FAIRFIELD COUNTY MATH LEAGUE 2023–2024
Match 2
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Factors and Multiples

1-1 How many positive integers n, $2 \leq n \leq 50$, have at most two prime factors? (Recall that 1 is not prime.)

1-2 What is the smallest positive integer that has the same number of factors as 160?

1-3 Let a, b, and c be integers greater than 1 such that $gcf(a, b) = 4$, $lcm(a, b) = 24$, and $gcf(ab, c) = 1$. What is the smallest possible value of $lcm(ab, c)$?

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Round 2: Polynomials and Factoring

2-1 Find the sum of all positive values of c such that the expression $x^2 + 7x + c$ is factorable into two binomials with integer coefficients.

2-2 Let a be the larger zero of $f(x) = x^2 - 11x + 24$, and let b be the largest integer such that $g(x) = x^2 + ax + b$ has two real irrational zeros. Find $f(b)$.

2-3 The polynomial $f(x) = 2x^3 + 4x^2 + px - 6$, where p is an integer, has at least one real rational zero. If A is the greatest possible value of p and B is the least possible value of p, find the value of $A - B$.

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Round 3: Area and Perimeter

3-1 If a square's area is ten times its perimeter, what is its perimeter?

3-2 A square is inscribed in an equilateral triangle with perimeter 36. The square has a side length of $a\sqrt{b} - c$ where a, b, and c are positive integers and b has no perfect square factors greater than 1. Find $a + b + c$.

3-3 An isosceles trapezoid is inscribed in a circle with area 36π such that the longer base of the trapezoid is a diameter of the circle. If the trapezoid has height $\sqrt{11}$, then its perimeter is $a + b\sqrt{c}$, where a, b, and c are positive integers and c has no perfect square factors greater than 1. Find $a + b + c$.

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Round 4: Absolute Value & Inequalities

4-1 Evaluate the expression: $\left|5-|5^2-5^3|\right|$

4-2 Consider the equation $|ax-8|=b$, where a and b are positive integer constants less than 100. If this equation has two solutions for x, x_1 and x_2, and $|x_1-x_2|=\frac{3}{2}$, find the number of ordered pairs (a,b).

4-3 The graph of the function $f(x)=mx$, where m is a positive constant, intersects the graph of the function $g(x)=|x-20|x-23||$ exactly three times. The largest $x-$coordinate of one of the points of intersection is $\frac{p}{q}$, where p and q are relatively prime integers. Find $p+q$.

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Round 5: Law of Sines and Cosines

5-1 In triangle ABC, $AB = 3(BC)$ and $m\angle B = 60°$. Find the value of $\left(\frac{AC}{BC}\right)^2$.

5-2 Consider triangle ABC, where $AB = 5$, $BC = 6$, and $\tan(B) = 2$. $(AC)^2 = p - q\sqrt{r}$, where p, q, and r are positive integers and r has no perfect square factors greater than 1. Find $p + q + r$.

5-3 Consider triangle FML with obtuse angle L. $FL = 8$ and the area of FML is 48. Point C lies on $\overline{FM}$ such that $\overline{FL} \perp \overline{CL}$ and $FC = 8CM$. Find FM.

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Round 6: Equations of Lines

6-1 A line with equation $3x - 8y = C$, where C is a constant, contains the point $(24, 20)$. What is the y-coordinate of the y-intercept?

6-2 Line l_1 has a slope of $\frac{5}{3}$ and a y −intercept of $(0, b)$, where b is a positive integer. Line l_1 is reflected across the x −axis to make line l_2, and the two lines intersect at $x = -21$. What is the value of b?

6-3 A line with equation $y = mx$, where m is a positive constant, has the property that decreasing the slope by 95% would reduce the measure of the angle made between the line and the x −axis in the first quadrant by 50%. Find the value of m^2.

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Match 2
Team Round

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1. The function $floor(x)$, also known as the greatest integer function, maps x to the greatest integer that is less than or equal to x. Consider the five-digit number $3abcd$, where the last four digits $a, b, c,$ and d are unknown. This number has the property that $floor(\frac{3abcd}{8}) = abcd$ (a four-digit number comprised of the same unknown four digits in order). What is the four-digit number represented by $abcd$?

2. The polynomial $x^4 + kx + 35$, where k is a positive integer, is factorable into two quadratic trinomial factors with integer coefficients. What is the value of k?

3. A rectangle has the property that its dimensions are integers and its area and perimeter are equal. Find the sum of all possible areas of the rectangle.

4. The figure enclosed on the xy-plane by the equation $|x + y| + 3|x - y| < 8$ has an area of $\frac{a}{b}$, where a and b are positive integers with no common factors greater than 1. Find $a - b$.

5. Two spotlights on level ground (assume elevation of 0) are aimed at a tightrope performer who stands on a rope that is stretched directly above the pathway between the lights. Light A makes angle A with the ground and light B makes angle B with the ground, and it is known that angle B is twice the measure of angle A. If the performer is 500 feet from light A and 350 feet from light B, then the height of the performer above the ground in feet is $\frac{a\sqrt{b}}{c}$, where $a, b,$ and c are positive integers, a and c have no common factors greater than 1 and b has no perfect square factors greater than 1. Find $ab + c$.

6. The parametric equations $x = \frac{3}{t-6} + 2$ and $y = \frac{2}{t-6} - 4$ produce a line on the xy −plane with a discontinuity at the point (a, b). Line l is perpendicular to this line and contains the point (a, b) and $(-12, c)$. What is the value of c?