FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Decimals and Base Notation

1-1 Express the product of 234_5 multiplied by 67_8 as a numeral in base 10.
[Answer: 3795]

$234_5 = 2 * 5^2 + 3 * 5 + 4 = 69$ in base 10, while $67_8 = 6 * 8 + 7 = 55$ in base 10, and $(69)(55) = 3795$.

1-2 If x is a positive integer such that $203_{x+1} + 210_{x-1} = 500_{x-1} + 31_x$, find the value of 2025_x as a numeral in base 10.
[Answer: 1045]

Turning the statement into an equation in terms of x yields $2(x+1)^2 + 3 + 2(x-1)^2 + x - 1 = 5(x-1)^2 + 3x + 1$. When expanded and written in standard form the quadratic becomes
$x^2 - 8x = 0$, so $x = 8$. Therefore the desired quantity is $2 * 8^3 + 2 * 8 + 5 = 1024 + 16 + 5 = 1045$.

1-3 The arithmetic shown was done in base B. The letters a and $b, c, d,$ and e denote missing digits in base B. Enter your answer as the sequence of digits $abcde$.
[Answer: 31345]

$$\begin{array}{r} a\,4 \\ \underline{\times\, b\,2} \\ 1\,1\,2 \\ \underline{+\, c\,4\,0} \\ d\,e\,2 \end{array}$$

The base must be larger than 4, and since $(4_B)(2_B) = 12_B$, it follows that $8 = B + 2$, so $B = 6$. Since $2a + 1 = 11_B = 7$, that means $a = 3$. Because $4b$ has to end in 4 in base 6, b has to be either 1 or 4, since $(4)(1) = 4_B$ and $4(4) = 24_B$. However $(34)(40)$ would have more than 3 digits in base 6, so $b = 1$ and $c = 3$. This means $d = 4$ and $e = 5$, making the desired sequence 31345.

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Round 2: Word Problems

2-1 Polly's sister, Tara, offers to do Polly's chores for a month in exchange for $\frac{1}{2}$ of what Polly has in her savings account. Polly says that she owes \$10 to her brother, Erik. Tara states that she'll take $\frac{3}{5}$ of the money left after Erik is paid. If Tara would have received the same amount of money with or without Erik getting paid back, how much money is in Polly's savings account?
[Answer: 60]

Let the amount in the savings account be x. From the problem, $\frac{1}{2}x = \frac{3}{5}(x - 10)$, so $\frac{1}{2}x = \frac{3}{5}x - 6$, or $\frac{1}{10}x = 6$, and thus $x = 60$.

2-2 On a work day, it takes Zach three times as long to drive to work as it does to come home. Assume the distances are the same. Zach compares his total average speed, *a* (for both trips to and from work), with his average speed driving home only, *h*. What percent of *h* is *a*?
[Answer: 50]

Let the distance traveled one way from home to work be d. This means $a = \frac{2d}{3\left(\frac{d}{h}\right)+\frac{d}{h}} = \frac{2d}{\frac{4d}{h}} = \frac{1}{2}h$. Therefore a is 50% of h.

2-3 Three candles marked $A, B,$ and C are to be lit and burned until exhausted. Assume all three candles burn at a constant (though not necessarily equal) rate. Candle A is lit first. Candle B is lit ten minutes after candle A is lit. Candle C is lit when candle A is exactly halfway through burning, and candle C finishes burning exactly 20 minutes after candle A finishes. Candle B finishes burning exactly 15 minutes after candle C finishes. If candle C takes the least time to burn and the total time for all three candles to burn separately is less than five hours, then the set of all possible times in minutes t that candle A takes to burn is defined by the interval $a < t < b$. Find $a + b$.
[Answer: 142]

Candle C will take $\frac{1}{2}t + 20$ minutes to burn, and candle B will take $t - 10 + 35 = t + 25$ minutes to burn. Candle B must take longer than candle A, so for C to take the least time it must take less than A, so $\frac{1}{2}t + 20 < t$, so $t > 40$. Also we have $t + (t + 25) +$

$\left(\frac{1}{2}t + 20\right) < 300$, so $\frac{5}{2}t < 255$, so $t < 102$. This means $40 < t < 102$, making the desired quantity $40 + 102 = 142$.

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Round 3: Polygons

3-1 Find the sum of all the values of n for which the measures of each exterior angle of a regular n-gon is a positive integer multiple of n.
[Answer: 9]

Setting up $\frac{360}{n} = kn$, or $\frac{360}{k} = n^2$, we find perfect square factors of 360, which are $1, 4, 9$, and 36. Since $n > 2$ and thus $n^2 > 4$, the only possible options are $n = 3$ and $n = 6$, making the desired quantity $3 + 6 = 9$.

3-2 The number of diagonals in a regular polygon is 45 more than 5 times the number of sides. Find the measure of one interior angle of the polygon.
[Answer: 160]

Let the number of sides be x. We have $\frac{x(x-3)}{2} = 45 + 5x$, which written as a quadratic in standard form becomes $x^2 - 13x - 90 = 0$, which factors into $(x - 18)(x + 5) = 0$, so $x = 18$. Therefore the desired quantity is $x = 180 - \frac{360}{18} = 160$.

3-3 A particular concave n-gon has 860 diagonals and x right angles. The polygon's remaining angles are all congruent with a measure of d degrees, where d is an integer. Find the smallest possible value of d.
[Answer: 207]

The value of n can be determined by $860 = \frac{n(n-3)}{2}$, so $n = 43$. The sum of all angle measures must be $(180)(41) = 7380 = 90x + d(43 - x)$. Isolating d yields $d = \frac{7380-90x}{43-x} = \frac{90(82-x)}{43-x}$. To minimize d, we want to find the smallest integer value of x that makes this quantity an integer. This will occur when $x = 13$, making $d = 3(69) = 207$.

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Round 4: Function and Inverses

Note: the inverse f^{-1} of a function is not necessarily a function.

4-1 Given $f(x) = \frac{5}{x-4}$, find the sum of all possible values of a such that $f(a) = f^{-1}(a)$.
[Answer: 4]

In order for $f(a) = f^{-1}(a)$, it follows that $a = f(a) = f^{-1}(a)$. Therefore $a = \frac{5}{a-4}$, so $a^2 - 4a - 5 = 0$, making the desired sum of the values 4.

4-2 The ordered pair (a, b) is a solution to the equation $p(x) = q(x)$ where $f(x) = p(x+1) = q(2x) = 8 + 6x$. Find the product ab.
[Answer: 28]

If $p(x+1) = 8 + 6x$, then $p(x) = 8 + 6(x-1) = 2 + 6x$. Similarly if $q(2x) = 8 + 6x$, then $q(x) = 8 + 6\left(\frac{x}{2}\right) = 8 + 3x$. Therefore $2 + 6a = 8 + 3a$, yielding $3a = 6$ and thus $a = 2$, making $p(2) = 2 + 6(2) = 14$ and the desired quantity $(2)(14) = 28$.

4-3 Consider the function $f(x) = \frac{7x+5}{ax-b}$, where a and b are positive integers and $a > b$. If $f(f(x))$ is undefined at $x = 7$, then the range of $f(x)$ is $\left(-\infty, \frac{p}{q}\right) \cup \left(\frac{p}{q}, \infty\right)$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.
[Answer: 39]

One possibility is that $f(7)$ is undefined, so $7a - b = 0$, but this means $b = 7a$ which does not comply with the condition $a > b$. Therefore $f(7) = \frac{b}{a}$, or $\frac{b}{a} = \frac{54}{7a-b}$, yielding $7ab - b^2 = 54a$. Solving for $a = \frac{b^2}{7b-54}$. Since $a > 0$, we have $7b > 54$, making the first few ordered pairs of positive integers (a, b) (32,8) and (9,9), but as a continues to decrease it is clear the only pair that works is (32,8). Therefore $f(x) = \frac{7x+5}{32x-8}$, and so the range is $y \neq \frac{7}{32}$, making the desired quantity $7 + 32 = 39$.

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Round 5: Exponents & Logarithms

5-1 If $\log_4 a = 2$ and $\log_4 b = a$ and $\log_b c = \frac{1}{3}$. Find $\log_4 c = \frac{p}{q}$ where p and q share no common divisors. Find $p + q$.
[Answer: 19]

Since $a = 4^2 = 16$, $4^{16} = b$, and $(4^{16})^{\frac{1}{3}} = c$, then $\log_4\left(4^{\frac{16}{3}}\right) = \frac{16}{3}$, making the desired quantity $16 + 3 = 19$.

5-2 The solution to the equation $2\log_{25} x + \log_{125} x^4 = 3$ is of the form $x = a^{\frac{b}{c}}$ with prime base a and with b and c relatively prime. Find $a + b - c$.
[Answer: 7]

Using the fact that $\log_{25}(x) = \frac{1}{2}\log_5(x)$ and $\log_{125}(x) = \frac{1}{3}\log_5(x)$, we have $\log_5(x) + \frac{4}{3}\log_5(x) = 3$, or $\frac{7}{3}\log_5(x) = 3$, yielding $x = 5^{\frac{9}{7}}$, making the desired quantity $5 + 9 - 7 = 7$.

5-3 If $9^{\log(4)} = \left(2^{\log(x)}\right)\left(12^{\log(8)}\right)$, then $x = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a^a + b^{\frac{1}{a}}$.
[Answer: 31]

We have $3^{2\log(4)} = \left(2^{\log(x)}\right)\left(2^{2\log(8)}\right)\left(3^{\log(8)}\right)$, and therefore $\frac{3^{2\log(4)}}{3^{\log(8)}} = \left(2^{\log(x)}\right)\left(2^{2\log(8)}\right)$, yielding $3^{\log(2)} = 2^{\log(64x)}$. Taking the logarithm of both sides yields $\log(2)\log(3) = \log(64x)\log(2)$, and so $64x = 3$, yielding $x = \frac{3}{64}$, making the desired quantity $3^3 + 64^{\frac{1}{3}} = 27 + 4 = 31$.

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Round 6: Matrices

6-1 Matrix $A = \begin{bmatrix} a & 2025a \\ \frac{1}{a} & a \end{bmatrix}$ has $\det(A) = 56a$. If $\det(A) > 0$, find a.
[Answer: 81]

We have $a^2 - 2025 = 56a$, or $a^2 - 56a - 2025 = 0$, which factors into $(a - 81)(a + 25) = 0$, but from the restriction in the problem, we have $a = 81$.

6-2 Matrix $A = \begin{bmatrix} \frac{x}{2} & 3 \\ 1 & x \end{bmatrix}$ and vector $b = \begin{bmatrix} 4 \\ m \end{bmatrix}$ satisfy $Ab = b$. If $x > 0$, find $\det(A)$.
[Answer: 5]

Setting up the system $4\left(\frac{x}{2}\right) + 3m = 4$ and $4 + mx = m$, we have $m = \frac{4}{1-x}$, and therefore $2x + \frac{12}{1-x} = 4$, yielding the quadratic $x^2 - 3x - 4 = 0$. (Note: Since $Ab = b$ means $Ab - b = 0$, then $(A - I)b = 0$, and therefore $\det(A - I) = 0$, which produces the same quadratic.) From the problem this means that $x = 4$, and therefore $\det(A) = (2)(4) - (3)(1) = 5$.

6-3 If $A = \begin{bmatrix} -\frac{3}{2} & 2 \\ 2 & -3 \end{bmatrix}$ and B is a 2x2 matrix such that $AB + I = 2A$, where I is the identity matrix, find the sum of the elements of B.
[Answer: 21]

Since $AB = 2A - I$, we have $B = A^{-1}(2A - I) = 2AA^{-1} - A^{-1} = 2I - A^{-1}$. Since $\det(A) = \frac{9}{2} - 4 = \frac{1}{2}$, it follows that $A^{-1} = 2\begin{bmatrix} -3 & -2 \\ -2 & -\frac{3}{2} \end{bmatrix} = \begin{bmatrix} -6 & -4 \\ -4 & -3 \end{bmatrix}$. Then $B = 2I - A^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} -6 & -4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 4 & 5 \end{bmatrix}$, making the desired quantity $8 + 4 + 4 + 5 = 21$.

FAIRFIELD COUNTY MATH LEAGUE 2025-2026
Match 3
Team Round

Please write your answers on the answer sheet provided.

1. Let B be the smallest positive integer (in base 10) such that the fraction $\frac{2}{2025}$ in base 10 can be written as a terminating decimal in base B. Let A be the integer in base 10 formed from the non-zero digits of $\frac{2}{2025}$ when written as a decimal in base B (for example, an answer of $.0271_B$ would mean $A = 271$). Find $A + B$ as expressed as a numeral in base 10.
[Answer: 50]

Because $2025 = (3^4)(5^2)$, the smallest base in which the fraction could be written as a terminating decimal has to allow for powers of 3 and 5 in the denominator, and therefore must be 15. Since $\frac{2}{2025} = \frac{2}{3^2(3^2 5^2)} = \frac{2(5^2)}{3^4 5^4} = \frac{50}{15^4}$, in base fifteen the numerator and denominator become $\frac{35}{10000}$, which as a decimal is $.0035$. Therefore $A = 35$, making the desired quantity $35 + 15 = 50$.

2. Mr. Zucca is teaching Mr. Hill and Mr. Forgette how to make giant snowflake decorations for the FCML party. First Mr. Zucca works with Mr. Hill and they complete the first snowflake decoration in 40 minutes. Then Mr. Zucca works with Mr. Forgette and they complete the second decoration in 35 minutes. After that, all three work together on the third decoration and it takes 32 minutes. Confident his pupils have learned, Mr. Zucca steps away to let Mr. Hill and Mr. Forgette complete the fourth decoration. How many minutes will it take them to do so, working together?
[Answer: 112]

Let Mr. Zucca's rate be z, Mr. Hill's rate be h, and Mr. Forgette's rate be f, all in snowflakes per minute. We have $z + h = \frac{1}{40}$, $z + f = \frac{1}{35}$, and $z + h + f = \frac{1}{32}$. Note that $2(z + h + f) = \frac{2}{32} = \frac{1}{16}$, and $h + f = 2(z + h + f) - (z + h) - (z + f) = \frac{1}{16} - \frac{1}{40} - \frac{1}{35} = \frac{35-14-16}{16*35} = \frac{1}{16*7} = \frac{1}{112}$, so Mr. Hill and Mr. Forgette will take 112 minutes.

3. If a and b are positive integers such that $a < b < 180$ and the measure of one interior angle of a regular a-gon, the measure of one interior angle of a regular b-gon, and the measure of one interior angle of a regular 180-gon form an arithmetic sequence. Find the smallest possible value of $a + b$.
[Answer: 56]

One way to solve this is to write b in terms of a and then find the smallest integer value of a that produces an integer value for b. Setting up $180 - \frac{360}{b} - \left(180 - \frac{360}{a}\right) = 180 - \frac{360}{180} - \left(180 - \frac{360}{b}\right)$, or $\frac{360}{a} - \frac{360}{b} = \frac{360}{b} - 2$. This means $\frac{360}{a} + 2 = \frac{720}{b}$, or $\frac{(180+a)}{a} = \frac{360}{b}$, yielding

$b = \frac{360a}{180+a}$. The smallest value of a that works is $a = 20$, yielding a value of $\frac{360(20)}{200} = \frac{360}{10} = 36$ for b, making the desired quantity $20 + 36 = 56$.

4. The function $g(x) = a\sqrt{x+b} - c$, where a, b, and c are positive constants, has a domain of $[-3, \infty)$. The function $g^{-1}(x)$ has a domain of $[-7, \infty)$ and intersects $g(x)$ at only point. Find $g(g(97))$.
[Answer: 17]

The domain of $g(x)$ tells us that $b = 3$, and the domain of $g^{-1}(x)$, i.e. the range of $g(x)$, tells us that $c = 7$. Therefore $g(x) = a\sqrt{x+3} - 7$. The intersection must occur when $g(x) = x$, so setting $x = a\sqrt{x+3} - 7$ yields $(x+7)^2 = a^2(x+3)$, or $x^2 + (14 - a^2)x + 49 - 3a^2$. Now setting $(14 - a^2)^2 - 4(49 - 3a^2) = 0$ yields $a^4 - 16a^2 = 0$, so $a = \pm 4$, but since $a > 0$, we know $g(x) = 4\sqrt{x+3} - 7$. Finally $g(g(97)) = g(33) = 17$.

5. The equation $3\log_8(x) = \log_x(64x)$ has solutions a and b. If $\log(2) \approx .301$, find the number of digits in 16^{a+b}.
[Answer: 10]

First note $\log_x(64x) = \log_x(64) + 1 = \frac{1}{\log_{64}(x)} + 1 = \frac{2}{\log_8(x)} + 1$. Therefore the equation becomes $3\log_8(x) = \frac{2}{\log_8(x)} + 1$, or $3(\log_8(x))^2 - \log_8(x) - 2 = 0$. This can be factored into $(3\log_8(x) + 2)(\log_8(x) - 1) = 0$, and thus our solution are found by solving $\log_8(x) = -\frac{2}{3}$ and $\log_8(x) = 1$, making the solutions $\frac{1}{4}$ and 8 respectively. Thus we have $16^{\frac{1}{4}+8} = 16^{\frac{33}{4}} = 2^{33}$, and $\log(2^{33}) = 33\log(2) \approx 33(.301) = 9.933$, meaning there are 10 digits in this value.

6. Consider matrices $A = \begin{bmatrix} 3 & 9 \\ u & v \end{bmatrix}$ and $B = \begin{bmatrix} w & 7.5 \\ x & y \end{bmatrix}$ such that…

- …u, v, w, x, and y are constants,
- …the product AB equals the element-wise product of A and B (the element-wise product is calculated by multiplying the corresponding matrix elements as in matrix addition),
- …$\det(AB) = 2025$.

Find the value of vw.
[Answer: 135]

Setting products of AB equal to element-wise products yields the equations $3w + 9x = 3w$, $22.5 + 9y = 67.5$, $uw + vx = ux$, and $7.5u + vy = vy$. The first equation yields $x = 0$ and the fourth equation yields $u = 0$. The second equation yields $y = 5$. Since $u = x = 0$, $\det(A) = 3v$ and $\det(B) = 5w$. Also, since $\det(AB) = \det(A)\det(B)$, it follows that $2025 = (3v)(5w)$, meaning $vw = \frac{2025}{15} = 135$.