FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 3
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Decimals and Base Notation

1-1 Let S be the set of 17 numbers that can be written as 31_b where b is an integer and $4 \leq b \leq 20$. The set S contains p prime numbers and n perfect square numbers. Find the value of $2p + 3n$.
[Answer: 21]

1-2 If $\frac{(5*10^3)^x(2^{3x})}{(2*10^6)^{x-2}} = 10^n$ for integers x and n, what is the value of n?
[Answer: 16]

1-3 For some integer base x, $210_x - 101_{x+1} = 202_{x-4}$. Express the value of 10_x as a numeral in base 10.
[Answer: 12]

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Round 2: Word Problems

2-1 Mr. Hill brought some money to the local bookstore. He spent $\frac{1}{3}$ of the money he brought on a science book and $\frac{2}{3}$ of the remaining money he brought on a history book. If the difference between the prices of the books was \$8.75, what was the cost in dollars of the history book?
[Answer: 35]

2-2 Mr. Allwood and Mr. Krupnikoff are raising racing slugs. They decide to race their slugs down a 36-inch yard stick. Mr. Allwood bets Mr. Krupnikoff that his slug is so fast his could win even if Mr. Krupnikoff's had a five minute head start. Mr. Krupnikoff's slug starts down the yard stick at a pace of 3 inches per minute, and five minutes later Mr. Allwood's slug starts down the yard stick at a pace of 5 inches per minute. How many inches apart are the two slugs when the first one crosses the finish line?
[Answer: 1]

2-3 A garbage can in a public park is put out in the morning every day, and fills three times faster from 9:00 AM on than it does before 9:00 AM. The fill rate every day prior to 9:00 AM is the same. On Monday, by the time the can was full, it had been out twice as long from 9:00 AM on as it had been prior to 9:00 AM. On Tuesday, the can is put out 45 minutes later than it was on Monday (though still prior to 9:00 AM) and was full by 12:05 PM. What time was the can put out on Monday morning? Enter your answer as a three-digit integer with no colon (for example: enter the time 6:17 as 617).
[Answer: 735]

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Round 3: Polygons

3-1 For how many values of $n < 100$ are the number of diagonals of the n −gon a positive integer multiple of n?
[Answer: 48]

3-2 A regular n-gon has the property that one of its interior angles measures exactly ten degrees more than the measure of one interior angle of a regular 28-gon. What is the value of n?
[Answer: 126]

3-3 Let n_1 be the number of sides of a polygon with the property that the number of diagonals is less than the number of diagonals of an $(n_1 + 2)$-gon by the measure in degrees of an exterior angle of a regular octagon. Let n_2 be the number of sides of a regular polygon with the property that it is the only regular polygon whose interior angle measures in degrees is a prime integer. Find the value of $2n_1 + n_2$.
[Answer: 406]

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Round 4: Function and Inverses

Note: the inverse f^{-1} of a function is not necessarily a function.

4-1 If $f(x) = 4x - \frac{8}{5}$, find $f^{-1}(f^{-1}(88))$.
[Answer: 6]

4-2 Let $f(x)$ be a one-to-one function that has domain $[-12,8]$ and range $[1,10]$. If $g(x) = 3f(2x + 6) - 7$, then $g(x)$ has a domain $[a, b]$ and a range of $[c, d]$, where a, b, c, and d are integers. Find the value of $b - 2a + d - 2c$.
[Answer: 50]

4-3 Consider functions $f(x) = \sqrt{x + 2}$ and $g(x) = \frac{1}{x^2-9}$ and real numbers c_1, c_2, and c_3. c_1 is in the domain of f but not in the domain of g. c_2 is in the domains of neither f nor g. c_3 is in the domains of both f and g but NOT in the domain of $g \circ f$. Find the value of $f^{-1}(c_1 + 2c_2 + 3c_3)$.
[Answer: 322]

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Round 5: Exponents & Logarithms

5-1 If $x = \log_7 14$ and $y = \log_7 245$, find the value of 7^{x+y-2}.
[Answer: 70]

5-2 If $(3^b)^{b-a} = 9^{\frac{1}{2}a^2 - \frac{1}{2}ab - 2}$ and $a + b = 12$, find the value of 64^{a-b}.
[Answer: 4]

5-3 If $6\log_3 x + 11 = 10\log_x 3$ and $x > 1$, find the value of $x^3 - 1$.
[Answer: 8]

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Individual Section

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Round 6: Matrices

6-1 If $\begin{bmatrix}5\\x\end{bmatrix}+\begin{bmatrix}2 & -1\\3 & y\end{bmatrix}\begin{bmatrix}x\\6\end{bmatrix}=\begin{bmatrix}13\\88\end{bmatrix}$, what is the value of y?
[Answer: 10]

6-2 Consider matrices $A=\begin{bmatrix}x & y\\-y & x\end{bmatrix}$ and $B=\begin{bmatrix}3x & y\\x & 3y\end{bmatrix}$. If $det(A)=18$ and $det(B)=28$, find the square of the sum of the elements of B.
[Answer: 400]

6-3 There are two values of x which make the matrix $\begin{bmatrix}6 & -1 & x\\x-1 & 4 & 14\\-x & 1 & 0\end{bmatrix}$ singular (non-invertible). If the two values are x_1 and x_2 and $x_1 > x_2$, find the value of $3x_1 - 5x_2$.
[Answer: 37]

FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 3
Team Round

Please write your answers on the answer sheet provided.

1. For how many integer bases $p > 6$ is the number $14641_p \leq 1.3 * 10^7$?
   [Answer: 53]

2. Randy and Katina have one hour to finish a huge pizza as part of a local challenge. On this day, Randy's eating rate is twice that of Katina's. After they have finished one third of the pizza, Randy needs to take a 20 minute break, after which he resumes eating and they finish the pizza together after another 24 minutes. When they finished the pizza, how many *seconds* were left until time was up?
   [Answer: 40]

3. Consider regular 20-gon $ABC \ldots . RST$. The angles formed by every diagonal with one endpoint A and $\overline{AB}$ are measured. What is the total sum in degrees of the measures of these angles?
   [Answer: 1377]

4. Let $f(x) = \frac{x}{\sqrt{x+3}-1}$ and $g(x) = \left|f(x) - \sqrt{x+3} - 1\right|$. If a is the smallest integer such that $g(x) \leq .01$ for all $x \geq a$, what is the value of a?
   [Answer: 40398]

5. Solve for the value of N:
   $$\sum_{k=1}^{N} \log\left(\frac{k}{k+1}\right) + \prod_{k=N+1}^{9999} \log_k(k+1) = 0$$

   Note: $\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_n$ and $\prod_{k=1}^{n} a_k = a_1 a_2 a_3 \ldots a_n$
   [Answer: 99]

6. Consider matrices $A = \begin{bmatrix} 12 & a \\ 4 & b \end{bmatrix}$ and $B = \begin{bmatrix} c & -8 \\ -1 & d \end{bmatrix}$ where a, b, c, and d are constants. If $A^{-1} = B$, find the determinant of the matrix $\begin{bmatrix} a & d \\ b & c \end{bmatrix}$.
   [Answer: 55]