FAIRFIELD COUNTY MATH LEAGUE 2023–2024
Match 2
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Factors and Multiples

1-1 How many positive integers n, $2 \leq n \leq 50$, have at most two prime factors? (Recall that 1 is not prime.)
[Answer: 47]

1-2 What is the smallest positive integer that has the same number of factors as 160?
[Answer: 60]

1-3 Let a, b, and c be integers greater than 1 such that $gcf(a, b) = 4$, $lcm(a, b) = 24$, and $gcf(ab, c) = 1$. What is the smallest possible value of $lcm(ab, c)$?
[Answer 480]

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Round 2: Polynomials and Factoring

2-1 Find the sum of all positive values of c such that the expression $x^2 + 7x + c$ is factorable into two binomials with integer coefficients.
[Answer: 28]

2-2 Let a be the larger zero of $f(x) = x^2 - 11x + 24$, and let b be the largest integer such that $g(x) = x^2 + ax + b$ has two real irrational zeros. Find $f(b)$.
[Answer: 66]

2-3 The polynomial $f(x) = 2x^3 + 4x^2 + px - 6$, where p is an integer, has at least one real rational zero. If A is the greatest possible value of p and B is the least possible value of p, find the value of $A - B$.
[Answer: 95]

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Round 3: Area and Perimeter

3-1 If a square's area is ten times its perimeter, what is its perimeter?
[Answer: 160]

3-2 A square is inscribed in an equilateral triangle with perimeter 36. The square has a side length of $a\sqrt{b} - c$ where a, b, and c are positive integers and b has no perfect square factors greater than 1. Find $a + b + c$.
[Answer: 63]

3-3 An isosceles trapezoid is inscribed in a circle with area 36π such that the longer base of the trapezoid is a diameter of the circle. If the trapezoid has height $\sqrt{11}$, then its perimeter is $a + b\sqrt{c}$, where a, b, and c are positive integers and c has no perfect square factors greater than 1. Find $a + b + c$.
[Answer: 29]

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Round 4: Absolute Value & Inequalities

4-1 Evaluate the expression: $\left|5-|5^2-5^3|\right|$
[Answer: 95]

4-2 Consider the equation $|ax-8|=b$, where a and b are positive integer constants less than 100. If this equation has two solutions for x, x_1 and x_2, and $|x_1-x_2|=\frac{3}{2}$, find the number of ordered pairs (a,b).
[Answer: 24]

4-3 The graph of the function $f(x)=mx$, where m is a positive constant, intersects the graph of the function $g(x)=|x-20|x-23||$ exactly three times. The largest x −coordinate of one of the points of intersection is $\frac{p}{q}$, where p and q are relatively prime integers. Find $p+q$.
[Answer: 239]

FAIRFIELD COUNTY MATH LEAGUE 2023–2024
Match 2
Individual Section

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Round 5: Law of Sines and Cosines

5-1 In triangle ABC, $AB = 3(BC)$ and $m\angle B = 60°$. Find the value of $\left(\frac{AC}{BC}\right)^2$.
[Answer: 7]

5-2 Consider triangle ABC, where $AB = 5$, $BC = 6$, and $\tan(B) = 2$. $(AC)^2 = p - q\sqrt{r}$, where p, q, and r are positive integers and r has no perfect square factors greater than 1. Find $p + q + r$.
[Answer: 78]

5-3 Consider triangle FML with obtuse angle L. $FL = 8$ and the area of FML is 48. Point C lies on $\overline{FM}$ such that $\overline{FL} \perp \overline{CL}$ and $FC = 8CM$. Find FM.
[Answer: 15]

FAIRFIELD COUNTY MATH LEAGUE 2023–2024
Match 2
Individual Section

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Round 6: Equations of Lines

6-1 A line with equation $3x - 8y = C$, where C is a constant, contains the point $(24, 20)$. What is the y-coordinate of the y-intercept?
[Answer: 11]

6-2 Line l_1 has a slope of $\frac{5}{3}$ and a y −intercept of $(0, b)$, where b is a positive integer. Line l_1 is reflected across the x −axis to make line l_2, and the two lines intersect at $x = -21$. What is the value of b?
[Answer 35]

6-3 A line with equation $y = mx$, where m is a positive constant, has the property that decreasing the slope by 95% would reduce the measure of the angle made between the line and the x −axis in the first quadrant by 50%. Find the value of m^2.
[Answer: 360]

FAIRFIELD COUNTY MATH LEAGUE 2023-2024
Match 2
Team Round

Please write your answers on the answer sheet provided.

1. The function $floor(x)$, also known as the greatest integer function, maps x to the greatest integer that is less than or equal to x. Consider the five-digit number $3abcd$, where the last four digits $a, b, c,$ and d are unknown. This number has the property that $floor(\frac{3abcd}{8}) = abcd$ (a four-digit number comprised of the same unknown four digits in order). What is the four-digit number represented by $abcd$?
[Answer: 4285]

2. The polynomial $x^4 + kx + 35$, where k is a positive integer, is factorable into two quadratic trinomial factors with integer coefficients. What is the value of k?
[Answer: 204]

3. A rectangle has the property that its dimensions are integers and its area and perimeter are equal. Find the sum of all possible areas of the rectangle.
[Answer: 34]

4. The figure enclosed on the xy-plane by the equation $|x + y| + 3|x - y| < 8$ has an area of $\frac{a}{b}$, where a and b are positive integers with no common factors greater than 1. Find $a - b$.
[Answer: 61]

5. Two spotlights on level ground (assume elevation of 0) are aimed at a tightrope performer who stands on a rope that is stretched directly above the pathway between the lights. Light A makes angle A with the ground and light B makes angle B with the ground, and it is known that angle B is twice the measure of angle A. If the performer is 500 feet from light A and 350 feet from light B, then the height of the performer above the ground in feet is $\frac{a\sqrt{b}}{c}$, where $a, b,$ and c are positive integers, a and c have no common factors greater than 1 and b has no perfect square factors greater than 1. Find $ab + c$.
[Answer: 6007]

6. The parametric equations $x = \frac{3}{t-6} + 2$ and $y = \frac{2}{t-6} - 4$ produce a line on the xy −plane with a discontinuity at the point (a, b). Line l is perpendicular to this line and contains the point (a, b) and $(-12, c)$. What is the value of c?
[Answer: 17]