FAIRFIELD COUNTY MATH LEAGUE 2025-2026
Match 5
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Fractions and Exponents

1-1 Find the sum of all values n such that $\frac{2^{n^2-1}}{8^{n+1}} = 64$.
[Answer: 3]

1-2 The value of $\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\frac{1}{48}+\cdots.\right)^2$ can be expressed as $\frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a+b$.
[Answer: 10]

1-3 If x and y are real numbers such that $\left(\frac{12^x}{288^y}\right)^x = 36$, find the integer value of $(64^y)^y$.
[Answer: 16]

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<u>Round 2: Rational Expressions and Equations</u>

2-1 The equation $\frac{x^2}{x-2} = \frac{k}{x-2} + 3$, with real constant k, has an extraneous solution. Solve for k.
[Answer: 4]

2-2 A 30% saline solution of unknown volume is added to 10mL of a 70% saline solution. The mixture is then diluted with an extra 1mL of pure water. The resultant mixture is a 40% saline solution. How many mL of the 30% saline solution were there initially?
[Answer: 26]

2-3 The graph of the function $f(x) = \frac{x+\frac{a}{x+1}}{x+5+\frac{b}{x+1}}$, where a and b are integer constants, has removable discontinuities (holes) at the points $(-1, y_1)$ and $(4, y_2)$ and a vertical asymptote at $x = k$. The quantity $|ky_1y_2|$ is $\frac{p}{q}$, where p and q are positive integers with no common factors greater than 1. Find $p + q$.
[Answer: 27]

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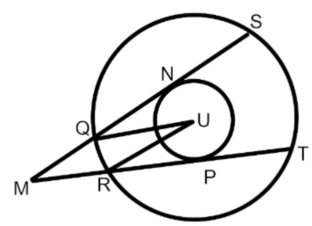
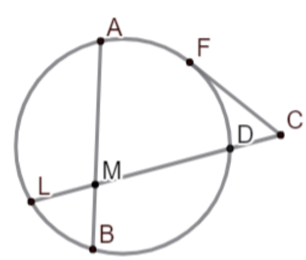
Round 3: Circles

3-1 A circle is inscribed in ΔGHI such that $\overline{GH}$ is tangent to the circle at point J, $\overline{HI}$ is tangent to the circle at point K and $\overline{GI}$ is tangent to the circle at point L. If $m\angle I = 52°$ and $m\widehat{JK} = 135°$, find $m\angle G$ in degrees.
[answer: 83]

3-2 See the diagram (not drawn to scale). Both circles have center U. $\overline{MS}$ is tangent to the smaller circle at N and $\overline{MT}$ is tangent to the smaller circle at P. If $m\,\widehat{ST}$ is double the $m\,\widehat{QR}$ and $m\angle QUR = 40°$, find $m\,\widehat{NP}$ in degrees.
[answer: 160]



3-3 See the diagram (not drawn to scale). Chords $\overline{AB}$ and $\overline{DL}$ intersect at point M, $\overline{FC}$ is tangent to the circle, and D lies on $\overline{CL}$. If DM and ML are solutions to the equation $x^2 - 40x + 51 = 0$, AM and MB are integers greater than 1, and $CF = 15$, find the value of $AB + CD$.
[Answer: 25]



FAIRFIELD COUNTY MATH23 LEAGUE 2025-2026
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Round 4: Quadratic Equations & Complex Numbers

4-1 A quadratic function with real number coefficients $f(x) = x^2 + bx + c$ has a zero of $x = 3 - 2i$. Find the value of $b + c$.
[Answer: 7]

4-2 The quadratic equation $(1 + i)z^2 + (a + bi)z + 6 = 0$ has a root at $z = 2$. What is the value of $|ab|$?
[Answer: 10]

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4-3 If $z = a + bi$ is a complex number with $|z| > 0$ such that $z^2 + z \cdot \bar{z} - (14a + (b^2 + 40)i) = 0$, find the sum of all possible values of $|z|^2$.
[Answer: 214]

FAIRFIELD COUNTY MATH LEAGUE 2025-2026
Match 5
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<u>Round 5: Trigonometric Equations</u>

5-1 If x is a quadrant 1 angle where $\cot^2(x) + \sec^2(x) - \tan^2(x) = \frac{10}{7}$ such that $\sin^2(x) = \frac{a}{b}$ where a and b share no common factors, determine $b^2 - a^2$.
[Answer: 51]

5-2 Let x be an acute angle such that if $\sin(x) + \cos(x) = \frac{\sqrt{5}}{2}$. If $\sin^3(x) + \cos^3(x) = \frac{a\sqrt{c}}{b}$, where c is prime and a and b share no common factors. Determine $a + b + c$.
[Answer: 28]

5-3 If θ is an angle in quadrant I such that $\frac{196}{\tan^2(2\theta)} = \sec^2(\theta) + 2\tan(\theta)$, then the sum of all possible values of $\tan(\theta)$ is $\frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.
[Answer: 73]

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Round 6: Sequences & Series

6-1 An arithmetic sequence has common difference d and first term a_1. If $a_{2026} = a_{20} + a_{26}$, find $\frac{a_1}{d}$.
[Answer: 1981]

6-2 The sum of an infinite geometric series is 5 and the first term is 2. If the sum of the squares of all the terms in this series can be represented as $\frac{a}{b}$, where a and b are mutually prime, what is $a - b$?
[Answer: 21]

6-3 A sequence of strictly increasing integers $a_1, a_2,\ldots$ follows the rule that $a_n = 2a_{n-1} + 2026$. What is the smallest possible value of a_{12}?
[Answer: 22]

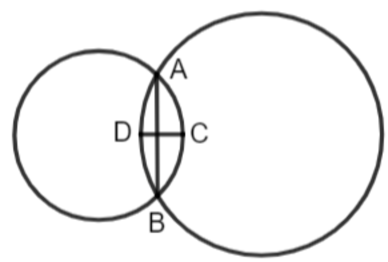
FAIRFIELD COUNTY MATH LEAGUE 2025-2026
Match 5
Team Round

Please write your answers on the answer sheet provided.

T-1 Let M be the set of all positive integers with 36 factors (including one and itself) that have both 2 and 5 as their only prime factors. Let N be the set of all positive integers with 12 factors (including one and itself) that have both 2 and 5 as their only prime factors. Let S be the set of all possible integer quotients of a number from set M and a number from set N. The product of the greatest and least elements in S is $a^b * 10^c$, where $a, b,$ and c are positive integers and a is prime. Find $a + b + c$.
[Answer: 17]

T-2 Rapido the rabbit races Vite the viper down a straight track. They both keep a constant speed, except when Rapido injures his foot halfway down the track, reducing his speed by half. This allows Vite to pass Rapido ¾ down the track. After Vite crosses the finish line, it takes Rapido another 20 seconds to run the last 30 feet of track. What is the length of the track in feet?
[Answer: 360]

T-3 See the diagram (not drawn to scale). Circles P with radius 6 and Q with radius 10 intersect each other at points A and B, with point C on P and point D on Q such that $\overline{CD}$ is a perpendicular bisector of $\overline{AB}$. If $\overline{AB}$ cuts $\overline{CD}$ into a 2:1 ratio, then $(AB)^2 = \frac{m}{n}$ where m and n are positive integers with no common factors greater than 1. Find $m + n$.
[Answer: 905]



T-4 There exists a complex number $z = a + bi$ where a and b are real numbers such that $a = 2b$ and $|(1 + i)(1 + z + z^2 + z^3 + \cdots)| = 3$. The product $ab = \frac{p}{q}$ where p and q are positive integers with no factors greater than 1. Find $p + q$.
[Answer: 11]

T-5 Consider right triangle AFL with right angle L. Points C and M lie on $\overline{FL}$ such that $\angle FAC \cong \angle CAM \cong \angle MAL$. If $AL = 12$ and $ML = 4$, then $FL = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.
[Answer: 55]

T-6 The sum of the first two terms of geometric series S_1 (first term 2025, ratio r) is 40% of the non-zero sum of the first eight terms. What is the sum of the first 4 terms of geometric series S_2 (first term 2026, ratio r^2)?
[Answer: 5065]