FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 2

Round 1: Factors and Multiples

1-1 If an integer is selected from the set of odd integers greater than 1 and less than 50, then the probability it is prime is $\frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $b - a$.
[Answer: 11]

1-2 The greatest common factor between x and 180 is 15. The least common multiple of x and 225 is 1575. Find the sum of all possible values of x.
[Answer: 630]

1-3 For two positive integers p and q, the product pq is the smallest whole number with exactly 14 factors, and the greatest common factor of p and q is 4. Find the least common multiple of p and q.
[Answer: 48]

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Round 2: Polynomials and Factoring

2-1 If $(2x+3)(ax^2+7x+b)=10x^3+cx^2+13x+d$ for integers $a, b, c,$ and d, find $a+b+c+d$.
[Answer: 18]

2-2 For how many distinct integer values of M is $27x^2+Mx+20$ factorable into two binomials with integer coefficients?
[Answer: 24]

2-3 The polynomial $f(x)=2x^3-24x^2+px-q$, where p and q are integers, has one zero of $x=1+\frac{3}{2}i$. What is the value of q?
[Answer: 65]

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Round 3: Area and Perimeter

3-1 A rectangle with integer dimensions has twice the area of a square with a perimeter of 24 units. What is the smallest possible perimeter of the rectangle?
[Answer: 34]

3-2 An isosceles trapezoid has a diagonal of length 10 and a height of $2\sqrt{5}$. What is the area of the trapezoid?
[Answer: 40]

3-3 Consider regular octagon $ABCDEFGH$. If the trapezoid $ABCD$ is reflected across its longer base, the resulting equilateral hexagon has area M. A regular hexagon with the same perimeter as the equilateral hexagon has area N. The ratio $\frac{M}{N}$ can be written in simplest radical form of $\frac{a\sqrt{b}+c\sqrt{d}}{f}$ where $a, b, c, d,$ and f are positive integers, b and d have no perfect square factors greater than 1, and $a, c,$ and f have no factors common to all of them greater than 1. Find $2(a+c)+3(b+d)-f$.
[Answer: 26]

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Round 4: Absolute Value & Inequalities

4-1 The inequality $|x+7|<22$ has a solution set of $p<x<q$. Find $2q-3p$.
[Answer: 117]

4-2 Consider the equation $|2x-a|=b$, where a and b are constants, and $b>0$, and suppose that the value of b is given. Then there are two values of a such that $x=18$ is a solution of the equation. Let these two values of a be a_1 and a_2. The sum of the other solution of the equation when $a=a_1$ and the other solution of the equation when $a=a_2$ is k, where k is independent of the initial choice of b. Find k.
[Answer: 36]

4-3 How many integers satisfy the inequality $\left|6-\frac{3}{4}x\right|-\left|\frac{1}{3}x+\frac{2}{3}\right|<4$?
[Answer: 24]

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Round 5: Law of Sines and Cosines

5-1 The Dutchman walks along a pier, and he spots a tugboat in the water 45 degrees north of west of his position. He walks 10 meters directly west along the pier and notices the tugboat is now 75 degrees north of west of his position. How far is the tugboat from his position in meters? Round your answer to the nearest whole meter.
[Answer: 14]

5-2 Triangle SCL has side lengths $SC = 9$, $SL = 6$, and $CL = 11$. The line segment $\overline{SL}$ is extended to a point N, with L between S and N, in such a way that triangle SCN is isosceles. $CN = a\sqrt{b}$ where a and b are integers and b has no perfect square factors greater than 1. Find $b - 2a$.
[Answer: 38]

5-3 Kite *ABCD* has right angles at *B* and *D*. The smallest angle in the kite is at *A*, and the measure of angle *A* is α, where $\cos\alpha = \frac{1}{8}$. The ratio of one of the shorter side lengths of the kite to one of its longer side lengths is $\frac{\sqrt{a}}{b}$, where a and b are integers and a has no perfect square factors larger than 1. Find $2a - b$.
[Answer: 11]

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Round 6: Equations of Lines

6-1 Line l_1 passes through the origin and has a slope of $\frac{3}{4}$. Line l_2 is perpendicular to l_1 and intersects l_1 at a distance of 10 units from the origin in quadrant I. If line l_2 has an equation in slope-intercept form of $y = mx + b$, find the value of $b - m$.
[Answer: 18]

6-2 A line with the equation $y = \frac{7}{3}x + a$ can be represented parametrically by the equations $x = 3t + b$ and $y = 7t + 11$, where a and b constants. What is the value of $3a + 7b$?
[Answer: 33]

6-3 Two perpendicular lines, one with a y-intercept of 17 and the other with a y-intercept of 4, intersect at the point $(6, k)$ where $k > 9$. What is the value of k?
[Answer: 13]

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Team Round

T-1 Let $f(x)$ be the number of positive integers less than or equal to x that are divisible by neither 6 nor 15. As x grows without bound, the value $\frac{f(x)}{x}$ approaches a rational number $\frac{p}{q}$, where p and q are positive integers with no common factors greater than 1. What is $p^2 + q^2$?
[Answer: 41]

T-2 A fourth degree polynomial $f(x)$ has the properties that its coefficients are real, the quartic coefficient is 1, $f(i) = 0$, dividing $f(x)$ by $x - 2$ produces a remainder of 50, and dividing $f(x)$ by $x + 1$ produces a remainder of -10. Find the value of $f(3)$.
[Answer: 190]

T-3 Quadrilateral $QUAD$ is inscribed in a circle. Both $\overline{QA}$ and $\overline{UD}$ are diameters of the circle. If the perimeter of $QUAD$ is 48 and the area of $QUAD$ is 67.5, then the area of the circle is $\frac{a}{b}\pi$ where a and b are integers with no common factors greater than 1. Find $a - b$.
[Answer: 437]

T-4 The equation $\left||x+9||x-5|-3\right| = k$, where k is a positive integer, has exactly three solutions when $k = a$ and exactly six solutions when $k = b$. Find the value of $|a - b|$.
[Answer: 43]

T-5 Consider triangle ABC with $AB = 3$, $AC = 7$, and $BC = 8$. If r is equal to the ratio of the sine of the smallest angle to the sine of the largest angle and t is the least numerical value of $\tan(A)$, $\tan(B)$, and $\tan(C)$, find the value of rt^2.
[Answer: 18]

T-6 Consider lines l_1 and l_2, where $l_1 \perp l_2$ and line l_1 has a positive slope. The lines intersect on the positive y-axis at point A, making a right triangle with the x-intercept of line l_1, B, and the x-intercept of line l_2, C. The y-axis intersects the hypotenuse of ABC at point D, and $CD = 4BD$. If the area of ABC is 245 square units, then the equation of l_2 can be written as $ax + by = c$ where a, b and c are positive integers that have no factors common

among all three of them greater than 1. Find the value of $3a + 9b + c$.
[Answer: 49]