

FAIRFIELD COUNTY MATH LEAGUE 2025–2026

Match 1

Individual Section

Please write your answers on the answer sheet provided.

Round 1: Percentages

- 1-1 You have three coupons that you can apply to purchasing two identical trophies. One coupon gives 25% off one item but cannot be combined with any other coupon for that item. A second coupon takes \$30 off of any full-priced item. A third coupon takes $k\%$ off any item. You end up paying \$65 in total. If each trophy originally cost \$80, what is the value of k ? Ignore sales tax.
[Answer: 90]

Each trophy began costing \$80. Taking 25% off the cost of one makes it \$60, meaning the second trophy must have ended up costing \$5. We take \$30 off the full price of \$80, making it \$50. Then we must take an additional \$45 off, which must be $k\%$ of \$50. Therefore the answer is

$$100 \left(\frac{45}{50} \right) = 90.$$

- 1-2 One week at an amusement park, the number of visitors on Monday is 150% greater than the number of visitors on Sunday. On Tuesday there are 500 more visitors than on Monday. On Wednesday there are 20% more visitors than on Tuesday. On Thursday it rains and there are 90% fewer visitors than on Wednesday. If the number of visitors on Thursday was 36% of the number of visitors on Sunday, then the number of visitors on Wednesday is $p\%$ greater than the number of visitors on Monday. Find the value of p .
[Answer: 44]

Let the number of visitors on Sunday be x . Monday's visitor count is $x + 1.5x = 2.5x$. Tuesday's count is $2.5x + 500$. Wednesday's count is $1.2(2.5x + 500) = 3x + 600$. Thursday's count is $.1(3x + 600) = .3x + 60$. Setting up $.3x + 60 = .36x$ yields $x = 1000$. This means Tuesday's and Thursday's counts were 2500 and 3600, respectively. Therefore $p = 100 \left(\frac{1100}{2500} \right) = 44$.

- 1-3 A local company is throwing a holiday party. Some employees say they plan to come, and the rest fail to respond. On the day of the party, every person who said they were coming shows up. Additionally, 20% of the people who failed to respond show up as well, increasing the number of people at the party by 8%. If no more than 200 people work at the company, find the greatest possible number of people at the party.
[Answer: 135]

Let k be the proportion of people who say they plan to come to the party. Therefore the proportion of people who do not respond is $1 - k$. The total proportion of people at the party is $k + .2(1 - k) = .2 + .8k$, and we also know this is equal to $1.08k$. Setting up $.2 + .8k = 1.08k$, we have $.28k = .2$, or $k = \frac{5}{7}$. This means the proportion of people at the party is $(1.08) \left(\frac{5}{7} \right) = \frac{27}{35}$. The desired quantity is therefore $\frac{27}{35}$ of the largest multiple of 35 smaller than 200, which is 175, or $\left(\frac{27}{35} \right) (175) = 135$.

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Round 2: Solving Equations

- 2-1 The equation $3x - 4a = x + 6$ has the same solution for x as the equation $\sqrt{x - a - 15} = 3$ where a is a constant in both equations. Find the value of a .
[Answer: 21]

Squaring both sides of the second equation yields $x - a - 15 = 9$, or $x = a + 24$. Substituting back into $2x - 4a = 6$, which can be simplified to $x - 2a = 3$, gives $a + 24 - 2a = 3$, or $a = 21$.

- 2-2 If r and s are positive integer constants such that $rx - 4s = 6x - 9$ and $r < s < 10$, how many ordered pairs (r, s) yield a positive solution for x ?
[Answer: 4]

Isolating x in terms of r and s yields $x = \frac{4s-9}{r-6}$. Since x must be positive, the numerator and denominator must both be positive or negative. Letting both be negative and restricting $r < s$ yields only one ordered pair, $(1, 2)$. Letting both be positive means $6 < r < s < 10$, yielding the ordered pairs $(7, 8)$, $(7, 9)$, and $(8, 9)$, making a total of 4 ordered pairs.

- 2-3 The equation $(x + a)^2 = (x + b)(x + 2b)$, where a and b are positive constants, has no solutions for x . If attempting to solve the equation algebraically leads to the false statement " $0 = 8$ ", find the value of a^2 .
[Answer: 72]

Expanding both sides yields $x^2 + 2ax + a^2 = x^2 + 3bx + 2b^2$, or $2ax + a^2 = 3bx + 2b^2$. Since all terms with x must be equal on both sides, we have $2a = 3b$, or $b = \frac{2}{3}a$. This yields

$a^2 = 2\left(\frac{2}{3}a\right)^2$, or $a^2 = \frac{8}{9}a^2$, or $\frac{1}{9}a^2 = 0$. Therefore $\frac{1}{9}a^2 = 8$, making our desired quantity $a^2 = 72$.

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Round 3: Triangles and Quadrilaterals

- 3-1 An isosceles triangle and an equilateral triangle, both with all integer side lengths, have the same perimeter. If one of the side lengths of the isosceles triangle is 13, find the smallest possible length of one side of the equilateral triangle.
[Answer: 9]

The smallest possible perimeter of the isosceles triangle would have the longest side with length 13 and the two congruent sides each having a length of 7, since $7 + 7 > 13$. This gives a perimeter of 27, which means each side of the equilateral triangle would have a length of $\frac{27}{3} = 9$.

- 3-2 The measure of one base angle of an isosceles trapezoid is 60° and each diagonal is perpendicular to one of the legs. If each leg has a length of 8 units, then the area of the trapezoid is $a\sqrt{b}$ where a and b are positive integers and b has no perfect square factors greater than 1. Find $a + b$.
[Answer: 51]

Each leg of the trapezoid is both the short leg of a 30-60-90 right triangle (with lengths of 8, $8\sqrt{3}$, and 16) whose hypotenuse is the base of the trapezoid as well as the hypotenuses of a 30-60-90 triangle (with lengths of 4, $4\sqrt{3}$, and 8) whose longer leg is an altitude of the trapezoid. This means that the base of the trapezoid has a length of 16 and the height of the trapezoid is $4\sqrt{3}$. Finally, the top base must have a length of $16 - 2(4) = 8$. Therefore the area of the trapezoid is $\frac{8+16}{2}(4\sqrt{3}) = 48\sqrt{3}$, making the desired quantity $48 + 3 = 51$.

- 3-3 Consider trapezoid $FCML$ with bases $\overline{FC}$ and $\overline{LM}$. Diagonal $\overline{CL}$ splits $\angle FLM$ such that $m\angle FLC$ and $m\angle CLM$ in degrees are both integers. If $m\angle LCM$ is fifteen degrees larger than $m\angle FLC$ and $m\angle LMC = 4m\angle CLM$, find the sum of the largest and smallest possible values of $m\angle CFL$.
[Answer: 162]

Let $m\angle FLC = x$ and $m\angle CLM = y$. Then $m\angle LCM = x + 15$, $m\angle LMC = 4y$, and by properties of parallelograms, $m\angle CFL = 180 - x - y$. Additionally, angles $\angle FCL$, $\angle CFL$, and $\angle FLC$ are supplementary, as well as angles $\angle FCL$, $\angle LCM$, and $\angle LMC$. This means $m\angle FCL = 180 - (180 - x - y + x) = 180 - (x + 15 + 4y)$, which simplifies to make the equation $x + 5y = 165$. To find the maximum and minimum values of $180 - x - y$, we must minimize and maximize the values of x and y . The minimum positive integer values of x and y (which maximize y) that work are (5,32), and the maximum values of x and y (which minimize y) are (160,1). Therefore, the maximum and minimum values of $m\angle CFL$ respectively are $180 - 5 - 32 = 143$ and $180 - 160 - 1 = 19$, making the desired quantity $143 + 19 = 162$.

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Round 4: Systems of Equations

- 4-1 The lines $x + 3y = 10$ and $3x + y = 2$ intersect at the point (a, b) . What is the value of $a + b$?
[Answer: 3]

Adding the two equations yields $4x + 4y = 12$, or $x + y = 3$. Alternatively, the system could be solved through either elimination or substitution to produce the ordered pair $\left(-\frac{1}{2}, \frac{7}{2}\right)$, whose coordinates have a sum of 3.

- 4-2 The system of equations given below has no solutions for what value of p ?
$$\begin{cases} 13x + y = p^2x - 2 \\ x + y = 4px + p \end{cases}$$

[Answer: 6]

Isolating y in both equations yields the outcome that $p^2 - 13 = 4p - 1$ to produce parallel lines. This means $p^2 - 4p - 12 = 0$, which yields solutions of $p = 6$ and $p = -2$. However, note that when $p = -2$ the intercepts are the same. Therefore the desired quantity is $p = 6$.

- 4-3 Two positive numbers x and y have the following property: increasing x by $y\%$ yields the same result as increasing y by 100% , and increasing y by $x\%$ is the same as increasing x by 200% . The sum $x + y = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.
[Answer: 878]

Transforming the description into equations yields $x\left(1 + \frac{y}{100}\right) = 2y$ and $y\left(1 + \frac{x}{100}\right) = 3x$. Distributing the variables in both equations and then subtracting to eliminate the term of $\frac{xy}{100}$ produces $x - y = 2y - 3x$, or $y = \frac{4}{3}x$. Substituting back into the first equation yields $x\left(1 + \frac{1}{75}x\right) = \frac{8}{3}x$, which can be solved to yield $x = 125$. Therefore $y = \frac{4}{3}(125) = \frac{500}{3}$, and $a + b = 125 + \frac{500}{3} = \frac{875}{3}$, making the desired quantity $875 + 3 = 878$.

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Round 5: Right Triangles

- 5-1 In right triangle ABC with right angle B , $\tan(A) = \frac{8}{15}$ and $AC = 425$. Find the perimeter of the triangle.
[Answer: 1000]

Based on the given information, $\sin(A) = \frac{8}{17} = \frac{BC}{425}$, and so $BC = (25)(8) = 200$ and $AB = (25)(15) = 375$, making the desired quantity $200 + 375 + 425 = 1000$.

- 5-2 Mr. Buchta is trying to find the height of a tree on his property. He plants a stake in the ground a certain distance away from the tree and calls it point A . He then walks 48 feet further away from the tree and plants another stake and calls it point B . If the angle of elevation from A to the top of the tree has a tangent of $\frac{3}{4}$ and the angle of elevation from B to the top of the tree has a sine of $\frac{5}{13}$, find the height of the tree in feet.
[Answer: 45]

Let the height of the tree be $15k$, so that $\tan(A) = \frac{3}{4} = \frac{15k}{20k}$, and $\tan(B) = \frac{5}{12} = \frac{15k}{36k}$. Since $36k - 20k = 48$, it follows that $k = 3$, making the height of the tree in feet $15(3) = 45$.

- 5-3 Consider isosceles right triangle ABC with right angle C , and square $FCML$ such that F is on $\overline{AC}$, M is on $\overline{BC}$, and L lies inside the triangle. If the area of triangle ABC is 2025 the distance from L to $\overline{AB}$ is 25, find the area of $FCML$.
[Answer: 200]

Since $2025 = \frac{1}{2}(AC)^2$, it follows that $AC = 45\sqrt{2}$. This means that the altitude, or the distance from C to the hypotenuse, is $\frac{AC}{\sqrt{2}} = 45$. This means that the diagonal of $FCML$ has a length of $45 - 25 = 20$, and therefore the area of $FCML$ is $\frac{1}{2}(20)^2 = 200$.

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Round 6: Coordinate Geometry

- 6-1 Find the area enclosed by the graph of $|x| + |y| \leq 2025$.
[Answer: 8201250]

The graph of the equation is a square with vertices at $(0, 2025)$, $(0, -2025)$, $(2025, 0)$, and $(-2025, 0)$. This means that the square has sides of length $2025\sqrt{2}$, making the area $2(2025)^2 = 8201250$.

- 6-2 Parallelogram $ABCD$ has coordinates $A(5, a)$, $B(b, 9)$, $C(11, c)$, and $D(d, 1)$. The diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point $E(x, y)$. Find the value of $x^2 + y^2$.
[Answer: 89]

Since the diagonals of a parallelogram bisect each other, they intersect at the midpoint of their respective coordinates, $\left(\frac{5+11}{2}, \frac{9+1}{2}\right) = (8, 5)$, making the desired quantity $8^2 + 5^2 = 89$.

- 6-3 The point (p, q) is reflected over the line $y = kx$, where k is a constant, to make the point $(p + 10, q - 8)$. If p and q are positive integers less than 100, find the greatest possible value of $p + q$.
[Answer: 170]

Note that the slope between the points (p, q) and $(p + 10, q - 8)$ is $-\frac{4}{5}$, which means that the slope of the line $y = kx$ must be $\frac{5}{4}$. The line must also pass through the midpoint of these points, which will have coordinates $(p + 5, q - 4)$. Therefore, we have $q - 4 = \frac{5}{4}(p + 5)$, which simplifies to $5p - 4q = -41$. Note that $(3, 14)$ is a valid ordered pair for (p, q) , and then successive pairs are of the form $(3 + 4k, 14 + 5k)$. Letting $k = 17$ creates the ordered pair $(71, 99)$, making the desired quantity $71 + 99 = 170$.

FAIRFIELD COUNTY MATH LEAGUE 2025–2026**Match 1****Team Round**

Please write your answers on the answer sheet provided.

1. The ordered pair of positive integers (a, b) has the property that increasing the price of an item by $(2a)\%$ and then decreasing the price by $a\%$ is equivalent to increasing the original price of the object by $b\%$. Find the sum of all possible values of b .
[Answer: 20]

From the problem description, $\left(1 + \frac{2a}{100}\right)\left(1 - \frac{a}{100}\right) = 1 + \frac{b}{100}$. Multiplying out the left side gives $1 + \frac{a}{100} - \frac{2a^2}{10000} = 1 + \frac{b}{100}$, or $\frac{a}{100} - \frac{a^2}{5000} = \frac{b}{100}$. Multiplying by 5000 yields $50a - a^2 = 50b$, or $a(50 - a) = 50b$. This means that $0 < a < 50$, and the product $a(50 - a)$ must produce an integer multiple of 50. This restricts a to being an even multiple of 5, giving ordered pairs of $(10, 8)$, $(20, 12)$, $(30, 12)$, and $(40, 8)$, making the sum of all possible values of b $8 + 12 = 20$.

2. The equation $(x - 3)(x + 3) = (x + c)(x + d)$, where c and d are positive integers, has the same solution for x as the equation $2 - 3(x + 7) = x + 29$. What is the largest possible value of $c + d$?
[Answer: 160]

First note that the solution for x in the second equation is $x = -12$. Substituting in the first equation yields $(-15)(-9) = (-12 + c)(-12 + d)$, and expanding yields $135 = 144 - 12c - 12d + cd$. Arbitrarily choosing to isolate c yields $c = \frac{12d-9}{d-12}$. Letting $d = 13$ yields $c = -(9 - (12)(13)) = 147$, making the desired quantity $147 + 13 = 160$. Note that increasing d to produce another valid result, such as letting $d = 15$, yields a significantly smaller value of $c = \frac{171}{3} = 57$, so this is the largest possible value of $c + d$ that we can obtain.

3. Consider an ordered triple (x, y, z) , where each value is the perimeter of a different triangle with integer side lengths. x is the perimeter of a right triangle, y is the perimeter of an isosceles (non-equilateral) triangle, and, z is the perimeter of a scalene triangle. There is only one such ordered triple (a, b, c) where $a + b + c = 50$ and both $\frac{a}{b}$ and $\frac{a}{c}$ are integers. Find abc .
[Answer: 2250]

There are a limited number of right triangles that have perimeters sufficiently small: $(3, 4, 5)$, $(5, 12, 13)$, $(6, 8, 10)$. The right triangle $(8, 15, 17)$ will have a perimeter of 40 which only allows 10 units across the remaining two triangles (which have minimum perimeters of 5—2, 2, and 1—and 9—2, 3, and 4—respectively), so that is already too large. The values of $a = 12$ and $a = 24$ do not work either; 12 is too small to add up to 50 with two of its factors, and the only triplet that adds to 50 containing 24 and two of its factors is $(24, 24, 2)$, which does not work as 2 is too small. This leaves $a = 30$. We already noted that $b = 5$ works, leaving $c = 15$. (Also note that c cannot equal 10, so $(30, 10, 10)$ is not a possibility.) This corresponds to the scalene triangle $(4, 5, 6)$, so our triple is $(30, 5, 15)$, making the desired quantity $30 * 5 * 15 = 2250$.

4. Consider the system $\begin{cases} \frac{6}{x+2y} = \frac{x}{1+y} \\ x+y = \sqrt{ky} \end{cases}$, where k is a positive constant. There exists a value of $k = a + b\sqrt{c}$, where a , b , and c are positive integers and c has no perfect square factors greater than 1, where the system has only one solution. Find $a + b + c$.
[Answer: 14]

Cross multiplying in the first equation and squaring the second equation yields a revised system of $6 + 6y = x^2 + 2xy$ and $x^2 + 2xy + y^2 = ky$. By substitution, we have $6 + 6y = ky - y^2$, or $y^2 + (6 - k)y + 6 = 0$. This system has only one solution when $(6 - k)^2 - 4(6) = 0$, or $k^2 - 12k + 12 = 0$. Using the quadratic formula, $k = \frac{12 \pm \sqrt{96}}{2} = 6 \pm 2\sqrt{6}$. However we are told all values in the number are positive, so $k = 6 + 2\sqrt{6}$, making the desired quantity $6 + 2 + 6 = 14$.

5. A right triangle has three sides of integer length and no common factors greater than 1. One of the legs has a length of 45. Find the smallest possible perimeter of the triangle.
[Answer: 126]

Since $45^2 + b^2 = c^2$, it follows that $45^2 = c^2 - b^2 = (c + b)(c - b)$. To minimize the values of c and b , we need to find factors of 2025 (45^2) that are as close to each other in magnitude as possible without being equal. The closest factors are 27 and 75, making $c = \frac{75+27}{2} = 51$ and $b = \frac{75-27}{2} = 24$. This triangle would have side lengths 24, 45, and 51, but this is a triple of 8, 15, and 17. The next closest factors are 25 and 81, making $c = \frac{81+25}{2} = 53$ and $b = \frac{81-25}{2} = 28$. This triangle would have side lengths 28, 45, and 53, which do not share any common factors, producing our desired quantity of $28 + 45 + 53 = 126$.

6. The point A has coordinates (a, b) where a and b are positive integers and $a > b$. Point A is reflected across the line $y = x$ to produce point B . Point B is rotated ninety degrees counterclockwise about the origin to produce point C . Point C is reflected across the origin to produce point D . If the area of quadrilateral $ABCD$ is 120, find the largest possible value of $a^2 + b^2$.
[Answer: 113]

See the diagram. Points A and B will be in Quadrant I. Point C will be in Quadrant II and is a reflection of point A over the y -axis. Point D will be in Quadrant IV and is a reflection of point A over the x -axis. The resulting quadrilateral $ABCD$ could be broken down into two triangles: triangle ABC with base length $AC = 2a$ and height $a - b$, and right triangle ACD with base length $AC = 2a$ and height $AD = 2b$. Therefore the area can be computed by

$\frac{1}{2}(2a)(a - b) + \frac{1}{2}(2a)(2b) = a(a + b)$. Since this product must equal 120 and $0 < b < a$, the only ordered pairs (a, b) that work are (10,2) and (8,7). The latter ordered pair maximizes $a^2 + b^2$, making the desired quantity $8^2 + 7^2 = 113$.

