{32}{77}$, making the desired quantity $32 + 77 = 109$.

6. Lines m and n are both tangent to the circle $(x-5)^2 + (y-3)^2 = 4$ and both lines have a y-intercept of $(0,2)$. The line that contains both points of tangency has the equation $Ax + By = C$ where A, B, and C are integers with no common factors greater than 1 and $A > 0$. Find $A + B + C$.
[Answer: 30]

Let The center of the circle be O, the point $(0,2)$ be M, and one of the points of tangency be N. Because the tangent line is at a right angle to the radius, it follows that $(MN)^2 + (NO)^2 = (MO)^2$. Therefore $(MN)^2 + 4 = 26$, and so $(MN)^2 = 22$. Both points of tangency would have to have a distance of $\sqrt{22}$ from M, and so would lie on the circle $x^2 + (y-2)^2 = 22$. Expanding the original circle equation and this one yields the equations $x^2 + y^2 - 10x - 6y = -30$ and $x^2 + y^2 - 4y = 18$. Since the two points would solve both of these equations, subtracting the equations give the equation of the line containing both: $10x + 2y = 48$, which simplifies to $5x + y = 24$, making the desired quantity $5 + 1 + 24 = 30$.