FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 6
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Lines and Angles

1-1 The sum of the measures of the interior angles of a regular convex polygon is four times the sum of its exterior angles. What is the difference between the measures of one interior angle and one exterior angle?
[Answer: 108]

1-2 A lattice point has integer coordinates. The corners of triangle ABC are lattice points. C is at the origin, B is on the positive x-axis, and A is in the first quadrant. The slope of $\overleftrightarrow{AC}$ is $\frac{5}{3}$. The slope of $\overleftrightarrow{AB}$ is $-\frac{3}{4}$. What is the minimum sum of A's coordinates?
[Answer: 24]

1-3 A certain convex polygon contains no right angles, and its number of obtuse angles is 2026 times its number of acute angles. Find the greatest possible number of sides of the polygon.
[Answer: 6081]

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 6
Individual Section

Please write your answers on the answer sheet provided.

Round 2: Literal Equations

2-1 Consider the simultaneous equations $a = \frac{1}{b} + 2$, $b = \frac{c+6}{c+1}$, and $c = a - 4$. The possible values of a are $\frac{m \pm \sqrt{n}}{p}$, where n has no perfect square factors greater than 1. Find the product mnp.
[Answer: 10]

2-2 If $3fg + fg^2 + 48 = 3g^2$ for positive real numbers f and g, then f can take any value in the interval $(0, a)$ and the smallest possible integer value of g is b. Find b^a.
[Answer: 125]

2-3 Every ordered pair (A, B) that satisfies the equation $y = \frac{2+3x}{5+x^2}$ also satisfies the equation $(2xy - a)^2 = (b + cy)(d - ey)$ for positive integers a, b, c, d, and e. Find $a + b + c + d + e$.
[Answer: 25]

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 6
Individual Section

Please write your answers on the answer sheet provided.

<u>Round 3: Solids & Volume</u>

3-1 Phil and Bobby each have a piece of 8” x 12” paper. Each rolled their paper to make a cylinder. Phil rolled his to make a tall, narrow cylinder, while Bobby rolled his to make a shorter, wider cylinder. The difference in the volumes of their cylinders is $\frac{a}{\pi}$ cubic inches. Find a.
[Answer: 96]

3-2 A composite figure is created as follows. The sculptor begins with a solid prism with 10 cm by 10 cm square base and 20 cm height, then carves a piece out of the bottom and adds it to the top. She removes a right square pyramid with height 12 cm and a base that perfectly aligns with the bottom base of the prism. Then she attaches the pyramid to the top of the prism so that there is no overlap. What is the surface area of the composite figure in square centimeters?
[Answer: 1320]

3-3 A particular solid has surface area S and volume V. If a similar solid has surface area $2S$ and volume $V + 10$, then $V = \frac{a+b\sqrt{c}}{d}$ where $a, b, c,$ and d are positive integers, c has no perfect square factors greater than 1, and $a, b,$ and d have no common factors greater than 1. Find $a + b + c + d$.
[Answer: 39]

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 6
Individual Section

Please write your answers on the answer sheet provided.

Round 4: Radical Expressions and Equations

4-1 Let $A = \sqrt{2026 + \sqrt{2026 + \sqrt{2026 + \cdots}}}$ and $B = \sqrt{2026 - \sqrt{2026 - \sqrt{2026 - \cdots}}}$
Find $A - B$.
[Answer: 1]

4-2 Find the sum of all real values of x that satisfy the equation $\sqrt[3]{x^2 - 4} = \frac{x+2}{\sqrt[3]{x+7}}$.
[Answer: 16]

4-3 Consider the function $f(x) = 3\sqrt{x+6} + 8$. The function $g(x) = af(x+b) + c$, where a, b, and c are integer constants, has a domain of $[3, \infty)$, has a range of $[-1, \infty)$, and contains the point $(4,5)$. If $f(x)$ and $g(x)$ both contain the point (m, n), find $m + n$.
[Answer: 42]

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 6
Individual Section

Please write your answers on the answer sheet provided.

Round 5: Polynomials and Advanced Factoring

5-1 The functions $f(x) = x^3 - 4x$ and $g(x) = 2x^2 - 8$ have the points (a, b) and (c, d) in common. Find $a^2 + b^2 + c^2 + d^2$.
[Answer: 8]

5-2 The polynomial $k(x) = x^4 + ax^3 + bx^2 + cx + 13$ with all rational coefficients has a zero of $2 + 3i$, and the sum of all zeros is 4. Find $a + b + c$.
[Answer: 6]

5-3 A particular cubic polynomial $g(x)$ with rational coefficients has a zero of $1 - 3i$ and a remainder of 162 when divided by $x - 4$. If one of the zeros is a positive integer, find the largest possible positive value of the leading coefficient.
[Answer: 9]

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 6
Individual Section

Please write your answers on the answer sheet provided.

Round 6: Counting and Probability

6-1 If A, B, and C are independent events occurring with probabilities 1, 0.3, and 0.8 respectively, the probability that exactly 2 of those events occur can be expressed as a reduced fraction $\frac{a}{b}$. Find $a + b$.
[Answer: 81]

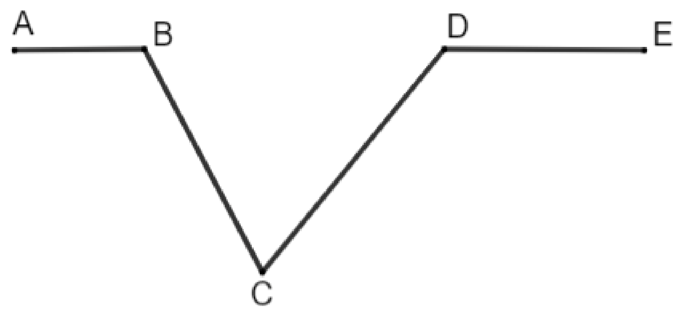
6-2 Arrange six consecutive integers from 2021 to 2026 in random order. The product of the first three numbers is p_1, and the product of the remaining three numbers is p_2. The probability that $p_1 + p_2$ is odd can be represented as $\frac{a}{b}$, where a and b are mutually prime. Find $a + b$.
[Answer: 11]

6-3 Consider independent events A, B, and C, each of which can occur with the same probability p. If $P(A \cup B \cup C)$ is 25% greater than $P(A \cup B)$, then $p = \frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers, b contains no perfect square factors greater than 1, and a and c are mutually prime. Find $a + b + c$.
[Answer: 32]

FAIRFIELD COUNTY MATH LEAGUE 2025-2026
Match 6
Team Round

Please write your answers on the answer sheet provided.

T-1 Consider the diagram (not drawn to scale). Points $A, B, D,$ and E are collinear. If $m\angle ABC, m\angle BCD,$ and $m\angle CDE$ in degrees (all greater than 0 and less than 180) form an arithmetic sequence of integers (not necessarily in that order), find the sum of the least and greatest possible values of the measure of the largest of the three angles in degrees.
[Answer: 300]



T-2 On the xy-plane, the equation $2x^3 + 2xy - 8x^2 - 6x - 8y + 24 = 0$ describes a line and a parabola that intersect at the point (a, b). Find the value of $|ab|$.
[Answer: 52]

T-3 Point B lies on the surface (not the base) of a hemisphere with a surface area (including the base) of 675π. Point B lies directly over diameter $\overline{AC}$, and point D lies on $\overline{AC}$ directly beneath B. If $AD = 2DB$, then the volume of the figure formed by rotating triangle ABC about the diameter $\overline{AC}$ is $k\pi$. Find the value of k.
[Answer: 1440]

T-4 If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x + 105}$, find the sum of all integers a such that $f(a)$ and $g(a)$ are both integers.

[Answer: 3040]

T-5 A polynomial $f(x)$ has the property that $f(0) = 2$ and for all x, $f(x + 1) - f(x) = x^3 + 8$. If the degree of $f(x)$ is n, the leading coefficient of $f(x)$ in standard form is A and the sum of the remaining coefficients of $f(x)$ is B, find $n + \frac{B}{A}$.
[Answer: 43]

T-6 In how many ways can we arrange letters of "bluegreen" such that the first four letters are the letters of "blue" and the last five letters are the letters of "green", but no two adjacent letters are the same?
[Answer: 756]