

FAIRFIELD COUNTY MATH LEAGUE 2026–2027

Match 1

Individual Section

Please write your answers on the answer sheet provided.

Round 1: Percentages

- 1-1 On Monday, 10% of the students at FCML High School were absent.
- On Tuesday, 10% of those absent on Monday were present, and the rest were still absent. Also, 10% of those who were present on Monday were absent, and the rest were still present.
 - On Wednesday, 25% of those absent on Tuesday were present, and the rest were still absent. Also, 25% of those who were present on Tuesday were absent, and the rest were still present.
- What percentage of the school's students were present on Wednesday?
- [Answer: 66]

Assume there are 1000 students at FCML High School. On Monday, $(0.9)(1000) = 900$ students are present and 100 students are absent. On Tuesday, the number of present students is $(0.9)(900) + (0.1)(100) = 810 + 10 = 820$, with 180 students absent. On Wednesday, The number of present students is $(0.75)(820) + (0.25)(180) = 615 + 45 = 660$, which is 66% of 1000.

- 1-2 If $\frac{31}{111}$ is expressed as a percentage, what digit occurs in the 100th place after the decimal point?
- [Answer: 9]

Using either long division or rewriting the fraction as $\frac{279}{999}$, we can see that the decimal expansion (after being multiplied by 100 to be expressed as a percent) is $27.\overline{927}$. This means the digits to the right of the decimal point cycle in 3's, and the 100th digit will be the first digit in a cycle as digits 97-99 will be 927, making the answer 9.

- 1-3 Consider the ordered pair (x, y) which has the following properties: increasing the quantity $y + 3$ by $x\%$ is the same as doubling y , and increasing the quantity $x + 1$ by $y\%$ is the same as quadrupling x . The quantity $y - 3x = \frac{a}{b}$, where a and b are positive whole numbers with no common factors greater than 1. Find $a + b$.
- [Answer: 301]

Writhing the descriptions as a system of equations, we have:
$$\begin{cases} 2y = \left(1 + \frac{x}{100}\right)(y + 3) \\ 4x = \left(1 + \frac{y}{100}\right)(x + 1) \end{cases}$$
 . Expanding the

products in each equation and subtracting y from both sides of the first and x from both sides of the second yields $y = \frac{3x}{100} + \frac{xy}{100} + 3$ and $3x = \frac{y}{100} + \frac{xy}{100} + 1$. Subtracting the second equation from the first yields $y - 3x = \frac{3x}{100} - \frac{y}{100} + 2 = -\frac{1}{100}(y - 3x) + 2$. Adding $\frac{1}{100}(y - 3x)$ to both sides yields $\frac{101}{100}(y - 3x) = 2$, which means $y - 3x = \frac{200}{101}$, making the desired quantity $200 + 101 = 301$.

FAIRFIELD COUNTY MATH LEAGUE 2026–2027

Match 1

Individual Section

Please write your answers on the answer sheet provided.

Round 2: Solving Equations

- 2-1 The number $a + 5$ has the same distance between the distinct numbers $4a - 19$ and $2a - 7$ for real number a . Find the value of a .
[Answer: 9]

We can set up the equation $(4a - 19) - (a + 5) = (a + 5) - (2a - 7)$, which simplifies to $3a - 24 = -a + 12$, or $4a = 36$, yielding $a = 9$.

- 2-2 There exists a two-digit number with its ten's digit triple that of the unit's digit. When the digits are reversed and the new number is added to twice the original number, the result is 150. Find the original two-digit number.
[Answer: 62]

A two-digit number can be written as $10a + b$, but since $a = 3b$, the number is also equal to $31b$. The number with the digits reversed is $10b + a$, which is equal to $10b + 3b = 13b$. Adding the second to twice the first yields $13b + 2(31b) = 75b = 150$, so $b = 2$, making $a = 6$, and so the desired number is 62.

- 2-3 Solve for x : $(2x^2 - 5)(x^2 + 3x + 1) = x(3 - x(5 - 2x(x + 1))) + (2x^2 - 1)(2x + 1) - 308$
[Answer: 19]

Expanding the left and right sides completely yields $2x^4 + 6x^3 + 2x^2 - 5x^2 - 15x - 5 = 2x^4 + 2x^3 - 5x^2 + 3x + 4x^3 + 2x^2 - 2x - 1 - 308$, and combining like terms on both sides yields $2x^4 + 6x^3 - 3x^2 - 15x - 5 = 2x^4 + 6x^3 - 3x^2 + x - 309$. This simplifies to $-12x - 5 = x - 309$, or $16x = 304$, which yields $x = 19$.

FAIRFIELD COUNTY MATH LEAGUE 2026–2027

Match 1

Individual Section

Please write your answers on the answer sheet provided.

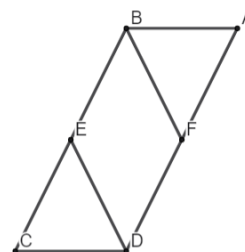
Round 3: Triangles and Quadrilaterals

- 3-1 The measure of the vertex angle of an isosceles triangle is double the measure of one of the base angles, and one side of the triangle has length 32 units. If a median is drawn from the vertex angle to the base, then the sum of the possible lengths of this median can be expressed $a + b\sqrt{c}$ where a , b , and c are positive integers and c has no perfect square factors greater than 1. Find $a + b + c$.
[Answer: 34]

Note that if a base angle has a measure x in degrees, then $x + x + (2x) = 180$, making $x = 45$, so this is an isosceles right triangle. The altitude length is therefore equal to half the length of the hypotenuse. If a leg has a measure of 32 units, then the hypotenuse has a measure of $32\sqrt{2}$ units and therefore the altitude has length $16\sqrt{2}$ units. If the hypotenuse has length 32 units, then the altitude will have length 16 units. This makes the sum of all possible lengths $16 + 16\sqrt{2}$ units, making the desired quantity $16 + 16 + 2 = 34$.

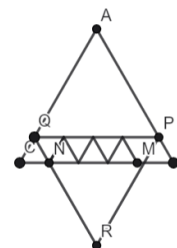
- 3-2 $ABCD$ is a parallelogram with point E at the midpoint of $\overline{BC}$ and F at the midpoint of $\overline{AD}$. $BEDF$ forms a rhombus with $BD = 16$ and $EF = 12$. The length of the altitude (height) from point B to $\overline{AD}$ is $\frac{a}{b}$ where a and b are whole numbers with no common factors greater than 1. Find $a + b$.
[Answer: 53]

See the diagram (not drawn to scale). The desired altitude length will be equal to the area of parallelogram $ABCD$ divided by AD (or equivalently BC). Note that $CD = EF = 12$, and thus the area of the parallelogram is $(CD)(BD) = (12)(16) = 192$. By the Pythagorean theorem, $DF = AF = 10$ units, so $AD = 20$. Therefore, the desired altitude length is $\frac{192}{20} = \frac{48}{5}$, making the desired quantity $48 + 5 = 53$.



- 3-3 Equilateral triangle ABC has side length 15. Points P and Q lie on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \parallel \overline{BC}$ and $PQ = 12$. Equilateral triangle PQR has sides $\overline{PR}$ which intersects $\overline{BC}$ at M and $\overline{QR}$ which intersects $\overline{BC}$ at N . The area of trapezoid $PQNM$ is $k\%$ of the area of triangle ABC . What is the value of k ?
[Answer: 28]

See the diagram. Because $\overline{PQ}$ is $\frac{4}{5}$ down the lengths of the sides of the triangle, the length of each side of triangles QNC and PBM is 3. Breaking up $\overline{PQ}$ and $\overline{MN}$ into intervals of length three, we can see that trapezoid $PQNM$ will consist of 7 of these equilateral triangles. We can also see that the entirety of triangle ABC will consist of $1 + 3 + 5 + 7 + 9 = 25$ of these equilateral triangles. This means the trapezoid comprises $\frac{7}{25}$ of the area of ABC , which is 28%.



FAIRFIELD COUNTY MATH LEAGUE 2026–2027

Match 1

Individual Section

Please write your answers on the answer sheet provided.

Round 4: Systems of Equations

- 4-1 A field is filled with healthy sheep and geese. There are 127 animals in the field with a total of 350 legs. How many geese are in the field?
[Answer: 79]

Let x represent the number of geese and y represent the number of sheep. Then the number of animals is represented by $x + y = 127$ and the number of legs is represented by $2x + 4y = 350$, which is simplified to $x + 2y = 175$. Subtracting the two equations yields $y = 48$, which means $x = 127 - 48 = 79$, which is our desired quantity.

- 4-2 What is the distance between the intersections of $f(x) = x^2$ and $g(x) = 2x + 4$?
[Answer: 10]

Setting $x^2 = 2x + 4$ means $x^2 - 2x - 4 = 0$, which is solvable with the quadratic formula to yield $x = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$, which are separated by a distance of $2\sqrt{5}$. The respective y -values are $2 + 2\sqrt{5}$ and $2 - 2\sqrt{5}$, which are separated by a distance of $4\sqrt{5}$. This means the desired quantity is $\sqrt{(2\sqrt{5})^2 + (4\sqrt{5})^2} = \sqrt{20 + 80} = 10$.

- 4-3 If x and y solve the system $\begin{cases} \frac{1}{x} + \frac{1}{y} = 9 \\ (x + y)^3 = x^3 + y^3 + 15 \end{cases}$, then $x^2 + y^2 = \frac{a-b\sqrt{c}}{d}$ where a, b , and d have no common factors greater than 1 and c has no perfect square factors greater than 1. Find $a + b + c + d$.
[Answer: 145]

Multiplying the top equation by xy yields $x + y = 9xy$, and expanding the cubed expression in the second equation and subtracting equal terms from both sides yields $3x^2y + 3xy^2 = 15$, or $xy(x + y) = 5$. Substituting $xy = \frac{5}{x+y}$ into the first equation yields $x + y = \frac{45}{x+y}$, or $(x + y)^2 = 45$, so $x + y = 3\sqrt{5}$ (we can take the positive value without losing generality). Therefore $xy = \frac{5}{3\sqrt{5}} = \frac{\sqrt{5}}{3}$. Finally, $x^2 + y^2 = (x + y)^2 - 2xy = 45 - \frac{2\sqrt{5}}{3} = \frac{135 - 2\sqrt{5}}{3}$, making the desired quantity $135 + 2 + 5 + 3 = 145$.

FAIRFIELD COUNTY MATH LEAGUE 2026–2027

Match 1

Individual Section

Please write your answers on the answer sheet provided.

Round 5: Right Triangles

- 5-1 In right triangle DEF with right angle E , the altitude drawn from E to hypotenuse $\overline{DF}$ divides $\overline{DF}$ into two segments of length 9 and 16. Find the perimeter of triangle DEF .
[Answer: 60]

If the length of the altitude from E to $\overline{DF}$ is x , then $x^2 = (9)(16) = 144$, so $x = 12$, which means the leg lengths (as hypotenuses of two smaller right triangles, one with legs of length 9 and 12 and the other with legs of length 12 and 16) must be 15 and 20, making the perimeter $15 + 20 + 25 = 60$.

- 5-2 A right triangle has integer side lengths that form an arithmetic sequence (the three sides increase by a constant common difference). If the perimeter is less than 100, find the largest possible area of the triangle.
[Answer: 384]

All right triangles whose side lengths form an arithmetic sequence are a multiple of a 3 – 4 – 5 triangle (this can be verified by setting up $x^2 + (x + k)^2 = (x + 2k)^2$, which yields $x = 3k$). Setting up $3x + 4x + 5x < 100$, we have $12x < 100$, making $x = 8$, and so the legs of the triangle have to be $3(8)$ and $4(8)$, and finally we have that the area of the triangle is $\frac{1}{2}(24)(32) = 384$.

- 5-3 There exists a right triangle with a leg of length 20 whose perimeter and area are both equal to k . Find the value of k .
[Answer: 45]

Let x be the length of unknown leg. Then setting the perimeter and area equal yields $x + 20 + \sqrt{x^2 + 400} = \frac{1}{2}(20)x$, or $9x - 20 = \sqrt{x^2 + 400}$. Squaring both sides yields $81x^2 - 360x + 400 = x^2 + 400$, or $80x^2 - 360x = 0$, so $x = \frac{360}{80} = \frac{9}{2}$, making the desired quantity $\frac{1}{2}(20)\left(\frac{9}{2}\right) = 45$.

FAIRFIELD COUNTY MATH LEAGUE 2026–2027

Match 1

Individual Section

Please write your answers on the answer sheet provided.

Round 6: Coordinate Geometry

- 6-1 Line $ax + 3y = 7$ is perpendicular to line $x - by = 5$. Find $\frac{a^2}{b^2}$.
[Answer: 9]

The slope of the first line is $-\frac{a}{3}$, and the slope of the second line is $\frac{1}{b}$. If $\left(-\frac{a}{3}\right)\left(\frac{1}{b}\right) = -1$, then $\frac{a}{b} = 3$, and so $\frac{a^2}{b^2} = 9$.

- 6-2 Given $A = (0,12)$ and $C = (25,12)$, find all points B on the x -axis such that $m\angle ABC = 90^\circ$. What is the product of the x -coordinates of such points?
[Answer: 144]

Let the x -coordinate of a point B be b . For $m\angle ABC$ to be 90° , the slope of $\overrightarrow{AB}$, which is $-\frac{12}{b}$, must be perpendicular to the slope of $\overrightarrow{BC}$, which is $\frac{12}{25-b}$. Therefore $\left(-\frac{12}{b}\right)\left(\frac{12}{25-b}\right) = -1$, and so $b^2 - 25b + 144 = 0$. This quadratic could be solved by factoring to yield $b = 9$ and $b = 16$, or we can recognize from properties of quadratics that the constant in the equation is the product of all values that solve the equation, either way making the desired quantity 144.

- 6-3 A point P is reflected about the line $y = x$, shifted left 34 units and down 14 units, rotated 90 degrees counterclockwise about the origin, and then reflected once more about the line $y = -x$ to make the point P' . If the coordinates of P and P' are the same, find the distance of point P from the origin.
[Answer: 26]

Let the coordinates of point P be (a, b) . The coordinates after each successive transformation are therefore: $(b, a) \rightarrow (b - 34, a - 14) \rightarrow (14 - a, b - 34) \rightarrow (34 - b, a - 14)$. Setting these new coordinates equal to (a, b) yields the equations $a = 34 - b$ and $a - 14 = b$, or $a + b = 34$ and $a - b = 14$. These can be solved to yield $a = 24$ and $b = 10$. This makes the desired quantity $\sqrt{24^2 + 10^2} = 26$.

FAIRFIELD COUNTY MATH LEAGUE 2026–2027

Match 1

Team Round

Please write your answers on the answer sheet provided.

1. I'm thinking of two whole numbers a and b . The value of b is $k\%$ of the value of a , The quantity $10a + b$ is 210% greater than the quantity $10b + a$. What is the value of k ?
[Answer: 23]

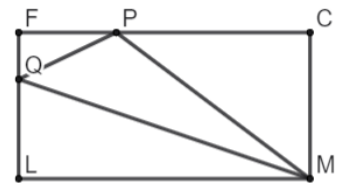
Writing the second relationship as an equation yields $10a + b = (1 + 2.10)(10b + a)$, and simplifying the right side yields $10a + b = 31b + 3.1a$. This simplifies to $30b = 6.9a$, which means $b = \frac{69}{300}a = \frac{23}{100}a$, making the desired quantity 23.

2. There is a number p such that the equation $(x - 4)^3 = (x + 2)(x - 5)(x - p)$ is linear in x . The solution to this linear equation is $x = \frac{a}{b}$, where a and b are whole numbers with no common factors greater than 1. Find $a + b + p$.
[Answer: 194]

Since the first two terms of the expanded expression on the left are $x^3 - 12x^2$ and the first two terms of the expanded on the expression on the right will be $x^3 + (2 - 5 - p)x^2$, it follows that $p = 9$. Knowing this, the full expansions of both sides are $x^3 - 12x^2 + 48x - 64 = x^3 - 12x^2 + 17x + 90$, which simplifies to $48x - 64 = 17x + 90$, or $31x = 154$, making $x = \frac{154}{31}$, which makes the desired quantity $154 + 31 + 9 = 194$.

3. Consider rectangle $FCML$ with point P on FC and point Q on FL . Triangle PQM occupies 20% of the area of $FCML$. If $FQ:QL = 2:9$, then $FP:PC = a:b$ where a and b are whole numbers with no common factors greater than 1. Find $a + b$.
[Answer: 15]

See the diagram. Let $FP = a$, $PC = b$, $FQ = 2$, and $QL = 9$. The area of the entire rectangle is $(a + b)(11) = 11a + 11b$, and the area of the four right triangles must be 80% of this total area. This means $\frac{1}{2}(a)(2) + \frac{1}{2}(b)(11) + \frac{1}{2}(9)(a + b) = \frac{4}{5}(11a + 11b)$. Therefore $a + \frac{11}{2}b + \frac{9}{2}a + \frac{9}{2}b = \frac{44}{5}a + \frac{44}{5}b \rightarrow \frac{11}{2}a + 10b = \frac{44}{5}a + \frac{44}{5}b \rightarrow \frac{6}{5}b = \frac{33}{10}a \rightarrow \frac{a}{b} = \frac{6}{5} * \frac{10}{33} = \frac{4}{11}$, making the desired quantity $4 + 11 = 15$.

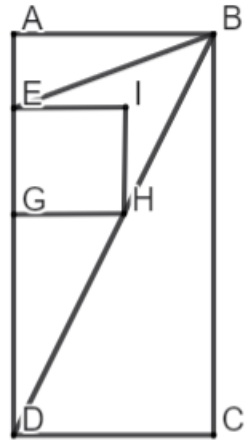


4. Let x and y be complex numbers such that the quantities $x + y$ and $x^3 + y^3$ are positive integers and $x^2 + y^2 = 100$. If the largest possible value of $x + y$ is a and $x^3 + y^3 = b$ when $x + y = a$, find $a + b$.
[Answer: 368]

Note $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$, which means $a(100 - xy) = b$. Also note that $xy = \frac{(x+y)^2 - x^2 - y^2}{2} = \frac{a^2 - 100}{2}$, so $a\left(\frac{200 - (a^2 - 100)}{2}\right) = b$, or $a\left(\frac{300 - a^2}{2}\right) = b$. If both a and b are positive

integers, then a must be the largest even number such that $a^2 < 300$, so $a = 16$. Therefore $b = 16 \left(\frac{300-256}{2} \right) = 16(22) = 352$, so the desired quantity is $16 + 352 = 368$.

5. See the diagram (not drawn to scale). $ABCD$ is a rectangle, $EIHG$ is a square, and H lies on $\overline{BD}$. If $AE = 3$, $\tan(\angle ABE) = \frac{3}{8}$, and the area of $EIHG$ is 36, then $\tan(\angle CBD) = \frac{a}{b}$ where a and b are whole numbers with no common factors greater than 1. Find $a + b$.
[Answer: 11]



We know $\tan(\angle ABE) = \frac{AE}{AB}$, so $AB = 8$. We also know $EG = GH = 6$. Since triangle BAD is similar to triangle HGD , we have $\frac{AB}{AD} = \frac{GH}{GD}$. Let $GD = x$, and we have $\frac{8}{3+6+x} = \frac{6}{x} \rightarrow 8x = 54 + 6x \rightarrow x = 27$. This means $AD = BC = 3 + 6 + 27 = 36$, and so the $\tan(\angle CBD) = \frac{BC}{DC} = \frac{8}{36} = \frac{2}{9}$, making the desired quantity $2 + 9 = 11$.

6. Consider circles C_1 , C_2 , and C_3 whose centers are all in Quadrant I and are increasing in distance away from the origin with the center of C_1 being closest and the center of C_3 being furthest away. C_1 is centered at $(1,1)$ and is tangent to the x - and y -axes. Circle C_2 is tangent to both positive axes and C_1 . Circle C_3 is is tangent to both positive axes and C_2 . The circumference of C_3 is $(a + b\sqrt{c})\pi$ where a , b , and c are positive integers and c has no perfect square factors greater than 1. Find $a + b + c$.
[Answer: 60]

Let the radius of circle C_2 be r . The center of C_2 is located at (r, r) . We also have that the distance from the center of C_2 to the origin must be the sum of r , the radius of C_1 , and the distance from the center of C_1 to the origin; in other words, the distance must be $r + 1 + \sqrt{2}$, and because this is the hypotenuse of an isosceles right triangle, this must also be equal to $r\sqrt{2}$. Therefore $r + 1 + \sqrt{2} = r\sqrt{2}$, so $r = \frac{\sqrt{2}+1}{\sqrt{2}-1}$, or after rationalization, $r = (\sqrt{2} + 1)^2 = 3 + 2\sqrt{2}$. Since the radii of the three circles are all proportional, it follows that the radius of C_3 must be $(3 + 2\sqrt{2})r = (3 + 2\sqrt{2})^2 = 17 + 12\sqrt{2}$, and therefore the circumference of C_3 is $(34 + 24\sqrt{2})\pi$, making the desired quantity $34 + 24 + 2 = 60$.