FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 6
Individual Section

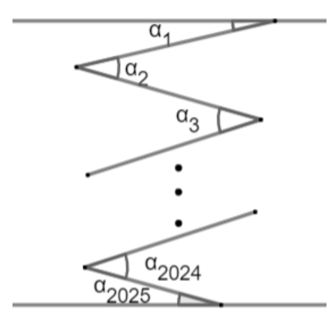
Please write your answers on the answer sheet provided.

Round 1: Lines and Angles

1-1 If acute angles A and B are complementary and $m\angle A = a°$ and $m\angle B = b°$, where $a < b$ and a and b are both prime integers, find the sum of the smallest and largest possible values of a.
[Answer: 50]

1-2 Consider isosceles triangle ABC, with $AB = AC$ and point D on $\overline{BC}$. If $m\angle BAD = m\angle DAC + 40°$, find $m\angle ADC$ in degrees.
[Answer: 110]

1-3 See the diagram (not drawn to scale). Between the parallel lines are 2025 angles labeled $\angle\alpha_1$ through $\angle\alpha_{2025}$. $m\angle\alpha_1 = m\angle\alpha_{2025} = a°$ and the measures of angles $\angle\alpha_1$ through $\angle\alpha_{2024}$ form an increasing arithmetic sequence. If the sum of the measures of all 2025 angles in degrees is ka, Find the value of k.
[Answer: 4048]



FAIRFIELD COUNTY MATH LEAGUE 2024–2025
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Round 2: Literal Equations

2-1 There is a constant value of A such that for all x and y, $y = \frac{Ax+2}{x+5}$ and $x = \frac{2-5y}{y-4}$ contain the same ordered pairs (x, y). If one of the ordered pairs is (A, B), find the value of B.
[Answer: 2]

2-2 Find the sum of all values of a such that $(a, 2025)$ is a solution to $9x^2 - 6xy + y^2 = 2025$.
[Answer: 1350]

2-3 If m and n are positive integers such that $\frac{10}{m} + \frac{45}{n} = \frac{1}{45}$, what is the sum of the smallest possible value of n and the smallest possible value of m?
[Answer: 2477]

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Round 3: Solids & Volume

3-1 A hemisphere with a circular base and a sphere have the same radius. If the hemisphere's surface area is 10π square inches less than the sphere's surface area, then the sphere's surface area is $k\pi$ square inches. What is the value of k?
[Answer: 40]

3-2 Mr. Bearse takes a piece of oaktag that is twice as long as it is wide. He cuts a one-inch square out of each corner and then folds the rectangular pieces upward along the dotted lines as shown to make a rectangular prism with no top face. If the resulting box has an exterior surface area of 46 square inches, find the volume of the box in cubic inches.
[Answer: 24]

3-3 The segment of the line $y = kx$, with $0 < k < 1$, that has domain $0 \le x \le a$ is rotated about the line $y = x$ to produce a cone with height 10 and volume 120π. If the same segment of $y = kx$ were rotated about the x-axis, the resulting cone would have a lateral surface area of $p\sqrt{q}\pi$ where p and q are positive integers and q has no perfect square factors greater than 1. Find $p + q$.
[Answer: 25]

FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 6
Individual Section

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Round 4: Radical Expressions and Equations

4-1 Solve for x: $\sqrt{x-1} = \frac{6}{\sqrt{x+4}}$
[Answer: 5]

4-2 Find the sum of all values of x that satisfy the equation $\sqrt[3]{x^2 - 4x + 4} - \sqrt[3]{27x - 54} = 4$.
[Answer: 67]

4-3 The function $f(x) = \sqrt{3x + k} - \sqrt{x^2 - 10x + 256}$ where k is a constant has a y-intercept of $(0,2)$, x-intercepts of $(a, 0)$ and $(b, 0)$, and a domain of $x \geq c$. Find the value of $|a| + |b| + |c|$.
[Answer: 129]

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Round 5: Polynomials and Advanced Factoring

5-1 A quadratic has the form $ax^2 - bx + b = 0$ where a and b are positive integers and that have no common factors greater than 1. If $x = 20$ is a solution of the quadratic, find $a + b$.
[Answer: 419]

5-2 Let $f(x)$ be a polynomial function that contains the points $(1,2), (2,4), (3,8)$, and $(4,17)$. If $n > 2$ is the degree of $f(x)$ and is the smallest degree possible, find the value of $n + f(6)$.
[Answer: 65]

5-3 What is the sum of the squares of the three complex zeros of
$f(x) = x^3 - 19x^2 + 83x - 2025$?
[Answer: 195]

FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 6
Individual Section

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Round 6: Counting and Probability

6-1 The local weather man loves probability. He says that the weather on each day this weekend will be either sunny or rainy. There is a $\frac{3}{5}$ probability that it will rain on Saturday. If it rains on Saturday there is a $\frac{2}{3}$ probability it will rain on Sunday as well. If it does not rain on Saturday, there is a $\frac{1}{4}$ probability it will rain on Sunday. The probability that it will be sunny both days is $\frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.
[Answer: 13]

6-2 Consider the weatherman's forecast from 6-1. If it is rainy on at least one day, the probability that it will be rainy on both days is $\frac{p}{q}$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.
[Answer: 11]

6-3 An AP Statistics class has 20 students, including k juniors. The students line up single file to collect their textbook. If the students line up in random order and the third student in line is a junior, the probability that the student is the first junior in line is P. Find the value of k that minimizes $\left|P - \frac{1}{2}\right|$.
[Answer: 6]

FAIRFIELD COUNTY MATH LEAGUE 2024-2025
Match 6
Team Round

Please write your answers on the answer sheet provided.

1. A particular n-gon has 3 right angles and the rest of its angles measure $179°$. What is the value of n?
[Answer: 93]

2. If for all $x \neq 0$, $y \neq 0$, and $z \neq \frac{1}{2}$, $\frac{1}{x} = \frac{3}{2y} - \frac{5}{6-\frac{3}{z}}$, then $y = \frac{Axz-Bx}{Cxz+Dz-E}$, where $A, B, C, D,$ and E are positive integers that have no factors common to all of them greater than 1. Additionally, there is a positive integer $F > 2$ such that $z \neq \frac{1}{F}$ if $x = 3$. Find $A + B + C + D + E + F$.
[Answer: 62]

3. Consider a right square pyramid with a base with edge length of 12 and a volume of $288\sqrt{2}$ cubic units. Point A lies on the midpoint of one of the edges of the base, and point B lies on the face opposite of A and is equidistant from all three vertices of its face. Find $(AB)^2$.
[Answer: 108]

4. The equation $\left(\sqrt{x} - \frac{6}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{15}{\sqrt{x}}\right) = \frac{7}{2}$ has positive solutions $x = a$ and $x = b$ where $a < b$. Find $10a + b$.
[Answer: 65]

5. How many of zeros of the polynomial $f(x) = x^{2025} + 2024x^{2024} - 2025$ will be complex non-real?
[Answer: 2022]

6. A box contains 30 fair coins of identical size and weight. Ten coins have heads on both sides, ten coins have tails on both sides, and ten coins have heads on one side and tails on the other. One coin is picked at random from the box and flipped twice. If both flips come up heads, the probability the coin has heads on both sides is $k\%$. Find the value of k.
[Answer: 80]