FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Decimals and Base Notation

1-1 Express the product of 234_5 multiplied by 67_8 as a numeral in base 10.
[Answer: 3795]

1-2 If x is a positive integer such that $203_{x+1} + 210_{x-1} = 500_{x-1} + 31_x$, find the value of
2025_x as a numeral in base 10.
[Answer: 1045]

1-3 The arithmetic shown was done in base B. The letters a and $b, c, d,$ and e denote
missing digits in base B. Enter your answer as the sequence of digits $abcde$.
[Answer: 31345]

$$\begin{array}{r} a\,4 \\ \times\,b\,2 \\ \hline 1\,1\,2 \\ +\,c\,4\,0 \\ \hline d\,e\,2 \end{array}$$

Please write your answers on the answer sheet provided.

Round 2: Word Problems

2-1   Polly's sister, Tara, offers to do Polly's chores for a month in exchange for $\frac{1}{2}$ of what Polly has in her savings account. Polly says that she owes $10 to her brother, Erik. Tara states that she'll take $\frac{3}{5}$ of the money left after Erik is paid. If Tara would have received the same amount of money with or without Erik getting paid back, how much money is in Polly's savings account?
[Answer: 60]

2-2   On a work day, it takes Zach three times as long to drive to work as it does to come home. Assume the distances are the same. Zach compares his total average speed, a (for both trips to and from work), with his average speed driving home only, h. What percent of h is a?
[Answer: 50]

2-3   Three candles marked $A, B,$ and C are to be lit and burned until exhausted. Assume all three candles burn at a constant (though not necessarily equal) rate. Candle A is lit first. Candle B is lit ten minutes after candle A is lit. Candle C is lit when candle A is exactly halfway through burning, and candle C finishes burning exactly 20 minutes after candle A finishes. Candle B finishes burning exactly 15 minutes after candle C finishes. If candle C takes the least time to burn and the total time for all three candles to burn separately is less than five hours, then the set of all possible times in minutes t that candle A takes to burn is defined by the interval $a < t < b$. Find $a + b$.
[Answer: 142]

Please write your answers on the answer sheet provided.

Round 3: Polygons

3-1   Find the sum of all the values of n for which the measures of each exterior angle of a
      regular n-gon is a positive integer multiple of n.
      [Answer: 9]

3-2   The number of diagonals in a regular polygon is 45 more than 5 times the number of sides.
      Find the measure of one interior angle of the polygon.
      [Answer: 160]

3-3   A particular concave n-gon has 860 diagonals and x right angles. The polygon's remaining
      angles are all congruent with a measure of d degrees, where d is an integer. Find the
      smallest possible value of d.
      [Answer: 207]

Please write your answers on the answer sheet provided.

Round 4: Function and Inverses

Note: the inverse f^{-1} of a function is not necessarily a function.

4-1 Given $f(x) = \frac{5}{x-4}$, find the sum of all possible values of a such that $f(a) = f^{-1}(a)$.
[Answer: 4]

4-2 The ordered pair (a, b) is a solution to the equation $p(x) = q(x)$ where
$f(x) = p(x + 1) = q(2x) = 8 + 6x$. Find the product ab.
[Answer: 28]

4-3 Consider the function $f(x) = \frac{7x+5}{ax-b}$, where a and b are positive integers and $a > b$. If
$f(f(x))$ is undefined at $x = 7$, then the range of $f(x)$ is $\left(-\infty, \frac{p}{q}\right) \cup \left(\frac{p}{q}, \infty\right)$ where p and q
are positive integers with no common factors greater than 1. Find $p + q$.
[Answer: 39]

Please write your answers on the answer sheet provided.

Round 5: Exponents & Logarithms

5-1 If $\log_4 a = 2$ and $\log_4 b = a$ and $\log_b c = \frac{1}{3}$. Find $\log_4 c = \frac{p}{q}$ where p and q share no common divisors. Find $p + q$.
[Answer: 19]

5-2 The solution to the equation $2\log_{25} x + \log_{125} x^4 = 3$ is of the form $x = a^{\frac{b}{c}}$ with prime base a and with b and c relatively prime. Find $a + b - c$.
[Answer: 7]

5-3 If $9^{\log(4)} = \left(2^{\log(x)}\right)\left(12^{\log(8)}\right)$, then $x = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a^a + b^{\frac{1}{a}}$.
[Answer: 31]

Please write your answers on the answer sheet provided.

<u>Round 6: Matrices</u>

6-1   Matrix $A = \begin{bmatrix} a & 2025a \\ \frac{1}{a} & a \end{bmatrix}$ has $\det(A) = 56a$. If $\det(A) > 0$, find a.

[Answer: 81]

6-2   Matrix $A = \begin{bmatrix} \frac{x}{2} & 3 \\ 1 & x \end{bmatrix}$ and vector $b = \begin{bmatrix} 4 \\ m \end{bmatrix}$ satisfy $Ab = b$. If $x > 0$, find $\det(A)$.

[Answer: 5]

6-3   If $A = \begin{bmatrix} -\frac{3}{2} & 2 \\ 2 & -3 \end{bmatrix}$ and B is a 2x2 matrix such that $AB + I = 2A$, where I is the identity
matrix, find the sum of the elements of B.

[Answer: 21]

Please write your answers on the answer sheet provided.

1. Let B be the smallest positive integer (in base 10) such that the fraction $\frac{2}{2025}$ in base 10 can be written as a terminating decimal in base B. Let A be the integer in base 10 formed from the non-zero digits of $\frac{2}{2025}$ when written as a decimal in base B (for example, an answer of $.0271_B$ would mean $A = 271$). Find $A + B$ as expressed as a numeral in base 10.
   [Answer: 50]

2. Mr. Zucca is teaching Mr. Hill and Mr. Forgette how to make giant snowflake decorations for the FCML party. First Mr. Zucca works with Mr. Hill and they complete the first snowflake decoration in 40 minutes. Then Mr. Zucca works with Mr. Forgette and they complete the second decoration in 35 minutes. After that, all three work together on the third decoration and it takes 32 minutes. Confident his pupils have learned, Mr. Zucca steps away to let Mr. Hill and Mr. Forgette complete the fourth decoration. How many minutes will it take them to do so, working together?
   [Answer: 112]

3. If a and b are positive integers such that $a < b < 180$ and the measure of one interior angle of a regular a-gon, the measure of one interior angle of a regular b-gon, and the measure of one interior angle of a regular 180-gon form an arithmetic sequence. Find the smallest possible value of $a + b$.
   [Answer: 56]

4. The function $g(x) = a\sqrt{x + b} - c$, where a, b, and c are positive constants, has a domain of $[-3, \infty)$. The function $g^{-1}(x)$ has a domain of $[-7, \infty)$ and intersects $g(x)$ at only point. Find $g(g(97))$.
   [Answer: 17]

5. The equation $3 \log_8(x) = \log_x(64x)$ has solutions a and b. If $\log(2) \approx .301$, find the number of digits in 16^{a+b}.
   [Answer: 10]

6. Consider matrices $A = \begin{bmatrix} 3 & 9 \\ u & v \end{bmatrix}$ and $B = \begin{bmatrix} w & 7.5 \\ x & y \end{bmatrix}$ such that…

   - …$u, v, w, x,$ and y are constants,
   - …the product AB equals the element-wise product of A and B (the element-wise product is calculated by multiplying the corresponding matrix elements as in matrix addition),
   - …$\det(AB) = 2025$.

   Find the value of vw.
   [Answer: 135]