**FAIRFIELD COUNTY MATH LEAGUE 2023–2024**
**Match 6**
Individual Section

**Please write your answers on the answer sheet provided.**

Round 1: Lines and Angles

1-1 An isosceles triangle has two angles whose measures differ by $15°$. What is the measure in degrees of the supplement of the largest possible angle in the triangle?

1-2 A convex decagon has one set of three congruent angles and another set of seven congruent angles. If all the angle measures in degrees are integers, what is the smallest possible measure in degrees of one angle of the decagon?

1-3 Consider triangle $ABC$ with point $D$ on $\overline{BC}$ such that $AB = AD = DC$. If $m\angle DAB > m\angle CAD$, find the largest possible integer value of $m\angle ABC$ in degrees.

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Round 2: Literal Equations

2-1 If $x = 10 + \frac{3}{w}$ and $y = 7x + 5$, then $w = \frac{a}{y-b}$ for some integers $a$ and $b$. Find $a + b$.

2-2 The equation $x^2 + 3y = \sqrt{x^4 + 5x^2y + 8y^2 + \frac{416}{81}}$ has as one of its solutions $\left(\frac{a}{b}, \frac{4}{9}\right)$ where $a$ and $b$ are positive coprime integers. Find $a + b$.

2-3 The equation $xz + 2xy = x^2 + 5y(z - 3y)$ is equivalent to the equation $z = Ax + By$ for integers $A$ and $B$ with certain restrictions on the ordered triple solutions. One such restricted solution is the ordered triple $(C, 10, D)$. Find $A + B + C + D$.

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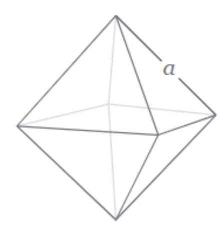
Round 3: Solids & Volume

3-1 A right cylinder has a base radius of 3 cm and the furthest distance from a point on one base to a point on the opposite base is $10$ cm. The volume of the cylinder is $V\pi$ cm$^3$. What is the value of $V$?

3-2 A particular hemisphere has the property that its volume is $x$ cubic units, its radius is $\frac{1}{2}x$ units, and its surface area is $y$ square units. What is the value of $y$?

3-3 A regular octahedron is a three-dimensional figure composed of 8 equilateral triangles. The volume of a regular octahedron is $ka^3$, where $a$ is the length of one edge of the octahedron and $k$ is a real constant. The volume of a regular octahedron with edge length $\frac{1}{k}$ is $\frac{p}{q}$, where $p$ and $q$ are positive integers with no common factors greater than 1. Find $p + q$.



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Round 4: Radical Expressions and Equations

4-1 Solve for $x > 8$: $\sqrt{x-8} = \frac{x-8}{11}$

4-2 If $\sqrt[6]{72000000} = (\sqrt{a})(\sqrt[3]{b})$ for positive integers $a > 1$ and $b > 1$, find the sum of the maximum possible value of $a$ and the maximum possible value of $b$.

4-3 What is the largest integer value of $p \leq 2024$ such that $\sqrt{x} + \sqrt{5} = \sqrt{x+p}$ has an integer solution for $x$?

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Round 5: Polynomials and Advanced Factoring

5-1 What is the smallest integer for which the polynomial $f(x) = x^3 - 10x^2 - 1$ has a positive value?

5-2 If the polynomial $x^4 - 65x^2 + p$, where $p$ is a constant, has four integer zeros, find the sum of all possible values of $p$.

5-3 If the polynomial $f(x) = x^3 - 18x^2 + kx - 96$ , where $k$ is a constant, has the property that its three real zeros form an arithmetic sequence, what is the value of $k$?

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<u>Round 6: Counting and Probability</u>

6-1 In how many ways can I arrange the letters of "GNOME" if the O must appear before the E (though it does not need to be next to it)?

6-2 On any given day, Mr. Forgette gives homework in his class with probability $p$ and Mr. Bearse gives homework in his class with probability $q$, and their homework assigning is independent. The probability that both teachers give homework on one day is $\frac{1}{18}$, and the probability that Mr. Forgette gives homework but Mr. Bearse does not on one day is $\frac{1}{6}$. The probability that neither give homework one day is $\frac{a}{b}$, wher $a$ and $b$ are integers with no common factors greater than 1. Find
$a + b$.

6-3 Wonka's Wilder Genes, a genetics testing company, has a test that is 80% effective in detecting the presence (or non-presence) of a certain gene. However, when someone tests positive, there is a note in the results saying that since the gene only appears in $p$ percent of the population, the probability the person has the gene is only 50%. Find the value of $p$.

**FAIRFIELD COUNTY MATH LEAGUE 2023-2024**
**Match 6**
Team Round

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1. An arithmetic sequence of positive integer angle measures starts at 6°, and there are exactly two pairs of distinct complementary angle measures in the sequence. What is the largest angle measure in the sequence less than 180°?

2. The graph of $2024y^2 + xy + x = 2024$ is of two lines which intersect at $(a, b)$. Find $a + b$.

3. Recall that for a convex polyhedron composed of $F$ faces with $E$ edges and $V$ vertices, $F + V = E + 2$. A particular polyhedron has the property that each vertex is formed from 4 equilateral triangles and one regular pentagon. How many faces will this polyhedron have?

4. If the function $f(x) = \sqrt{x} + \sqrt{x - 201}$ includes the integer pair $(a, b)$, find the sum of all possible values of $b$.

5. A cubic polynomial $f(x)$ has all integer coefficients less than 100 and a positive leading coefficient. If $f(1) = 4$, $f(2) = 10$, and $f(3) = 20$, find the largest possible sum of the absolute values of the coefficients.

6. On a standardized multiple choice exam on which each question had five options, $\frac{3}{10}$ of all takers got the hardest question correct. A team of analysts believes that there was a proportion of test takers $p$ who knew the correct answer, and another proportion of test takers $q$ who were "educated guessers" who were able to get the question right after narrowing the options down to two. The team is assuming that everyone else who got the question correct guessed randomly out of the five options. The values $p$ and $q$ satisfy the equation $Ap + Bq = 1$, where $A$ and $B$ are integers. Find $A + B$.