FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 6
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Lines and Angles

1-1 Angle A has the property that its measure is exactly nine times the measure of its complementary angle. Find the measure of angle A in degrees.

1-2 Mr. Zucca is preparing a geometry exam. He has parallelogram $FCML$ and he wrote in that $m\angle F = (x^2 - 7x + 78)°$. He had written that one of the other angles had a measure of $(x^2 - 4x + 39)°$, but he does not remember which angle that was. He does remember that all the angle measures were supposed to be integers. Find the sum of all possible values of $m\angle F$.

1-3 Angles $A, B, C,$ and D all have positive integer degree measures less than 180. The measure of angle B is twice that of the measure of angle A. Angle C is supplementary to angle A and angle D is supplementary to angle B. If $m\angle A < m\angle D < m\angle B < m\angle C$, find the sum of the least and greatest possible values of $m\angle A$.

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Round 2: Literal Equations

2-1 In the equation $x(z-3)+5z=y(4-2z)$, $z=\frac{Ax+By}{x+Cy+D}$ for integers $A, B, C,$ and D. Find $A-B-2C+3D$.

2-2 The equation $x=\sqrt{y}+\sqrt{y-3}$, when solved for y, yields $y=\frac{x^A+Bx^C+D}{Ex^2}$, where $A, B, C, D,$ and E are integers. Find $A+B+C+D+E$.

2-3 The equation $16y=12x^5-(6+2y)x^3+96x^2-48$ is equivalent to $y=Ax^2+B$ for integers A and B, but only when x is not equal to a particular integer C. Find $A+BC$.

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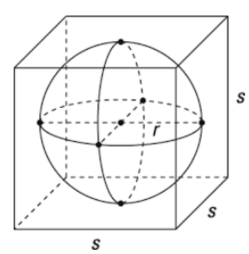
Round 3: Solids & Volume

3-1 The volume of a sphere with a surface area of π units is $\frac{a}{b}\pi$ units, where a and b are positive integers with no common factors greater than 1. Find $b - a$.

3-2 A triangle on the xy-plane with vertices $(0,0)$, $(15,8)$, and $(0,44)$ is rotated around the y-axis to form a three-dimensional figure. The surface area of this figure is $k\pi$ for some integer k. Find the value of k.

3-3 A sphere is inscribed in a cube (see diagram) and the shortest distance from one vertex of the cube to the surface of the sphere is 5 centimeters. The surface area of the cube in square centimeters is $a + b\sqrt{c}$, where a, b, and c are positive integers and c has no perfect square factors greater than 1. Find $a + bc$.



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Round 4: Radical Expressions and Equations

4-1 What is the extraneous solution to the equation $2 - x = \sqrt{5x - 4}$?

4-2 The quantity $\left(\sqrt[3]{4}\right)(\sqrt{6})$ can be written in simplest radical form as $a\sqrt[b]{c}$, where a, b, and c are integers. Find the value of $2a - b + c$.

4-3 The equation $(3x + 1)^{\frac{2}{3}} + 28 = (3993x + 1331)^{\frac{1}{3}}$ has two real solutions, x_1 and x_2, where
$x_1 > x_2$. Find the value of $2x_1 + x_2$.

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Round 5: Polynomials and Advanced Factoring

5-1 The polynomials $f(x) = x^3 + k$ where k is a constant and $g(x) = 3x + 12$ share an ordered pair when $x = 2$. What is the value of k?

5-2 The polynomial $Ax^3 + 11x^2 + Bx - 4$ can be factored into $(3x + C)(2x^2 + 3x + D)$, where $A, B, C,$ and D are integers. what is the value of $A + 2C - B$?

5-3 A polynomial function $f(x) = x^4 - x^3 + ax^2 + bx - 30$ where a and b are integers has one zero of $x = -2 + i$. Find the value of $|a| + |b|$.

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Match 6
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Round 6: Counting and Probability

6-1 How many different four-digit integers can I make by rearranging the numbers in 2023, including 2023? (Note: zero can not be the leading digit in a four-digit number.)

6-2 An unfair ten-sided die has sides labled 1-10. The probability of rolling at most 4 in a single roll is $\frac{4}{9}$. The probability of rolling at least 4 in a single roll is $\frac{3}{5}$. The probability of rolling a 4 is $\frac{a}{b}$, where a and b are positive integers with no common factors greater than 1. Find $b - a$.

6-3 Jess Masters wants to play a game. She places a chess piece is in the bottom left square of a 4x4 board. The piece may move one square at a time up, down, left, or right. She wants to move the piece from the bottom left square to the top right moving *only* up or right, and then she wants to move the piece back to the bottom left moving *only* down or left. In addition, in the path from the top right back to the bottom left, she does not want the piece to enter any other square in the top-most row or left-most column. How many different paths from the bottom left to the top right and back again can Jess take?

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Match 6
Team Round

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1. Let n be the number of regions into which 10 distinct lines will divide a plane. Find the difference between the greatest and least possible values of n.

2. For positive real numbers x and y such that $x > y$, the equation $\frac{x}{y} - \frac{y}{x} = 4\sqrt{6}$ is true when $y = (a - b\sqrt{c})x$ where $a, b,$ and c are positive integers and c has no perfect square factors greater than 1. Find $a + \frac{c}{b}$.

3. A right square pyramid has a base with side lengths s and a height h perpendicular to the base. This pyramid has a surface area A_1. The pyramid is then sheared in half by a plane through the vertex, with the length of the plane parallel to a side of the base. One half of the pyramid is then discarded, and the surface area of the remaining half is A_2. If $s = h$, the quantity $\frac{A_1}{A_2}$ is $a - b\sqrt{c}$, where a, b, and c are positive integers and c has no perfect square factors greater than 1. Find $a + 2b + 3c$.

4. For a particular value of x, the expression $\sqrt{2x + \sqrt{2x+1}}$ is greater than the expression $\sqrt{2x+1}$ by $\frac{1}{3}$. This value of x can be written as $\frac{a}{b}$ where a and b are integers with no common factors greater than 1. Find $a + 2b$.

5. The Sophie Germaine Identity states $x^4 + 4y^4 = (x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$. What is the largest value of the integer $k < 10000$ such that $x^4 + ky^4 = (x^2 + axy + by^2)(x^2 - axy + by^2)$ where a and b are positive integers?

6. Ivana Vinmunny has \$1 to play "Prime & Punishment". The game costs \$1 to pay. Ivana must roll a fair 20-sided die with sides labeled with the integers 1 through 20. Ivana will lose \$1 if she rolls a prime number, but will win \$1 otherwise. Ivanna will keep playing forever unless she runs out of money. The probability Ivana will go broke is $\frac{a}{b}$, where a and b are integers with no common factors greater than 1. Find $10a + b$.