Name: ___________________________________

Grade:___________________________________

School: _________________________________

CONNECTICUT ARML QUALIFICATION TEST
Thursday, March 9, 2023

Pre-test instructions:

- When you have accessed this test, print it out and staple it. Do not read it.
- Then gather all you will need for the test: pencils, scrap paper. (Graph paper and protractors are not allowed.)
- Put your phone, all calculators, and any other electronic devices apart from your computer on a surface way out of reach. (You will need your phone at the end of the test in order to take images of your scratchwork.)
- Go to the bathroom now, so that you won't need to during the test.
- Your computer should still be signed on to the Zoom meeting you have been given for this test. Position your computer camera on your desk about 20 inches to the side of the place you will be working. Your entire work area should be visible to the proctors, and it must **not** be possible for other participants to read what you are writing.
- Read the directions below and wait to be told to start the test.

Test directions:

- You will be given 1 hour 45 minutes in which to answer 25 questions.
- All questions carry equal weight.
- All answers are positive integers.
- Write your work in the space provided. You will be required to submit your work at the end of the test in order to confirm the authenticity of your answers. This scratchwork will not be graded.
- All the usual rules for testing apply to this test, including the fact that no communication of any sort with any person is allowed, except with a proctor of this test. (You will be asked to sign a pledge at the end of the test.)
- Books, class notes, etc. are **not** allowed.
- Calculators and the Internet are **not** allowed.

1. Let $i = \sqrt{-1}$. The value of

$$\frac{1}{1+\frac{1}{2-i}}$$

is $a + bi$, where a and b are real. Find $100(a^2 + b^2)$.
[Answer: 50]

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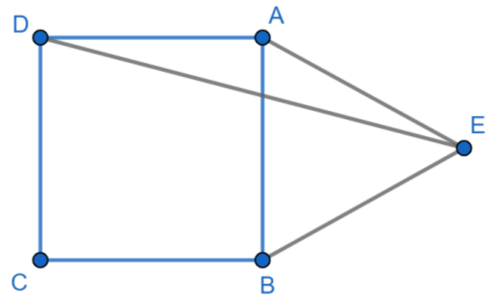
2. In the diagram below, $ABCD$ is a square, ABE is an equilateral triangle, and point E lies outside square $ABCD$. Find the degree measure of $\angle CDE$.
(In this test, do not attempt to include units in your answers.)



[Answer: 75]

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3. Evaluate

$$\left(\frac{27}{3^{\sqrt{3}}}\right)^{(3+\sqrt{3})}$$

[Answer: 729]

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4. Let $P(x) = ax^7 + bx^3 + cx - 7$, where a, b, c are constants. Given that $P(3) = 5$, find $|P(-3)|$.
[Answer: 19]

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5. Find the number of integer solutions to the equation $|2x + 7| + |2x - 1| = 8$.
[Answer: 4]

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6. Regular polygon $ABCDE\ldots$ has the property that $\angle ACD = 120°$. How many sides does the polygon have?
[Answer: 9]

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7. Let x be a real number such that $\sec x - \tan x = 2$. Then $\sec x + \tan x = \frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.
[Answer: 3]

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8. Let the finite sequence $a_1, a_2, a_3, \dots, a_{20}$ be defined by $a_1 = 72$ and $a_n = \phi(a_{n-1})$ for $2 \le n \le 20$, where $\phi(n)$ is the number of positive integer divisors of *n*. Find the sum of the twenty terms of the sequence.
[Answer: 127]

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9. Find the number of integers *n*, with $-10 \le n \le 10$, such that *n* lies in the domain of the function *f*, where

$$f(x) = \frac{\sqrt{x^2 - 5x - 6}}{x^2 + x - 30}$$

[Answer: 14]

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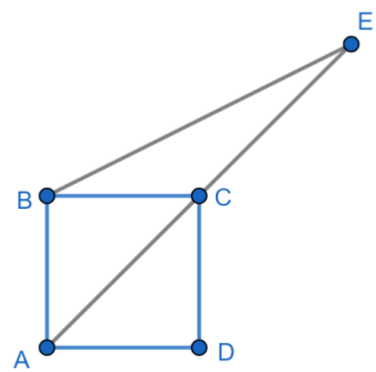
10. In the diagram below, the area of square *ABCD* is 1. Point *E* is on ray $\overrightarrow{AC}$ such that $CE = AC$. Find BE^2.



[Answer: 5]

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11. Find the largest solution *x*, in degrees, with $0 \le x \le 360°$, of the equation $\sin x \cos x = \frac{1}{4}$.
[Answer: 255]

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12. Let *a*, *b*, *c* be real numbers. Find the minimum possible value of

$$3a^2 + 27b^2 + 5c^2 - 18ab - 30c + 217$$

[Answer: 172]

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13. Alice and George have a fair 8-sided die with the numbers 0, 1, 2, 9, 2, 0, 1, 1 written on the faces. If Alice and George each roll the die once, the probability that Alice rolls a larger number than George does is $\frac{a}{b}$, where *a* and *b* are relatively prime positive integers. Find $a + b$.
[Answer: 87]

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14. Let x, y, and z be the roots of the equation $t^3 - 3t^2 + 2t - 4 = 0$.
Find $(x + 1)(y + 1)(z + 1)$.
[Answer: 10]

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15. If $\sin x + \cos x = \frac{1}{3}$, then $\sin^3 x + \cos^3 x = \frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.
[Answer: 40]

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16. Suppose that $x + \log_8 3$, $x + \log_4 3$, $x + \log_2 3$ are consecutive terms of a geometric sequence. Find the common ratio of the sequence.
[Answer: 3]

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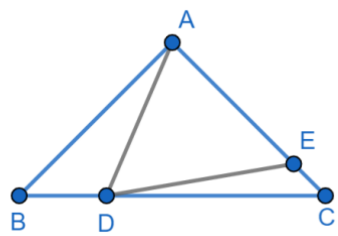
17. In the diagram below, $AB = AC$, $m\angle BAD = 30°$ and $AE = AD$. Find the degree measure of $\angle CDE$.



[Answer: 15]

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18. Find the remainder when 2022^{2022} is divided by 5.
[Answer: 4]

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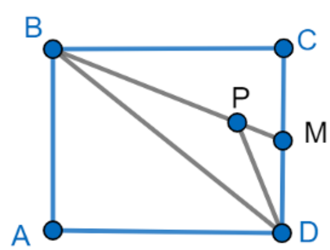
19. The diagram below shows rectangle $ABCD$ with $AB = 10$ and $BC = 12$. Let M be the midpoint of side CD and P be the point on line segment BM such that $BP = BC$. Then the area of quadrilateral $ABPD$ is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.



[Answer: 1153]

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20. How many integers n, with $1 \leq n \leq 100$, can be expressed as the difference of two perfect squares? (Note: 0 is a perfect square.)
[Answer: 75]

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21. Evaluate $\sqrt{97 \cdot 98 \cdot 99 \cdot 100 + 1}$
[Answer: 9701]

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22. Let $f(x) = \sqrt{2x + 1 + 2\sqrt{x^2 + x}}$. Evaluate

$$\frac{1}{f(1)} + \frac{1}{f(2)} + \cdots + \frac{1}{f(24)}$$

[Answer: 4]

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23. Let $f(x) = \frac{2}{4^x + 2}$. The value of

$$f\left(\frac{1}{2022}\right) + f\left(\frac{2}{2022}\right) + \cdots + f\left(\frac{2021}{2022}\right)$$

is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.
[Answer: 2023]

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24. For any set P, let $|P|$ denote the number of elements in P, and let $n(P)$ denote the number of subsets of P, including the empty set and P itself. Suppose, now, that A, B, and C are sets for which $n(A) + n(B) + n(C) = n(A \cup B \cup C)$ and $|A| = |B| = 100$. What is the minimum possible value of $|A \cap B \cap C|$?
[Answer: 97]

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25. For complex constant c and real constants p and q there are three distinct complex values of z that satisfy the equation $z^3 + cz + p(1 + qi) = 0$. Suppose that c, p, and q are chosen so that all three complex roots z satisfy $\frac{5}{6} \leq \frac{\mathrm{Im}(z)}{\mathrm{Re}(z)} \leq \frac{6}{5}$, where $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ denote the real and imaginary parts of z, respectively. The largest possible value of $|q|$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 649]

END OF TEST

Post-test directions. You have 10 minutes to complete steps 1–3 below.

1. Enter your answers on the Google Form provided in the Zoom chat. Check that you have typed your answers correctly, and then submit the form. (You have 3½ minutes to do this.)

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2. Please complete the following pledge by crossing out the words that do not apply, and add your signature.

   I did / did not abide by exam rules exactly as is expected in a classroom.
   I did / did not complete this test without the help of any person or source.
   I did / did not complete this test without the use of a calculator and/or the Internet.

   Signed:______________________________

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3. Get your phone, and using the Adobe Scan app, create a PDF consisting of all the pages of this test, including the front page (which includes your name) and this page (which includes your signed pledge). For this document, select share, email, and send the link by email to CTARMLTeam@gmail.com. Please include your name in the subject line.

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4. Please remain on the Zoom meeting until you are told that you may leave.
