### FAIRFIELD COUNTY MATH LEAGUE 2025–2026 Match 1

Individual Section

Please write your answers on the answer sheet provided.

#### Round 1: Percentages

- 1-1 You have three coupons that you can apply to purchasing two identical trophies. One coupon gives 25% off one item but cannot be combined with any other coupon for that item. A second coupon takes \$30 off of any full-priced item. A third coupon takes k% off any item. You end up paying \$65 in total. If each trophy originally cost \$80, what is the value of k? Ignore sales tax.
- 1-2 One week at an amusement park, the number of visitors on Monday is 150% greater than the number of visitors on Sunday. On Tuesday there are 500 more visitors than on Monday. On Wednesday there are 20% more visitors than on Tuesday. On Thursday it rains and there are 90% fewer visitors than on Wednesday. If the number of visitors on Thursday was 36% of the number of visitors on Sunday, then the number of visitors on Wednesday is p% greater than the number of visitors on Monday. Find the value of p.
- 1-3 A local company is throwing a holiday party. Some employees say they plan to come, and the rest fail to respond. On the day of the party, every person who said they were coming shows up. Additionally, 20% of the people who failed to respond show up as well, increasing the number of people at the party by 8%. If no more than 200 people work at the company, find the greatest possible number of people at the party.

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#### **Round 2: Solving Equations**

2-1 The equation 3x - 4a = x + 6 has the same solution for x as the equation  $\sqrt{x - a - 15} = 3$  where a is a constant in both equations. Find the value of a.

2-2 If r and s are positive integer constants such that rx - 4s = 6x - 9 and r < s < 10, how many ordered pairs (r, s) yield a positive solution for x?

2-3 The equation  $(x + a)^2 = (x + b)(x + 2b)$ , where a and b are positive constants, has no solutions for x. If attempting to solve the equation algebraically leads to the false statement "0 = 8", find the value of  $a^2$ .

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3-1	An isosceles triangle and an equilateral triangle, both with all integer side lengths, have the same perin If one of the side lengths of the isosceles triangle is 13, find the smallest possible length of one side of equilateral triangle.						
3-2	The measure of one base angle of an isosceles trapezoid is $60^{\circ}$ and each diagonal is perpendicular to one of the legs. If each leg has a length of 8 units, then the area of the trapezoid is $a\sqrt{b}$ where $a$ and $b$ are positive integers and $b$ has no perfect square factors greater than 1. Find $a + b$ .						
3-3	Consider trapezoid $FCML$ with bases $\overline{FC}$ and $\overline{LM}$ . Diagonal $\overline{CL}$ splits $\angle FLM$ such that $m\angle FLC$ and $m\angle CLM$ in degrees are both integers. If $m\angle LCM$ is fifteen degrees larger than $m\angle FLC$ and $m\angle LMC = 4m\angle CLM$ , find the sum of the largest and smallest possible values of $m\angle CFL$ .						

### FAIRFIELD COUNTY MATH LEAGUE 2025–2026

#### Match 1

**Individual Section** 

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#### Round 4: Systems of Equations

- 4-1 The lines x + 3y = 10 and 3x + y = 2 intersect at the point (a, b). What is the value of a + b?
- 4-2 The system of equations given below has no solutions for what value of p?  $\begin{cases}
  13x + y = p^2x 2 \\
  x + y = 4px + p
  \end{cases}$
- 4-3 Two positive numbers x and y have the following property: increasing x by y% yields the same result as increasing y by 100%, and increasing y by x% is the same as increasing x by 200%. The sum  $x + y = \frac{a}{b}$  where a and b are positive integers with no common factors greater than 1. Find a + b.

## FAIRFIELD COUNTY MATH LEAGUE 2025–2026 Match 1

**Individual Section** 

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#### Round 5: Right Triangles

- 5-1 In right triangle ABC with right angle B,  $tan(A) = \frac{8}{15}$  and AC = 425. Find the perimeter of the triangle.
- 5-2 Mr. Buchta is trying to find the height of a tree on his property. He plants a stake in the ground a certain distance away from the tree and calls it point A. He then walks 48 feet further away from the tree and plants another stake and calls it point B. If the angle of elevation from A to the top of the tree has a tangent of  $\frac{3}{4}$  and the angle of elevation from B to the top of the tree has a sine of  $\frac{5}{13}$ , find the height of the tree in feet.
- 5-3 Consider isosceles right triangle ABC with right angle C, and square FCML such that F is on  $\overline{AC}$ , M is on  $\overline{BC}$ , and L lies inside the triangle. If the area of triangle ABC is 2025 the distance from L to  $\overline{AB}$  is 25, find the area of FCML.

## **FAIRFIELD COUNTY MATH LEAGUE 2025–2026 Match 1**

**Individual Section** 

Please write your answers on the answer sheet provided.

Round	6:	Coordinate	Geometry
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6-1 Find the area enclosed by the graph of  $|x| + |y| \le 2025$ .

6-2 Parallelogram *ABCD* has coordinates A(5, a), B(b, 9), C(11, c), and D(d, 1). The diagonals  $\overline{AC}$  and  $\overline{BD}$  intersect at point E(x, y). Find the value of  $x^2 + y^2$ .

6-3 The point (p, q) is reflected over the line y = kx, where k is a constant, to make the point (p + 10, q - 8). If p and q are positive integers less than 100, find the greatest possible value of p + q.

#### FAIRFIELD COUNTY MATH LEAGUE 2025-2026

#### Match 1

Team Round

Please write your answers on the answer sheet provided.

- 1. The ordered pair of positive integers (a, b) has the property that increasing the price of an item by (2a)% and then decreasing the price by a% is equivalent to increasing the original price of the object by b%. Find the sum of all possible values of b.
- 2. The equation (x-3)(x+3) = (x+c)(x+d), where c and d are positive integers, has the same solution for x as the equation 2-3(x+7) = x+29. What is the largest possible value of c+d?
- 3. Consider an ordered triple (x, y, z), where each value is the perimeter of a different triangle with integer side lengths. x is the perimeter of a right triangle, y is the perimeter of an isosceles (non-equilateral) triangle, and, z is the perimeter of a scalene triangle. There is only one such ordered triple (a, b, c) where a + b + c = 50 and both  $\frac{a}{b}$  and  $\frac{a}{c}$  are integers. Find abc.
- 4. Consider the system  $\begin{cases} \frac{6}{x+2y} = \frac{x}{1+y}, & \text{where } k \text{ is a positive constant. There exists a value of } k = a + b\sqrt{c}, \\ x+y=\sqrt{ky}, & \text{where } a, b, \text{ and } c \text{ are positive integers and } c \text{ has no perfect square factors greater than 1, where the system has only one solution. Find } a+b+c.$
- 5. A right triangle has three sides of integer length and no common factors greater than 1. One of the legs has a length of 45. Find the smallest possible perimeter of the triangle.
- 6. The point A has coordinates (a, b) where a and b are positive integers and a > b. Point A is reflected across the line y = x to produce point B. Point B is rotated ninety degrees counterclockwise about the origin to produce point C. Point C is reflected across the origin to produce point D. If the area of quadrilateral ABCD is 120, find the largest possible value of  $a^2 + b^2$ .