

FAIRFIELD COUNTY MATH LEAGUE 2024–2025

Match 4

Individual Section

Please write your answers on the answer sheet provided.

Round 1: Basic Statistics

- 1-1 A set of five positive integers has a median of 8, a unique mode of 2, and a range of 13. If the arithmetic mean of the numbers is an integer, what is the sum of the numbers?
[Answer: 40]
- 1-2 The Mean Absolute Deviation (MAD) of a set of numbers is the arithmetic mean of the absolute difference of each number and the arithmetic mean of the set. A set of four integers $a, b, c,$ and d has an arithmetic mean \bar{x} . If $a < b < \bar{x} < c < d$, $\bar{x} = 26$ and the MAD of the set is 15, find the largest possible value of d .
[Answer: 55]
- 1-3 Set A consists of 50 positive 1-digit, 2-digit, and 3-digit integers. The elements of set B are formed by placing the digit “1” in front of each element of set A. The arithmetic mean of the elements of set B is 409.6 greater than the arithmetic mean of the elements of set A. Find the largest possible number of total digits of all the elements of set A.
[Answer: 110]

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Round 2: Quadratic Equations

2-1 The quadratic function $f(x) = x^2 - (3k + 1)x + 64$ has only one distinct positive real zero. Find $f(k)$.

[Answer: 9]

2-2 The quadratic equation $4x^2 - 12x + 7 = 0$ has zeros $x = m$ and $x = n$. The quadratic equation $ax^2 + bx + c = 0$, where a, b , and c are relatively prime integers and $a > 0$, has zeros of $x = 2m + 1$ and $x = 2n + 1$. Find $|a| + |b| + |c|$.

[Answer: 23]

2-3 There are two positive values of p such that the equations $y = 3x - 2$ and $x = py^2 + 4py + 4$ share only one solution (x, y) . The larger of the two values of p is $\frac{a+\sqrt{b}}{c}$, where a, b , and c are positive integers and b has no perfect square factors greater than 1. Find $a + b + c$.

[Answer: 53]

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Round 3: Similarity

3-1 Ivana Nicegarten is filling her decorative pool, which is shaped like a cone with the vertex pointed downward into the ground. Water is pouring into the pool at a constant rate. After 12 minutes, the water in the pool has depth d . How many total minutes will it take for the depth to be $2d$?

[Answer: 96]

3-2 Consider trapezoid $GEOM$, with $\overline{GE} \parallel \overline{OM}$, angles G and M are right angles, and $m\angle GME = m\angle EOM$. If $GM = 12$ and $GE = 16$, find the perimeter of the trapezoid.

[Answer: 68]

3-3 Consider parallel lines l_1 and l_2 . One transversal intersects l_1 at A and l_2 at B . A second transversal intersects l_1 at C and l_2 at D , and the two transversals intersect at point E which is between points C and D . If $AC = 12$, $CE = 8$, $DE = 6$, and the total area of triangles ACE and DBE is T square units, then the distance between lines l_1 and l_2 is $\frac{a}{b}T$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.

[Answer: 89]

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Round 4: Variation

4-1 If y varies directly as x and $x = 40$ when $y = 50$, find the value of x when $y = 95$.

[Answer: 76]

4-2 A relationship where z varies directly as the 1.5 power of x and inversely as the square of y contains the ordered triple (p, q, r) . Increasing p by 300% and decreasing q by $33\frac{1}{3}\%$ produces the new z -value s . Find $\frac{s}{r}$.

[Answer: 18]

4-3 If y varies inversely as the n th power of x for some positive number n , and $y = 32$ when $x = 1$ and $y = 1$ when $x = 4$, then $y = k$ when $x = 36$. Find $\frac{1}{k}$.

[Answer: 243]

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Round 5: Trig Expressions & DeMoivre's Theorem

5-1 A complex number z_1 has an argument of 342° and is one of the complex n -th roots of the complex non-real number z_2 . If z_2 has no complex n -th roots with a negative imaginary component and a real component greater than the real component of z_1 , find the largest possible value of n .

[Answer: 19]

5-2 If $z = \left(\frac{1+7i}{2+bi}\right)^4$ where b is a real number and z has the same modulus as $3.2 + 2.4i$, find b^2 .

[Answer: 21]

5-3 If A is an angle such that $\sin\left(A + \frac{3\pi}{4}\right) = \frac{\sqrt{5}}{8}$, then $\sin(2A) = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.

[Answer: 59]

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Round 6: Conic Sections

6-1 One of the foci of the ellipse $\frac{(x-3)^2}{50} + \frac{(y+7)^2}{14} = 1$ has a positive x -coordinate a . What is the value of a ?

[Answer: 9]

6-2 A hyperbola has an asymptote with an equation of $y = \frac{1}{2}x + \frac{7}{2}$ and a range of $(-\infty, 2] \cup [6, \infty)$. The largest y -value of the hyperbola where $x = 4$ is $\frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.

[Answer: 15]

6-3 An ellipse is centered at origin and has a focus at $(0, \sqrt{2})$ and an area of $4\sqrt{3}\pi$. A circle is also centered at the origin and intersects the ellipse at points that lie on $y = x$ and $y = -x$. The square of the radius of the circle is $\frac{m}{n}$ where m and n are positive integers with no common factors greater than 1. Find $m + n$. (Note: the area of an ellipse is πab where a and b are semi-major and semi-minor axis lengths.)

[Answer: 55].

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Team Round

Please write your answers on the answer sheet provided.

1. The geometric mean of a set of n numbers is the n th root of the product of the numbers. A set of three distinct positive integers greater than 1 has the property that its arithmetic and geometric means are both integers. What is the smallest possible value of the arithmetic mean of the set?
[Answer: 7]
2. The quadratic equations $x^2 - 8x + p = 0$ and $x^2 - 2x + q = 0$, where p and q are real constants, each have two positive solutions for x . They share one solution and the other solutions are reciprocals of each other. If $p = a + b\sqrt{c}$, where a, b , and c are positive integers and c has no perfect square factors greater than 1, find $a + b + c$.
[Answer: 17]
3. Consider rectangle $FCML$, with $FC = 8$ and $CM = 10$. Point N is drawn on diagonal \overline{FM} such that the distance from N to \overline{ML} is 7. Point T is drawn on \overline{FL} such that \overline{CT} contains point N . $LT = \frac{a}{b}$, where a and b are positive integers with no common factors greater than 1. Find $a + b$.
[Answer: 47]
4. If y varies directly as the second power of x and the ordered triple (a, b, c) has the properties that a, b , and c are all different positive integers and both (a, b) and (b, c) fit this particular variation relationship, find the smallest possible value of $a + b + c$ when $b = 30$.
[Answer: 165]
5. If k is a positive number such that $\arctan\left(\frac{1}{3}\right) + \arctan(k) = \arctan\left(\frac{2}{3}\right)$, then $k = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.
[Answer: 14]
6. The circle $x^2 + y^2 - 16x - 6y + 48 = 0$ and the line $x = k$, which lies to the left of the center of the circle, intersect at points A and B such that $AB = 8$. A particular conic section represents the set of all points equidistant from the center of the circle and the line. Find the x -coordinate of the two intersection points between the conic section and the circle.
[Answer: 10]