FAIRFIELD COUNTY MATH LEAGUE 2023–2024
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Decimals and Base Notation

1-1 For how many integers $n, 2 \leq n \leq 50$, is $\frac{1}{n}$ a terminating decimal?

1-2 If b is an integer base such that $301_b + 58_{10} = 413_b$, what is the value of 10_b in base 10?

1-3 For an integer base $x > 10$, where A is the digit for ten in that base, the sequence $13_x, 13.A_x, 14.8_x$ is arithmetic. If this arithmetic sequence continues, the next integer term will have value y. Express the sum $x + y$ as a numeral in base 3.

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Round 2: Word Problems

2-1 If 3 bakers could decorate 4 cakes in 5 minutes, how many minutes would it take 10 bakers to decorate 80 cakes? Assume bakers work at a constant pace independently.

2-2 Jen, Kaitlyn, and Debbie are comparing pet rock collections. They find that the ratio of pet rocks owned by Jen, Kaitlyn, and Debbie is $5:2:3$, respectively. Jen decides to give 85 total of her pet rocks to Kaitlyn and Debbie so they can all have the same number. How many pet rocks do they have combined?

2-3 An empty bus starts its route by picking up some passengers on Avon Street. It travels to Brookfield Lane, where it drops off a third of its passengers, but picks up two more than twice the number that it dropped off. From there it heads to Canton Drive, where it drops off 1 more than one-third of its passengers and picks up one-fourth of one more than it dropped off. Finally at Danbury Road it drops off all 22 of its passengers. How many passengers did the bus pick up on Avon Street?

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Round 3: Polygons

3-1 There exists a regular k −gon such that the number of diagonals is exactly 180 times the number of sides. What is the value of k?

3-2 Consider a regular n −gon for which the degree measure of one angle is divisible by 9. What is the largest possible value of n?

3-3 An equilateral quadrilateral with perimeter 40 is inscribed in an equiangular quadrilateral with area 120 such that the diagonals of the equilateral quadrilateral are parallel with the sides of the equiangular quadrilateral. What is the square of the perimeter of the equiangular quadrilateral?

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Round 4: Function and Inverses

Note: the inverse f^{-1} of a function is not necessarily a function.

4-1 If $g(x+1) = 3x + 7$ and $g(k) = 10$, find $g(5k)$.

4-2 If $f(x)$ is a linear function with y-intercept $(0,4)$ and $f^{-1}(x)$ crosses the y −axis at $(0,-2)$, find
$(f + f^{-1})(60)$.

4-3 Consider the functions $f(x) = \frac{a}{3x-b}$ and $g(x) = \sqrt{3x-c}$ with real constants a, b, and c. If $(f \circ g)^{-1}$ has a domain of $\left(-\infty, -\frac{1}{2}\right] \cup (0, \infty)$ and a range of $\left[\frac{5}{3}, 7\right) \cup (7, \infty)$, find
$(f \circ g)^{-1}\left(\frac{2}{7}\right)$.

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Round 5: Exponents & Logarithms

5-1 If $\log_2 x = 6$, $\log_2 y = x$, and $\log_y z = \frac{3}{8}$, find $\log_2 z$.

5-2 If m and n are positive integers less than 100 such that $\frac{36^{m-1}}{6^{3n+2}} = 6$, what is the largest possible value of n?

5-3 The equation $16^x - 8^{x+\frac{2}{3}} = 6 * 4^{x+2}$ has the solution $x = a + \log_2 b$, where a and b are positive integers and b is odd. Find $a + b$.

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Match 3
Individual Section

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Round 6: Matrices

6-1 If the matrix $\begin{bmatrix} 3 & -2 \\ k & 8 \end{bmatrix}$ does not have an inverse, find the determinant of the matrix $\begin{bmatrix} k & 2 \\ k & k \end{bmatrix}$.

6-2 If $\begin{bmatrix} 4 & 3 & 0 \\ m & 5 & 1 \\ 5 & 2 & m \end{bmatrix}$ is a singular matrix, the sum of the possible values of m is $\frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.

6-3 Consider matrix $A = \begin{bmatrix} 5 & x \\ 2 & 7 \end{bmatrix}$, where x is an integer. If the matrix A^{-1} has all integer entries, find the maximum possible value of the sum of the entries of A^{-1}.

FAIRFIELD COUNTY MATH LEAGUE 2023-2024
Match 3
Team Round

Please write your answers on the answer sheet provided.

1. Your computer programmer friend has asked you to take care of her dog while she is on a vacation. She texts you on the day she leaves: "Your pay is in the safe. Good luck." When you arrive at the home, you find a cryptic note by a safe in the kitchen: $FED_x - BED_x = ME_y$. On the back of the note are two scrawls: "$y = 3x + 8$" and "Combination: $xxyy$". You realize that *x* and *y* are integer bases and you know that when a base is larger than 10, letters are used to represent values larger than 9, with A = 10, B = 11, etc. If this combination is a four-digit number, with the digits of *x* in base ten being the first two digits and the digits of *y* in base ten being the second two, what is the combination to the safe?

2. Mike and Andrew are competing in a problem solving race to see who can solve a certain number of problems the fastest. Andrew starts out solving the problems at a consistent pace that is $33\frac{1}{3}\%$ faster than Mike's pace. However, after solving half his problems, Andrew blows a neuron and his pace slows to half of what it was originally, while Mike's pace stays consistent. After the race has gone on for two hours, Mike overtakes Andrew and stays ahead through the last fifteen problems to win the race. On average, how many seconds did Mike spend on each problem?

3. What is the largest possible value of n such that the degree measure of one angle of a regular n-gon is exactly 2 degrees greater than the degree measure of one angle of a regular m-gon?

4. Consider the functions $f(x) = 5 + \frac{18}{2+e^x}$ and $g(x) = \log_2(\log_3(4x + 1) - 4)$. The function $f(g(x))$ has a domain of $x > a$ and a range of $b < y < c$ for real numbers a, b, and c. What is $a + b + c$?

5. The sum of all real solutions to the equation $(\log_9(x^2))^2 + 3\log_x(9) = 1 + 6\log_3(x)$ is $\frac{a}{b}$, where a and b are positive integers with no common factors greater than 1. Find $a + b$.

6. The transpose of an $m\ x\ n$ matrix A, noted A^T, is an $n\ x\ m$ matrix in which row 1 of A is column 1 of A^T, row 2 of A is column 2 of A^T, etc. Consider matrix $A = \begin{bmatrix} 4 & x \\ 3 & y \end{bmatrix}$, where x and y are positive integers, and matrix $B = AA^T$. If $\det(A) < 0$, $\det(B) = 4$, and the entries of B have a sum of 625, find the value of x.