FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Decimals and Base Notation

1-1 The decimal $.20\overline{24}$, when written as a fraction in simplest terms, is $\frac{a}{b}$ where a and b are relatively prime integers. Find $a + b$.

1-2 If x and y are positive integer bases such that $24_x + 63_y = 41_{x+y}$, then write the value of 10001_{x-y} as a numeral in base 10.

1-3 Let m, n, and q be positive integers such that $(16^m)(15^n) = 8.1 * 10^q$. If m, n, and q are then used as digits in the base $(q + 1)$ number mnq_{q+1}, express this number as a numeral in base 10.

FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 2: Word Problems

2-1 Mr. Bearse spent $\frac{1}{3}$ of his holiday bonus check on a new pair of shoes. He then spent $\frac{1}{3}$ of the remaining money on a new briefcase. Finally he spent $\frac{3}{4}$ of what now remained of his bonus on a new pet lizard. If he now had \$125 left over to buy gifts for friends, how much in dollars did the briefcase cost?

2-2 Mr. Zucca is grading tests from three classes, each with the same number of tests. He grades each class on a different day: one on Monday, one on Tuesday, and one on Wednesday. His grading on Tuesday is twice as fast as his grading on Monday, and on Wednesday, he grades 50% faster than his grading on Tuesday. If it takes him a total of 2 hours and 12 minutes to grade all three classes' tests, how many minutes did he spend grading tests on Monday?

2-3 During a 2-hour period from 9:00 AM to 11:00 AM at FCML world, Connecticut's mathiest amusement park, patrons enter at a rate of 20 people per minute and leave at a rate of 8 people per minute. At 9:00 AM, 70 patrons are in line for the Graphinator ride, and the line is increasing at a rate of 25% of the net change in the park's population. The rate of people joining the line increases additionally every 20 minutes by an additional 10% of the park's net change in population. The ride processes people at a rate of 5 patrons per minute. Let m and n be the smallest and largest number of people in line for the Graphinator during the 2-hour window, respectively. Find $m + n$.

FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 3: Polygons

3-1 For how many values of $n < 2024$ does an n −gon have an odd number of sides but a positive even number of diagonals?

3-2 A regular n-gon contains the adjacent vertices A, B, and C. If $m\angle B = 10m\angle BAC$, find the value of n.

3-3 A regular n-gon has the property that the number of its sides added to the measure of one interior angle measure in degrees is equal to the measure in degrees of one interior angle of an $\frac{8}{3}n$-gon. Find the value of n.

FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 4: Function and Inverses

Note: the inverse f^{-1} of a function is not necessarily a function.

4-1 Let $f(x)$ be a linear function such that $f(6) = 26$ and $f^{-1}(6) = 1$. Find $f(10)$.

4-2 Let $f(x) = \sqrt{x-7}$ and $g(x) = 3f(x) - 10$. There exist values a and b such that $f^{-1}(a) = g^{-1}(a) = b$. Find the value of b.

4-3 Consider the functions $f(x) = \frac{1}{x^2-a}$ and $g(x) = \log_2(3x + b)$ where a and b are positive constants. If the domain of $f \circ g$ is $\left(-\frac{11}{3}, k\right) \cup \left(k, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$, then $|k| = \frac{p}{q}$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.

FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 5: Exponents & Logarithms

5-1 If x, y, and z are integers such that $(12^x)(18^y) = 24^z$, find $-\frac{x}{y}$.

5-2 Consider the equation $\log_4(x^2 + x + 6) - \log_4(x - 3) = 2$. If the equation has exactly one real solution p, find the value of p^2. If the equation has two real solutions p and q, find the value of $p^2 + q^2$. If the equation has no real solutions, write your solution as 2024.

5-3 If $u = \log_2(3)$ and $v = \log_3(2)$, then the expression $(\log_2(9) + \log_3(8))(\log_9(16) + \log_8(27))$ is equivalent to $au^2 + bv^2 + c$ where a, b, and c are positive integers. Find $a + b + c$.

FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 6: Matrices

6-1 If $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$ has determinant k, find is the determinant of $\begin{bmatrix} k & 1 \\ 2 & 3 \end{bmatrix}$.

6-2 If $A = \begin{bmatrix} 4 & 2 \\ 6 & 9 \end{bmatrix}$ and B is a $2x2$ matrix such that $AB = \begin{bmatrix} -2 & 4 \\ -9 & 6 \end{bmatrix}$, find the determinant of $A + B$.

6-3 The matrix $A = \begin{bmatrix} x & y & x \\ 2 & 2 & 1 \\ 7 & 9 & 10 \end{bmatrix}$, where x and y are positive integers less than 100, has an inverse with a determinant of $\frac{1}{2}$, find the largest possible value of x.

FAIRFIELD COUNTY MATH LEAGUE 2024-2025
Match 3
Team Round

Please write your answers on the answer sheet provided.

1. If x is an integer base such that $62_{3x} = 205_x + 45_{x+1}$, express the value of 2024_x as a numeral in base 10.

2. For a fall banquet, Andrew and Mike cut out decorative paper leaves over three days and make the same total number of leaves each day. Their work rates are constant but not necessarily equal. On day one, they work together the entire time, but on day two, Mike arrives ten minutes late, causing the total time Andrew has to make leaves that day to increase by 25%. Frustrated, Andrew refuses to show up on day three, leaving Mike to make all the remaining alone. If Mike ended up making 198 of the 360 total leaves across the three days, how many leaves did Mike make the second day?

3. There are 12 ordered pairs $(m, n), n > m$, such that the difference in degrees of one interior angle of a regular n-gon and one interior angle of a regular m-gon is 10 degrees. The smallest possible measure of an exterior angle in degrees of one of the n-gons is $\frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.

4. A function $f(x)$ has the property that for all $a > -1$, $f(a) = f^{-1}(2a + 1)$. If $f(1) = 2$, find the value of $f(63)$.

5. The equation $(\log_4(16x))\left(\log_4\left(\frac{x^2}{4}\right)\right) = \log_x(64)$ has three positive real solutions a, b, and c where $a < b < c$. Find $\frac{c}{ab}$.

6. If the value of x is such that the matrices $A = \begin{bmatrix} 2x & 1 \\ -1 & 4x^2 \end{bmatrix}$ and $B = \begin{bmatrix} 2x+1 & -13 \\ 2x+1 & 11x \end{bmatrix}$ have equal determinants, find the largest possible value of the determinant of A.