

FAIRFIELD COUNTY MATH LEAGUE 2022–2023

Match 3

Individual Section

Please write your answers on the answer sheet provided.

Round 1: Decimals and Base Notation

1-1 Let S be the set of 17 numbers that can be written as 31_b where b is an integer and $4 \leq b \leq 20$. The set S contains p prime numbers and n perfect square numbers. Find the value of $2p + 3n$.

1-2 If $\frac{(5 \cdot 10^3)^x (2^{3x})}{(2 \cdot 10^6)^{x-2}} = 10^n$ for integers x and n , what is the value of n ?

1-3 For some integer base x , $210_x - 101_{x+1} = 202_{x-4}$. Express the value of 10_x as a numeral in base 10.

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Round 2: Word Problems

- 2-1 Mr. Hill brought some money to the local bookstore. He spent $\frac{1}{3}$ of the money he brought on a science book and $\frac{2}{3}$ of the remaining money he brought on a history book. If the difference between the prices of the books was \$8.75, what was the cost in dollars of the history book?
- 2-2 Mr. Allwood and Mr. Krupnikoff are raising racing slugs. They decide to race their slugs down a 36-inch yard stick. Mr. Allwood bets Mr. Krupnikoff that his slug is so fast his could win even if Mr. Krupnikoff's had a five minute head start. Mr. Krupnikoff's slug starts down the yard stick at a pace of 3 inches per minute, and five minutes later Mr. Allwood's slug starts down the yard stick at a pace of 5 inches per minute. How many inches apart are the two slugs when the first one crosses the finish line?
- 2-3 A garbage can in a public park is put out in the morning every day, and fills three times faster from 9:00 AM on than it does before 9:00 AM. The fill rate every day prior to 9:00 AM is the same. On Monday, by the time the can was full, it had been out twice as long from 9:00 AM on as it had been prior to 9:00 AM. On Tuesday, the can is put out 45 minutes later than it was on Monday (though still prior to 9:00 AM) and was full by 12:05 PM. What time was the can put out on Monday morning? Enter your answer as a three-digit integer with no colon (for example: enter the time 6:17 as 617).

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Round 3: Polygons

- 3-1 For how many values of $n < 100$ are the number of diagonals of the n -gon a positive integer multiple of n ?
- 3-2 A regular n -gon has the property that one of its interior angles measures exactly ten degrees more than the measure of one interior angle of a regular 28-gon. What is the value of n ?
- 3-3 Let n_1 be the number of sides of a polygon with the property that the number of diagonals is less than the number of diagonals of an $(n_1 + 2)$ -gon by the measure in degrees of an exterior angle of a regular octagon. Let n_2 be the number of sides of a regular polygon with the property that it is the only regular polygon whose interior angle measures in degrees is a prime integer. Find the value of $2n_1 + n_2$.

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Round 4: Function and Inverses

Note: the inverse f^{-1} of a function is not necessarily a function.

4-1 If $f(x) = 4x - \frac{8}{5}$, find $f^{-1}(f^{-1}(88))$.

4-2 Let $f(x)$ be a one-to-one function that has domain $[-12,8]$ and range $[1,10]$. If $g(x) = 3f(2x + 6) - 7$, then $g(x)$ has a domain $[a, b]$ and a range of $[c, d]$, where a, b, c , and d are integers. Find the value of $b - 2a + d - 2c$.

4-3 Consider functions $f(x) = \sqrt{x+2}$ and $g(x) = \frac{1}{x^2-9}$ and real numbers c_1, c_2 , and c_3 . c_1 is in the domain of f but not in the domain of g . c_2 is in the domains of neither f nor g . c_3 is in the domains of both f and g but NOT in the domain of $g \circ f$. Find the value of $f^{-1}(c_1 + 2c_2 + 3c_3)$.

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Round 5: Exponents & Logarithms

5-1 If $x = \log_7 14$ and $y = \log_7 245$, find the value of 7^{x+y-2} .

5-2 If $(3^b)^{b-a} = 9^{\frac{1}{2}a^2 - \frac{1}{2}ab - 2}$ and $a + b = 12$, find the value of 64^{a-b} .

5-3 If $6 \log_3 x + 11 = 10 \log_x 3$ and $x > 1$, find the value of $x^3 - 1$.

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Round 6: Matrices

6-1 If $\begin{bmatrix} 5 \\ x \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & y \end{bmatrix} \begin{bmatrix} x \\ 6 \end{bmatrix} = \begin{bmatrix} 13 \\ 88 \end{bmatrix}$, what is the value of y ?

6-2 Consider matrices $A = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$ and $B = \begin{bmatrix} 3x & y \\ x & 3y \end{bmatrix}$. If $\det(A) = 18$ and $\det(B) = 28$, find the square of the sum of the elements of B .

6-3 There are two values of x which make the matrix $\begin{bmatrix} 6 & -1 & x \\ x-1 & 4 & 14 \\ -x & 1 & 0 \end{bmatrix}$ singular (non-invertible). If the two values are x_1 and x_2 and $x_1 > x_2$, find the value of $3x_1 - 5x_2$.

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Team Round

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1. For how many integer bases $p > 6$ is the number $14641_p \leq 1.3 * 10^7$?
2. Randy and Katina have one hour to finish a huge pizza as part of a local challenge. On this day, Randy's eating rate is twice that of Katina's. After they have finished one third of the pizza, Randy needs to take a 20 minute break, after which he resumes eating and they finish the pizza together after another 24 minutes. When they finished the pizza, how many *seconds* were left until time was up?
3. Consider regular 20-gon $ABC \dots RST$. The angles formed by every diagonal with one endpoint A and \overline{AB} are measured. What is the total sum in degrees of the measures of these angles?
4. Let $f(x) = \frac{x}{\sqrt{x+3}-1}$ and $g(x) = |f(x) - \sqrt{x+3} - 1|$. If a is the smallest integer such that $g(x) \leq .01$ for all $x \geq a$, what is the value of a ?

5. Solve for the value of N :

$$\sum_{k=1}^N \log\left(\frac{k}{k+1}\right) + \prod_{k=N+1}^{9999} \log_k(k+1) = 0$$

Note: $\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$ and $\prod_{k=1}^n a_k = a_1 a_2 a_3 \dots a_n$

6. Consider matrices $A = \begin{bmatrix} 12 & a \\ 4 & b \end{bmatrix}$ and $B = \begin{bmatrix} c & -8 \\ -1 & d \end{bmatrix}$ where $a, b, c,$ and d are constants. If $A^{-1} = B$, find the determinant of the matrix $\begin{bmatrix} a & d \\ b & c \end{bmatrix}$.