FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 4
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Basic Statistics

1-1 The mean absolute deviation of a set of numbers is the average (arithmetic mean) of the positive differences between each number and the arithmetic mean of the data set. If a is the mean absolute deviation of the first ten positive integers and b is the median of the first 50 positive odd integers, find the product ab.
[Answer: 125]

1-2 A set of four numbers, three of which of are $2, 8,$ and 12, has the property that its average (arithmetic mean) is equal to its median. Find the sum of all possible values of the fourth number.
[Answer: 22]

1-3 The geometric mean of n numbers is the n^{th} root of the product of the numbers. Set A contains three distinct positive integers. Set B contains three elements that are each exactly 30 more than a corresponding element in set A. Set C contains three elements that are each exactly 20 times a corresponding element in set A. The geometric mean of the sum of the elements of A and the sum of the elements of B is equal to the average (arithmetic mean) of the elements of sets A and C. What is the largest possible value of an element of set B?
[Answer: 35]

FAIRFIELD COUNTY MATH LEAGUE 2022–20
Match 4
Individual Section

Please write your answers on the answer sheet provided.

Round 2: Quadratic Equations

2-1 The functions $f(x) = x^2 - 8x + 15$ and $g(x) = x^2 + px - 36$, where p is a constant, both have integer zeros, one of which is common to both functions. Find the value of $f(p)$.
[Answer: 24]

2-2 A quadratic function $f(x)$ has the same vertex as $g(x) = 2x^2 - 24x + 70$ and one zero of $6 - 2\sqrt{2}$. What is the y −coordinate of the y −intercept of $f(x)$?
[Answer: 7]

2-3 The quadratic equation $z^2 - (3 + pi)z + 60 = pi$, where p is a real constant and i is the imaginary constant, has one solution of $z = 3 + bi$ for some real number b. Find the value of b^2.
[Answer: 240]

FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 4
Individual Section

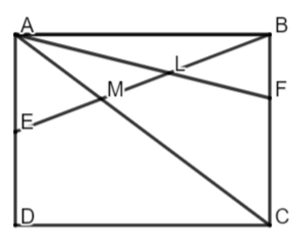
Please write your answers on the answer sheet provided.

Round 3: Similarity

3-1 A polygon has a perimeter of 10 units and an area of 20 square units. A similar polygon has an area of 40 square units and a perimeter of d units. Find the area in square units of a square with diagonal length d.
[Answer: 100]

3-2 Refer to the diagram. A trough shaped like an isosceles triangular prism is filled completely with water. The rectangular top of the trough has dimensions of 12 inches by 20 inches, and the trough holds 2880 cubic inches of water. A slow leak causes the trough to lose water at a rate of 4 cubic inches per minute. If the leak continues, after how many minutes will the water in the trough have dropped by eight inches?
[Answer: 400]

3-3 Consider rectangle $ABCD$ (see the diagram, not necessarily to scale) with point E on $\overline{AD}$ such that $AE = ED$, and point F on $\overline{BC}$ such that $FC = 2BF$. $\overline{BE}$ intersects $\overline{AC}$ at point M and $\overline{BE}$ intersects $\overline{AF}$ at point L. The ratio of the area of $FCML$ to the area of $ABCD$ is $\frac{p}{q}$ where p and q are integers with no common factors greater than 1. Find $2q - p$.
[Answer: 26]



FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 4
Individual Section

Please write your answers on the answer sheet provided.

Round 4: Variation

4-1 If y varies directly as the square of x and $y = 2$ when $x = \frac{1}{3}$, find the sum of the squares of all possible values of x when $y = 288$.
[Answer: 32]

4-2 Assume that z varies directly with x and inversely as the square of y, and $z = 5$ when $x = 3$ and $y = 4$. For how many ordered pairs (x, y) where x and y are positive integers less than 150 is $z = 5$?
[Answer: 7]

4-3 Dr. Dootiny is studying animal populations in an isolated ecosystem. There are white mice and brown mice competing for resources and a species of lizard that only eats brown mice. She makes a model saying that the population of white mice is inversely proportional to the population of brown mice, and the population of lizards varies directly as the square root of the population of brown mice. In her first week, she counts 2000 white mice and 450 lizards. After 6 months the population of lizards had doubled and there were 14,400 brown mice. If her model is correct, the product of the populations of the two types of mice at any time is n thousand $(n * 10^3)$ for some integer n. Find the value of n.
[Answer: 7200]

FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 4
Individual Section

Please write your answers on the answer sheet provided.

Round 5: Trig Expressions & DeMoivre's Theorem

5-1 If A and B are angles in quadrant one such that $\cos(A) = \frac{5}{13}$ and $\cos(B) = \frac{3}{4}$, then $\sin(A + B) = \frac{a+b\sqrt{c}}{d}$ where a, b, c, and d are integers, c has no perfect square factors greater than 1, and a, b, and d have no common factors greater than 1. Find the value of $a + 2b + 3c - d$.
[Answer: 15]

5-2 One of the complex twelfth roots of a complex number z has an argument of $\frac{13}{10}\pi$. If each of the twelfth roots of z is written with an argument between 0 and 2π, then the smallest of these arguments is $\frac{a}{b}\pi$, where a and b are positive integers with no common factors greater than 1. The argument of z is $\frac{c}{d}\pi$, where c and d are positive integers with no common factors greater than 1 and $0 \leq \frac{c}{d} < 2$. Find the value of $a + 2b + 3c + 4d$.
[Answer: 76]

5-3 If $\tan\left(x + \frac{\pi}{3}\right) = 5\sqrt{3}$, then $\sec^2(x) = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + 2b$.
[Answer: 51]

FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 4
Individual Section

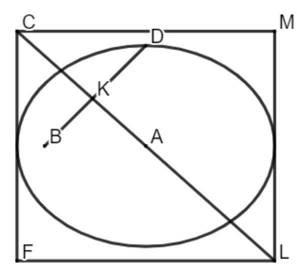
Please write your answers on the answer sheet provided.

Round 6: Conic Sections

6-1 A particular conic section is the set of all points equidistant from the point $(8,2)$ and the line $x = 6$. What is the x −value of the point on this conic where $y = 8$?
[Answer: 16]

6-2 A circle has a center that lies on $x = -4$, has a radius of 3, and contains the point $(-5,10)$. Find the product of all possible y −coordinates of the center of this circle.
[Answer: 92]

6-3 Refer to the diagram (not necessarily drawn to scale) that shows an ellipse with the same center as a rectangle FCML where $\overline{FC}$ and $\overline{ML}$ are tangent to the ellipse. The ellipse has a major axis of length 18 and a minor axis of length 14. The center of the ellipse is point A and one of the foci is labeled point B, so $\overline{FL}||\overline{AB}$. The point on the ellipse with the largest y-value is point D. A diagonal of the rectangle intersects $\overline{BD}$ at point K so that $BK = AK$. The area of the rectangle is $\frac{a\sqrt{b}}{c}$ where b has no perfect square factors greater than 1 and a and c have no common factors greater than 1. Find $a - b + c$.
[Answer: 567]



FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 4
Team Round

Please write your answers on the answer sheet provided.

1. A set of three distinct integers has the property that doubling the largest number would increase the range by 40% and increase the average (arithmetic mean) by 12. What is the positive difference between the largest and smallest possible values of the median of the original set?
[Answer: 88]

2. There are two values of the constant m such that the equation $\frac{21}{m^3}x^2 + \frac{10}{m}x = \frac{7-4m}{m}x - \frac{9m}{4} - \frac{10}{m^2}x^2$ has exactly one real solution for x. If the two values of m are m_1 and m_2 such that $m_1 > m_2$, find the value of $-\frac{5m_1}{m_2}$.
[Answer: 16]

3. A particular irregular prism has a height of 10 units, a surface area of 25 square units, and a volume of 40 cubic units. For how many different integer heights h such that $10 < h < 100$ would a similar prism have an integer surface area in square units or an integer volume in cubic units?
[Answer: 53]

4. For some positive number n, z varies directly as x to the power of n and inversely as y to the power of $n + 1$. If $z = \frac{1}{16}$ when $x = 1$ and $y = 16$, and $z = 16$ when $x = 256$ and $y = 1$, find z when $x = 128$ and $y = 4$.
[Answer: 2]

5. If $\cos\left(x - \frac{\pi}{4}\right) = \frac{7}{8}$, find the value of $32\sin(2x)$.
[Answer: 17]

6. The eccentricity of a hyperbola is defined as the ratio of the distance between the center and one focus to the distance between the center and one vertex. A hyperbola with equation $Ax^2 + By^2 + Cx + Dy + E = 0$ for real constants $A, B, C, D,$ and E has an eccentricity of 2.6, one focus at (5,2), and another focus in quadrant II 14 units away horizontally from the first focus. If the equation of the asymptote with a positive slope is $f(x)$, find $f(13)$.
[Answer: 38]