FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 2
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Factors and Multiples

1-1 Consider all positive integers from 1 to 100 inclusive that have an odd number of factors (including themselves). Set A contains all of the distinct factors of these numbers with nothing repeated. Find the number of elements in set A.
[Answer: 23]

Since only perfect squares have an odd number of factors, set A contains all the factors of the perfect squares up to 100 with no repeats: $\{1,4,2,9,3,16,8,25,5,36,18,12,6,49,7,64,32,81,27,100, 50,20,10\}$

1-2 If n is an integer such that $lcm(n, 2025) = 20250$ and $lcm(n, 360) = 9000$, find the sum of all possible values of n.
[Answer: 3250]

Since $lcm(n, (3^4)(5^2)) = (2)(3^4)(5^3)$, we know n must have at 2 and 5^3 in its factorization, as well as any integer power of 3 from 0 to 4. Then since $lcm(n, (2^3)(3^2)(5)) = (2^3)(3^2)(5^3)$, we know n must have a power of 3 no higher than 2. This makes the sum of all possible values of n $(2)(5^3)(3^0 + 3^1 + 3^2) = (2)(125)(13) = 3250$.

1-3 A positive integer n has exactly 2025 factors, including four distinct prime factors, and ends in four zeros. The number $\frac{n}{10!}$ is an odd integer that is divisible by 21 but not 245. Find the largest possible integer value of k such that 3^k is a factor of n.
[Answer: 14]

The number n must consist of at least $2^4 * 5^4$. Since $10! = (2^8)(3^4)(5^2)(7)$ and $\frac{n}{10!}$ is an odd integer, n must have $2, 3, 5,$ and 7 as prime factors, and also we know 8 is the highest power of 2 in the factorization of n and $\frac{n}{10!}$ has at least 5^2 in the quotient. Since $\frac{n}{10!}$ is divisible by $3 * 7$ but not $7^2 * 5$, it follows that n must have at least 3^5 and 7^2 in its factorization but not 7^3. This means we know that $n = (2^8)(3^x)(5^y)(7^2)$, where $x \geq 5$ and $y \geq 4$. We also know $(9)(x + 1)(y + 1)(3) = 2025$, so $(x + 1)(y + 1) = 75$. Letting $y = 4$, its minimum value, gives the maximum possible value of $x = 14$, which is our desired quantity.

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 2
Individual Section

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Round 2: Polynomials and Factoring

2-1 Given that -13 is a zero of the polynomial $g(x) = x^3 + 3x^2 - 106x + 312$, find the product of the remaining zeros.
[Answer: 24]

You can factor out $x + 13$ from $g(x)$ to get $g(x) = (x + 13)(x^2 - 10x + 24)$, or recognize that the product of the zeros must be -312, so the answer is $\frac{312}{13} = 24$.

2-2 Assume $h(x) = 2x^2 + 5x - 42$ and $j(x) = x^2 + bx - 1219$, with $j(x)$ having integer zeros. If a and b are integers such that $h(a) = 0$ and b is the largest positive value it can be, find $a + b$.
[Answer: 1212]

Note that $h(x)$ factors into $(x + 6)(2x - 7)$, meaning it has zeros of -6 and $\frac{7}{2}$, but since a is an integer we have $a = 0$. Also b must be the integer sum of two numbers that multiply to -1219, and the largest b can be is $-1 + 1219 = 1218$. Therefore the desired quantity is $-6 + 1218 = 1212$.

2-3 If $f(x) = x^4 + ax^2 + 2025$, where a is a nonzero constant, is a polynomial with four integer zeros, find the sum of all possible values of $f(1)$.
[Answer: 3712]

In order for the polynomial $f(x)$ to have the form it does while having four integer zeros, it must be the product of two polynomials of the form $(x^2 - a^2)(x^2 - b^2)$. Considering all cases where a^2 and b^2 are squares of integers that multiply to 2025, we have $(x^2 - 1)(x^2 - 2025)$, $(x^2 - 9)(x^2 - 225)$, and $(x^2 - 81)(x^2 - 25)$. Considering the values each product produces when $x = 1$, we get 0, $(8)(224)$, and $(80)(24)$ respectively, making the desired quantity $0 + 1792 + 1920 = 3712$.

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 2
Individual Section

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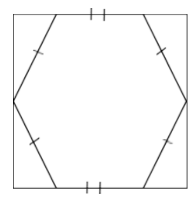
Round 3: Area and Perimeter

3-1 A rhombus has diagonals in a ratio of $15:8$ and an area of 240 square units. What is the perimeter of the rhombus?
[Answer: 68]

Since the area of a rhombus is $\frac{1}{2}d_1d_2$, this rhombus has an area $\frac{1}{2}(15k)(8k) = 240$, so $k = 2$ and the diagonals have length 30 and 16. Therefore the rhombus is made of four right triangles with legs of length 8 and 15, making the side lengths of the rhombus 17, and the perimeter is therefore $4(17) = 68$.

3-2 An equiangular hexagon is inscribed in a square as shown in the diagram. If the perimeter of the square is 300 units, then the perimeter of the hexagon is $a + b\sqrt{c}$ where $a, b,$ and c are positive integers and c has no perfect square factors greater than 1. Find $a + b + c$.
[Answer: 203]



Because each angle of the hexagon has a measure of $120°$, the right triangles are congruent $30-60-90$ triangles. If the shorter leg of a triangle has length x, then the perimeter of the hexagon is $4(2x) + 2(75 - 2x)$. Noting $2\left(x\sqrt{3}\right) = 75$, we have $2x = 25\sqrt{3}$. Therefore the perimeter is $4\left(25\sqrt{3}\right) + 2\left(75 - 25\sqrt{3}\right) = 150 + 50\sqrt{3}$, making the desired quantity $150 + 50 + 3 = 203$.

3-3 A semicircle with radius 10 is inscribed in a trapezoid such that the shorter base of the trapezoid is the diameter of the semicircle and the longer base is tangent to the semicircle. If the semicircle takes up exactly half of the area of the trapezoid, then the longer base is k units longer than the shorter base. Find the value of k rounded to the nearest integer.
[Answer: 23]

The area of the trapezoid must be $(2)\left(\frac{1}{2}\pi r^2\right) = \pi(10)^2 = 100\pi$ square units. The trapezoid will have one base of 20 units, one base of $20 + k$ units, and a height of 10 units. Setting up $\frac{1}{2}(10)(20 + 20 + k) = 200 + 5k = 100\pi$ yields $k = 20\pi - 40 = 20(\pi - 2) \approx 20(1.14) \approx 23$.

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 2
Individual Section

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Round 4: Absolute Value & Inequalities

4-1 Find the sum of all integer solutions to $|3x-5|<10$.
[Answer: 9]

Writing the inequality as $\left|x-\frac{5}{3}\right|<\frac{10}{3}$ yields a solution interval to be $\left(-\frac{5}{3},5\right)$, which contains the integers $-1, 0, 1, 2, 3$, and 4, which have a sum of 9.

4-2 The equations $y-3x=11$ and $|x+2|+|y-5|=4$ intersect at (a,b) and (c,d). What is the value of $|abcd|$?
[Answer: 48]

The most direct way to solve this is to substitute $y=3x+11$ into the absolute value equation to yield $|x+2|+|3x+11-5|=4$, which simplifies to $|x+2|+3|x+2|=4$, or $4|x+2|=4$, making $|x+2|=1$, yielding $x=-1$ and $x=-3$. Plugging back into $y=3x+11$ produces $y=8$ when $x=-1$ and $y=2$ when $x=-3$. Therefore the desired quantity is $|(-1)(8)(-3)(2)|=48$.

4-3 Given the function $f(x)=|x^2+6x-4|$, there is a set of three and only three values $a, b,$ and c such that $f(a)=f(b)=f(c)$. Find the product abc.
[Answer: 51]

The function $y=x^2+6x-4$ graphs as a parabola with a vertex at $(-3,-13)$. The graph of the function $f(x)$ has all negative points reflected above the x-axis, making the point of the reflected vertex $(-3,13)$. This means the only y-value that has three corresponding x-values will be the y-value of the reflected vertex, which will have two other corresponding points on the quadratic. So one of the values in question is $x=-3$. To find the others, we set $x^2+6x-4=13$, which can be rewritten as $x^2+6x-17=0$. The x-values that solve this equation have a product of -17. Therefore the desired quantity is $(-3)(-17)=51$.

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 2
Individual Section

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Round 5: Law of Sines and Cosines

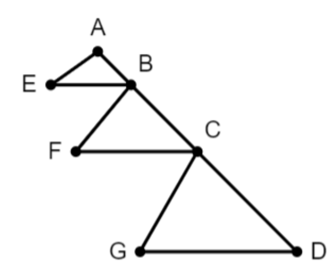
5-1 Consider the acute triangle XYZ with an area of 14. If $XY = 7$ and $XZ = 5$, find the value of $(YZ)^2$.
[Answer: 32]

Since the area of the triangle is $\frac{1}{2}(XY)(XZ)\sin(X)$, we have $\frac{1}{2}(7)(5)\sin(X) = 14$, yielding $\sin(X) = \frac{28}{35} = \frac{4}{5}$. Because the triangle is acute, $\cos(X) > 0$ and therefore $\cos(X) = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$. Therefore, $(YZ)^2 = 5^2 + 7^2 - 2(5)(7)\left(\frac{3}{5}\right) = 25 + 49 - 42 = 32$.

5-2 Consider triangle ABC with point D on $\overline{AC}$. If $AB = 8$, $DC = 3$, the area of triangle ABD is 21, and the area of triangle ABC is 30, find AD.
[Answer: 7]

Let the segment length $AD = x$. The area of triangle ABD is $\frac{1}{2}(AB)(AD)\sin(A) = \frac{1}{2}(8)(x)\sin(A) = 4x\sin(A)$, and the area of triangle ABC is $\frac{1}{2}(AB)(AC)\sin(A) = \frac{1}{2}(8)(x+3)\sin(A) = 4(x+3)\sin(A)$. Since the area of triangle ABD is $\frac{7}{10}$ of the area of triangle ABC, we have $4x\sin(A) = \frac{7}{10}(4)(x+3)\sin(A)$, yielding $40x = 28x + 84$, or $x = 7$.

5-3 See the diagram (not drawn to scale). Points B and C lie on line $\overline{AD}$ such that $AB = 2$, $BC = 3$, and $CD = 4$. $\overline{EB}||\overline{FC}||\overline{GD}$, and $AE = 1$, $BF = 2$, and $CG = 3$. If $\sin(D) + \sin(E) + \sin(F) + \sin(G) = 1$, then $\sin(E) = \frac{a}{b}$ where a and b are positive integers with no perfect square factors greater than 1. Find $a + b$.
[Answer: 47]



Because $\overline{EB}||\overline{FC}||\overline{GD}$, $m\angle ABE = m\angle BCF = m\angle CDG$, which means their sine values are also equal. Using the law of sines, $\frac{\sin(D)}{3} = \frac{\sin(G)}{4}$, $\frac{\sin(D)}{2} = \frac{\sin(F)}{3}$, and $\frac{\sin(D)}{1} = \frac{\sin(E)}{2}$. Therefore, $\sin(D) = \frac{3}{4}\sin(G) = \frac{2}{3}\sin(F) = \frac{1}{2}\sin(E)$. Writing the equation in the problem in terms of $\sin(E)$ gives $\frac{1}{2}\sin(E) + \sin(E) + \frac{3}{4}\sin(E) + \frac{2}{3}\sin(E) = 1$, and therefore $\frac{35}{12}\sin(E) = 1$, making $\sin(E) = \frac{12}{35}$, so the desired quantity is $12 + 35 = 47$.

FAIRFIELD COUNTY MATH LEAGUE 2025–2026
Match 2
Individual Section

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Round 6: Equations of Lines

6-1 The points $(-2, s)$ and $(-8, t)$ are on a line perpendicular to $y = \frac{2}{5}x + 2$. What is the value of $t - s$?
[Answer: 15]

A line perpendicular to the given line has a slope of $-\frac{5}{2}$, meaning $\frac{t-s}{-8-(-2)} = -\frac{5}{2}$, and multiplying both sides by -6 yields $t - s = 15$.

6-2 A triangle is formed by the three lines $y = 1$, $x = -3$, and $y = -1 + \frac{\sqrt{2}}{6}x$. The area of the triangle is $a + \frac{b\sqrt{c}}{d}$, where $a, b, c,$ and d are positive integers, b and d have no common factors greater than 1, and c has no perfect square factors greater than 1. Find $a + b + c + d$.
[Answer: 39]

The triangle is a right triangle and the point that where the legs intersect at a right angle is $(-3,1)$. The y-value of the other point where $x = -3$ is found by evaluating $y = -1 + \frac{\sqrt{2}}{6}(-3) = -1 - \frac{\sqrt{2}}{2}$, making the point $\left(-3, -1 - \frac{\sqrt{2}}{2}\right)$. The x-value of the other point where $y = 1$ is found by solving $1 = -1 + \frac{\sqrt{2}}{6}x$, which yields $x = \frac{12}{\sqrt{2}} = 6\sqrt{2}$, making the point $\left(6\sqrt{2}, 1\right)$. One leg therefore goes from $x = -3$ to $x = 6\sqrt{2}$, having a length of $3 + 6\sqrt{2}$. The other leg goes from $(-3,1)$ to $\left(-3, -1 - \frac{\sqrt{2}}{2}\right)$, therefore having a length of $2 + \frac{\sqrt{2}}{2}$. The area of the triangle is therefore $\frac{1}{2}\left(3 + 6\sqrt{2}\right)\left(2 + \frac{\sqrt{2}}{2}\right) = \frac{1}{2}\left(12 + \frac{27\sqrt{2}}{2}\right) = 6 + \frac{27\sqrt{2}}{4}$, making the desired quantity $6 + 27 + 2 + 4 = 39$.

6-3 A particular line can be defined parametrically such that there exist integer constants a and b where $x = abt + b$ and $y = at - \frac{1}{b}$. If the line intersects the line $y = x$ when $x = -\frac{13}{12}$, find the value of b.
[Answer: 25]

Note that $by = abt - 1 = x - b - 1$, so $by = x - b - 1$. Setting $y = x$ we have $bx = x - b - 1$, or $\frac{1+b}{1-b} = x$. Setting $\frac{1+b}{1-b} = -\frac{13}{12}$ and solving $12 + 12b = 13b - 13$ yields the answer $b = 25$.

FAIRFIELD COUNTY MATH LEAGUE 2025-2026
Match 2
Team Round

Please write your answers on the answer sheet provided.

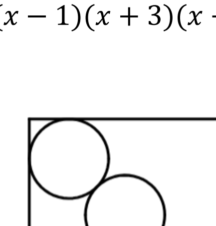
T-1 There are 10 different positive numbers who have 16 total factors, including 1 and themselves, and whose distinct prime factors add to 21. The least common multiple of these 10 numbers has 4^k total factors for some integer k. Find the value of k.
[Answer: 8]

First consider different combinations of prime factors that add to 21. There are 4: $(2,19)$, $(3,5,13)$, $(3,7,11)$, and $(2,3,5,11)$. For each combination we can consider exponents that would produce 16 total factors: $(2^7)(19), (2)(19^7), (2^3)(19^3), (3^3)(5)(13), (3)(5^3)(13), (3)(5)(13^3)$, $(3^3)(7)(11),\ (3)(7^3)(11), (3)(7)(11^3)$, and $(2)(3)(5)(11)$, making our 10 total numbers. The least common multiple of these would consist of all of the used prime factors to their highest exponents, so $(2^7)(3^3)(5^3)(7^3)(11^3)(13^3)(19^7)$, which would have $8*4*4*4*4*4*8 = 2^{16} = 4^8$ total factors, so $k = 8$.

T-2 Mike and Andrew were playing a rousing game of "Guess My Polynomial". Andrew tells Mike, "My polynomial is a cubic with a leading coefficient of 1, a constant of -18, and three integer zeros." Mike says, "I need more information than that." Andrew replies, "Okay, the quadratic coefficient is my favorite number, which I told you last week." Mike says, "That's still not enough information." Andrew answers, "Fine, three of the coefficients are positive." "Ah," Mike says. "Now I know it." What is the value of the largest positive coefficient of Andrew's cubic?
[Answer: 9]

From the first clue, we have $x^3 + ax^2 + bx - 18$, which means the product of the three integer zeros must be 18. The second clue means that a, the opposite of sum of the zeros, must be a value obtainable by more than one combination of zeros that multiply to 18. The third clue reveals that both a and b must be positive, which means the sum is negative and only one possible combination should produce a polynomial with three positive coefficients. Narrowing down possible combinations of zeros that multiply to 18 and add to a negative number, we have: $(1,-1,-18), (-1,2,-9), (1,-2,-9),\ (-1,3,-6),\ (1,-3,-6), (-2,3,-3)$, and $(2,-3,-3)$, which have sums of $-18, -8, -10, -4, -8, -2$, and -4, respectively. Therefore the correct combination must be one of the four sets that that share a sum with another set. However, only one of the four will produce three positive coefficients when expanded: Expanding $(x-1)(x+3)(x+6)$ yields $x^3 + 8x^2 + 9x - 18$, making the desired quantity 9.

T-3 See the diagram. Three congruent circles are arranged in a square such that one circle's center is in the center of the square and the other two circles are tangent to the sides of the square as well as tangent to the center circle. If one circle has an area of 2025π, then the area of the square is $a + b\sqrt{c}$ where a, b, and c are positive integers and c has no perfect square factors greater than 1. Find $a + b + c$.
[Answer: 40502]



Let the radius of one circle be r. A line segment drawn from the center of the top left circle to the center of the bottom right circle with have a length of $4r$, and will also be the hypotenuse of an isosceles right triangle with side lengths $2\sqrt{2}r$. Therefore the length of one side of the square must be $2r + 2\sqrt{2}r$, or $(2 + 2\sqrt{2})r$. The area of the square then is $\left((2 + 2\sqrt{2})r\right)^2 = (12 + 8\sqrt{2})r^2$. Since each circle has an area of 2025π, it follows that $r^2 = 2025$ making the area of the square $(12 + 8\sqrt{2})(2025)$, making the desired quantity $(12)(2025) + 8(2025) + 2 = (20)(2025) + 2 = 40500 + 2 = 40502$.

T-4 Consider the sequence of functions $f_1(x),\ f_2(x), \dots,\ f_{2025}(x)$, where $f_1(x) = |x|$ and for all $n > 1$, $f_n(x) = |f_{n-1}(x) - n|$. Find the sum $f_1(0) + f_2(0) + f_3(0) + \cdots + f_{2025}(0)$.
[Answer: 1026168]

First, $f_1(0) = 0$. Then $f_2(x) = ||x| - 2|$, which has a y-intercept of 2. To find successive values of interest, we keep subtracting the next integer, making the difference positive if it comes out negative. In other words, $f_3(0) = |2 - 3| = 1$, $f_4(0) = |1 - 4| = 3$, $f_5(0) = |3 - 5| = 2$, $f_6(0) = |2 - 6| = 4$, etc.
Therefore we have as a sequence: $\{0,2,1,3,2,4,3,5, \dots ,1013,1012\}$. Note that every two consecutive terms adds to a multiple of 2, making the sum $2 + 4 + 6 + 8 + \cdots + 2024 + 1012$. To sum this quickly we can use the formula $2\sum_{n=1}^{1012} n + 1012$, or $\frac{2(1012)(1013)}{2} + 1012 = 1012(1013) + 1012 = (1012)(1014) = 1026168$.

T-5 Right triangle ABC has area 120 and a hypotenuse length $BC = 29$. Triangle DEF has $AB = DE$, $AC = DF$, $EF = 21$, and area k. Find the value of k^2.
[Answer: 4400]

We know that $\frac{1}{2}(AB)(AC) = 120$ and $(AB)^2 + (AC)^2 = 29^2 = 841$. Triangle DEF has area $k = \frac{1}{2}(DE)(DF)\sin(D) = \frac{1}{2}(AB)(AC)\sin(D) = 120\sin(D)$. By the law of cosines, $21^2 = (DE)^2 + (DF)^2 - 2(DE)(DF)\cos(D) = (AB)^2 + (AC)^2 - 2(AB)(AC)\cos(D) = 841 - 480\cos(D)$. Therefore $441 = 841 - 480\cos(D)$, so $\cos(D) = \frac{400}{480} = \frac{5}{6}$. Therefore $\sin^2(D) = 1 - \frac{25}{36} = \frac{11}{36}$, and $k^2 = (120\sin(D))^2 = 120^2\sin^2(D) = 14400\left(\frac{11}{36}\right) = 400(11) = 4400$.

T-6 The coefficients $A, B,$ and C of a linear equation $Ax + By = C$ are chosen independently and randomly from a list of 100 consecutive integers that includes 0. The probability that the resulting line has a positive slope and a negative y-intercept has a maximum value that can be expressed as $\frac{k}{20{,}000}$. Find the value of k.
[Answer: 4851]

Let p be the number of integers in the list that are positive (making $99 - p$ negative integers). The line will have a positive slope if A and B have opposite signs, and it will have a negative y-intercept if B and C have opposite signs. Therefore either $A > 0, B < 0$, and $C > 0$, or $A < 0$, $B > 0$, and $C < 0$. The first scenario has a probability of $\left(\frac{p}{100}\right)\left(\frac{99-p}{100}\right)\left(\frac{p}{100}\right) = \frac{p^2(99-p)}{1000000}$ and the second scenario has a probability of $\left(\frac{99-p}{100}\right)\left(\frac{p}{100}\right)\left(\frac{99-p}{100}\right)=\frac{p(99-p)^2}{1000000}$. Since these are mutually exclusive, their combined probability is the sum, which is $\frac{99p(99-p)}{1000000}$. This value is maximized whenever $p=49$ or $p=50$, yielding $\frac{99(49)(50)}{1000000}=\frac{(99)(49)}{20000}$, meaning $k=(99)(49)=4851$.