FAIRFIELD COUNTY MATH LEAGUE 2023–2024
Match 4
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Basic Statistics

1-1 A set of 100 elements has a median of 6 and the property that doubling every element would not change the standard deviation of the set. What is the sum of the elements of the set?
[Answer: 600]

1-2 A sequence of four numbers has a mean of 25, a median of 13, and a range of 50. What is the largest number in the sequence?
[Answer: 62]

1-3 How many sets of three distinct positive integers less than 100 have the property that their mean is equal to their median?
[Answer: 2401]

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Round 2: Quadratic Equations

2-1 A quadratic function $f(x)$ has one distinct real zero at $x = 6$ and a y-intercept of $(0,24)$. What is $f(15)$?
[Answer: 54]

2-2 The equation $2x^2 + ax + 2024 = 0$, where a is a constant, has solutions $x = p$ and $x = q$. The equation $x^2 - 68x + b = 0$, where b is a constant, has solutions $x = p + 2$ and $x = q + 2$. What is the value of b?
[Answer: 1144]

2-3 Consider a sequence of quadratic equations of the form $x^2 + p_n x + q_n = 0$ for $n = 1, 2, 3, ...$, where p_n and q_n are constants. This sequence has the property that for all $n > 1$, the zeros for the nth equation are the sum and positive difference of the zeros of the $(n-1)$th equation. If $p_1 = -10$ and $q_1 = 9$, for how many values of n is $q_n < 10000$?
[Answer: 10]

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Round 3: Similarity

3-1 Right triangle ABC has legs of length 16 and 30. It is similar to triangle DEF that has one leg of length 24. What is the sum of all possible perimeters of DEF?
[Answer: 184]

3-2 A sculptor is making a piece in the shape of a right square pyramid by pouring and shaping material from the bottom up. He pauses once he has a frustum with a top base area that is 36% smaller than the bottom base area. He finds he has used 305 cubic feet of material so far. How many cubic feet of material will the sculptor require for the entire sculpture?
[Answer: 625]

3-3 A rhombus is inscribed in a rectangle such that the rhombus and the rectangle share a diagonal of length 40. If the rhombus has area 400, what is the area of the rectangle?
[Answer: 640]

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Round 4: Variation

4-1 If z varies jointly as x and y and $z = 24$ when $x = 5$ and $y = 3$, find the value of xy when $z = 40$.
[Answer: 25]

4-2 Studies of a new dinosaur species *Reallybigosaur fairfieldi* have shown that the length of an individual varied as the 1.1 power of its tooth length, and that its weight varies as the $\frac{10}{3}$ power of its individual length. If specimen A has a tooth length that is 8 times the tooth length of specimen B, then we would expect that specimen A would weigh k times as much as specimen B. What is the value of k?
[Answer: 2048]

4-3 For some real number $n > 2$, z varies directly as the nth power of x and the $(n-2)$th power of y. If $z = 720$ when $x = 6$ and $y = 2$, what is the value of z when $x = 2$ and $y = 6$?
[Answer: 80]

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Round 5: Trig Expressions & DeMoivre's Theorem

5-1 If $z = 5 - i\sqrt{11}$, find the value of $|z^3|$.
[Answer: 216]

5-2 If $\sin(A) > 0$ and $\tan(A) = -2.4$, then $\sin\left(A - \frac{\pi}{3}\right) = \frac{a+b\sqrt{c}}{d}$, where a, b, and d have no common factors greater than 1 and c has no perfect square factors greater than 1. Find $a + b + c + d$.
[Answer: 46]

5-3 Given the equation $\arctan(x + 2) - \arctan(x) = \arctan\left(\frac{18}{25}\right)$, the sum of the squares of all possible values of x is $\frac{a}{b}$ where a and b are relatively prime integers. Find $a + b$.
[Answer: 77]

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Round 6: Conic Sections

6-1 If the circle with the equation $(x + a)^2 + (y + a)^2 = 2024$ passes through the origin, what is the value of a^2?
[Answer: 1012]

6-2 Let D represent the shortest distance from any point on the conic section $x^2 + y^2 - 16x - 30y + 271 = 0$ to the point on the conic section $2y^2 - x - 4y + 8 = 0$ that has the smallest x-value. What is D^2?
[Answer: 98]

6-3 A 10-meter by 6-meter rectangle is inscribed in an ellipse. The midpoint of each six meter side is one meter away from the nearest vertex of the ellipse. The distance in meters between the midpoint of each ten meter side and the nearest vertex is $\frac{a\sqrt{b}-c}{d}$, where a, c, and d are relatively prime positive integers and b is a positive integer with no perfect square factors greater than 1. Find $a + b + c + d$.
[Answer: 73]

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Match 4
Team Round

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1. Set A consists of five distinct positive integers. Set B is created by tripling every element of set A and then subtracting 12 from every element. The mean of set B is the same as that of set A, but the median of set B is six less than the median of set A. The range of set B is 20% of the square of the range of set A. What is the second largest element of set B?
[Answer: 12]

2. The quadratic equation $az^2 + bz = 8 - 15i$, where a is a nonzero complex constant and b is a real constant, has one distinct complex zero z_0. If $|z_0| = \frac{1}{3}$, find $|b|$.
[Answer: 102]

3. A rectangle is inscribed in a right triangle such that the triangle and the rectangle share a right angle, and one vertex of the rectangle lies on the hypotenuse of the right triangle. The triangle has legs of length 5 and 40, and the rectangle has area 32. Find the sum of the squares of all possible lengths of one diagonal of the rectangle.
[Answer: 1105]

4. Assume y varies inversely as the nth power of x. If $y = 112$ when $x = 3$ and $y = 7$ when $x = 24$, then the value of y when $x = 16$ is $\frac{a\sqrt[3]{b}}{c}$, where a, b, and c are positive integers, a and c are relatively prime, and b contains no perfect cube factors greater than 1. Find $a + b + c$.
[Answer: 37]

5. How many complex numbers $z = a + bi$, where a and b are both positive, have the property that $z^{2024} = z$?
[Answer: 505]

6. A hyperbola whose equation can be written in the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ has one asymptote with the equation $y = 2x + 3$ and one focus at $(8,9)$. Find the value of $h + k + a^2$.
[Answer: 17]