FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 4
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Basic Statistics

1-1 A set of 6 positive integers has a mean of 12, a unique mode of 8, and a median of 10. What is the largest possible value of the largest number in the set?

1-2 Alvin is playing 15 games as part of a bowling season. His average in games 7-10 is 25 greater than his average in games 1-6. His average in games 11-15 is $33\frac{1}{3}\%$ greater than his average in games 7-10, making his season average 30 points greater than his average in games 1-6. What was his season average?

1-3 The geometric mean of a set of n numbers is defined as the n^{th} root of the product of the numbers. Two numbers a and b have a geometric mean of 30. Increasing both a and b by a number k increases their arithmetic mean by 6 and increases their geometric mean by 12. Find $a + b$.

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Round 2: Quadratic Equations

2-1 The quadratic $y = ax^2 + bx + c$ where $a, b,$ and c are rational numbers has a zero of $-4 + 3i$ and contains the point (1, 17). If a, *b,* and *c* are real numbers, find the product abc.

2-2 Consider the functions $f(x) = 2mx^2 + 2mx + 5$ and $g(x) = mx^2 + 3x + 3$, where m is a real number. If $f(x)$ and $g(x)$ intersect at one point, then k is the sum of all possible values of m. Find $24k$.

2-3 What is the smallest positive integer a such that the equation $x^2 - ax + a + 2025 = 0$ has integer solutions?

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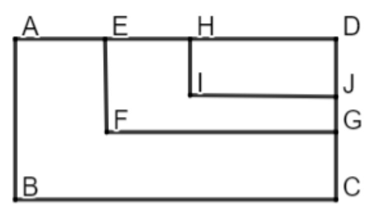
Round 3: Similarity

3-1 Jerry stands near a 32 foot tall tree. He is 6 feet tall. His shadow is 42 inches long. The tips of his shadow and that of the tree are at the same point. Exactly how close (in inches) to the tree does he stand?

3-2 There exists an isosceles triangle such that the height is 8 and the numerical values of its perimeter and area are the same. There exists a similar triangle whose area is twice the area of the first triangle. Find the perimeter of the second triangle.

3-3 See the diagram (not drawn to scale). A rectangular pen $ABCD$ is to be constructed such that there are similar rectangular pens, $EFGD$ and $HIJD$, and $\frac{\text{area } ABCD}{\text{area } EFGD} = \frac{\text{area } EFGD}{\text{area } HIJD}$. If the amount of material used to construct the required additional sides for $EFGD$ and $HIJD$ is equal to half the material required to construct $ABCD$, then $\frac{\text{area } HIJD}{\text{area } ABCD} = \frac{u - v\sqrt{w}}{z}$, where u, v, and w are positive integers and u and w have no common factors greater than 1. Find $u + v + w + z$.



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Round 4: Variation

4-1 The value of y varies directly with the square of x, and $y = 4$ when $x = 4$. What is the positive value of x when $y = 100$?

4-2 The value of a varies directly with b. Using the same positive constant of proportionality, the value of b varies inversely with c. There exists an integer x such that if $c = x$ then $a = x + 5$. What is the constant of proportionality?

4-3 Three positive values x, y, and z are related such that z varies with directly with x^3 and inversely with y to a power of $n > 0$. However, x and y are not independent; y varies directly with x to the power of $2n + 1$, which consequently allows z to be expressed (without y) as varying directly with x^2. If $z = 45$ when $y = 50$ and $x = 30$, then when $z = 54$, $\frac{x}{y} = \frac{\sqrt{a}}{b}$ where a and b are positive integers and a contains no perfect square factors greater than 1. Find $a + b$.

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Round 5: Trig Expressions & DeMoivre's Theorem

5-1 Let $z = \cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)$. If $z^4 = a + \frac{c\sqrt{d}}{f}i$ where c and f are positive integers no common factors greater than 1 and d is a positive integer with no perfect square factors greater than 1, find $af + cd$.

5-2 Let A and B be angles such that $\tan(A) = -1$ and $\tan(B) = 2$ with $\frac{\pi}{2} < A < \pi$ and $\pi < B < \frac{3\pi}{2}$. Then $\sin(2A) + \cos(2B) + \frac{c}{g} = 0$, where c and g are positive integers with no perfect square factors greater than 1. Find cg.

5-3 If θ is an angle in quadrant I such that $\tan\left(2\theta + \frac{\pi}{4}\right) = 5$, then $\theta = \arctan\left(\frac{\sqrt{a}-b}{c}\right)$, where $a, b,$ and c are positive integers and a has no perfect square factors greater than 1. Find $a + b + c$.

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Round 6: Conic Sections

6-1 Points $A(2,3)$ and $B(7,8)$ are on a circle with center (h,k). Find $h+k$.

6-2 Points $A(x_1,y_1)$ and $B(x_2,y_2)$ are both on the parabola with vertex $(0,0)$ and focus $(1,0)$. If $x_1x_2 + y_1y_2 = 12$, find the maximum value of x_1x_2.

6-3 For given values of a and b, consider an ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and a hyperbola with equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The ellipse has a focus $(0,6)$ and an area, given by πab, of $k\pi$. The hyperbola has a focus $\left(2\sqrt{15},0\right)$ and an asymptote that passes through the point $(10,p)$. Find $k+p^2$.

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Match 4
Team Round

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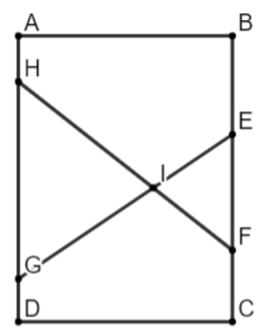
1. Sets A and B both consist of 5 distinct positive integers. Q_1, Q_2 and Q_3 of set A are equal to Q_0, Q_1, and Q_3 of set B, respectively. Set B has the property that both its mean and median are equal to 10, which is also the amount by which the range of set A is larger than the range of set B. Find the number of possible sums of the elements of A.

2. A particular quadratic equation $ax^2 - bx + c = 0$ has the property that $a, b,$ and c are positive prime integers less than 100 and the solutions to the equation are rational numbers greater than or equal to 1. Find the largest possible value of $a + b + c$.

3. See the diagram (not drawn to scale). For rectangle $ABCD$, $CF = 2AH$, $BE = 3DG$, $AB = EF$, the area of triangle EFI is 50 square units, and the area of triangle GHI is 450 square units. If AH and DG are both positive integers, find the largest possible value of the area of $ABCD$.



4. The variable w varies jointly as x to the power of a and y to the power of b, and inversely as z to the power of c, where $a, b,$ and c are positive integers. The units of x are oingos, the units of y are oingo-boingos, the units of z are boingos, and the units of w are oingos cubed per boingo. If $w = 100$ when $x = 40$, $y = 20$, and $z = 36$, find the sum of all possible values of w when $x = 80, y = 120$, and $z = 72$. Assume the proportionality constant is unitless.

5. A complex number z has one complex n^{th} root with an argument of $112°$ and another complex n^{th} root with an argument of $172°$. Given the maximum possible value of $n \leq 50$, find the argument of z expressed in degrees between 0 and 360.

6. Consider an ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ that contains the point $(2,4)$ and whose area, given by πab, is 36π. If we define S as the sum of all possible positive values of a, find S^2.