FAIRFIELD COUNTY MATH LEAGUE 2022–2023
Match 1

Round 1: Percentages

1-1 I'm thinking of a three digit number, all of whose digits are nonzero. The hundreds digit is 75% of the tens digit, and the ones digit is 20% less than the sum of the tens digit and twice the hundreds digit. What is the number?
[Answer: 348]

1-2 How many positive integers have the property that increasing them by $66\frac{2}{3}\%$ and then decreasing that result by 65% produces a value greater than 100 but less than 1000?
[Answer: 1543]

1-3 There is a positive number x such that increasing x by $x\%$ is equivalent to doubling the sum of x and $\frac{3}{8}$. If x can be written in simplest radical form as $a + b\sqrt{c}$, find the value of $2a - b + c$.
[Answer: 198]

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Round 2: Solving Equations

2-1 Solve for x: $3x - 5\left(x + 7\big(2x - 6(x+1)\big)\right) = 6(25x + 1)$
[Answer: 17]

2-2 The ordered pair $(2,7)$ is one ordered pair (x, y) that solves the equation $ax^2 - by = 1$ where a and b are positive integers less than 100. What is the largest possible value of b?
[Answer: 53]

2-3 Find the sum of the squares of all real solutions to the equation
$\frac{3x+12}{x-2} + 3x^2 - 2 = \frac{3x^3-6}{x-2} + 5x.$
[Answer: 1]

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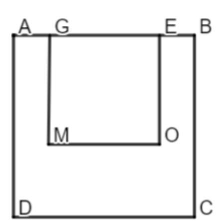
<u>Round 3: Triangles and Quadrilaterals</u>

3-1 If a is the measure of one angle of an equilateral triangle in degrees, b is the largest possible integer degree measure of a base angle of an isosceles triangle, and c is the degree measure of the largest angle in a right triangle, find $2a + 3b + c$.
[Answer: 477]

3-2 A rhombus has diagonals of integer length and a perimeter of 100. What is the largest possible area of the rhombus?
[Answer: 600]

3-3 Consider the diagram to the right (not drawn to scale) showing squares $ABCD$ and $GEOM$, with points G and E on $\overline{AB}$ such that $AG = EB$. If the ratio of the areas of the squares is $\frac{3}{8}$, then the ratio of the area of trapezoid $DMOC$ to the area of square $ABCD$ is $\frac{p}{q}$ where p and q are relatively prime integers. Find $p + q$.
[Answer: 21]



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Round 4: Systems of Equations

4-1 At Clog O' Burger, 6 burgers and 4 orders of fries is $46.50, and 4 burgers and 6 orders of fries is $43.50. What is the total cost of 2 burgers and 2 orders of fries at Clog O' Burger in dollars?
[Answer: 18]

4-2 Given the system $\begin{cases} \frac{8}{x} + \frac{3}{y} = 24 \\ \frac{2}{x} - \frac{9}{y} = -7 \end{cases}$, find the value of $10xy$.
[Answer: 3]

4-3 The graphs of $x^2 - y^2 + 3x + 4y - 31 = 0$ and $x + y = 10$ intersect at the point (a, b). Find the value of $20a + b$.
[Answer: 101]

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Round 5: Right Triangles

5-1 A right triangle has the properties that the tangent of one of its acute angles has a value of $\frac{4}{5}$ and the area of the triangle is 50 square units. What is the square of the length of the hypotenuse?
[Answer: 205]

5-2 A balloon is flying between two points on level ground, point A and point B. If the tangent of the angle of elevation to the balloon from point A is $\frac{2}{3}$ and the tangent of the angle of elevation to the balloon from point B is $\frac{3}{8}$ and the ground distance between points A and B is 900 feet, find the height above the ground of the balloon in feet.
[Answer: 216]

5-3 A right triangle has all integer side lengths. One of the legs has a length of 48. What is the largest possible length of the hypotenuse?
[Answer: 577]

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Round 6: Coordinate Geometry

6-1 The point $(5, -8)$ is rotated 90 degrees counterclockwise about the origin, then shifted up 7 units, and then reflected across the line $y = x$ to produce the new point with coordinates (p, q). Find the value of $2p + 3q$.
[Answer: 48]

6-2 The point $(2,7)$ is exactly three units away from the point $(2,10)$ which lies on the line $y = 2x + 6$. There is one other point on the line $y = 2x + 6$ that lies three units away from $(2,7)$. If its coordinates are (a, b), find the value of $a + 2b$.
[Answer: 10]

6-3 The points $(2,3)$, $(6,7)$, and $(4,1)$ all lie on a circle with center (h, k) and radius R. Find $\frac{kR^2}{h}$.
[Answer: 8]

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Team Round

T-1 Two positive integers a and b have the property that a% of b is equal to the sum of 60% of a and 30% of b. What is the second largest possible value of b?
[Answer: 960]

T-2 The equation $\frac{5}{x+1}+\frac{1}{x+25}+2=\frac{2x}{x-2}$ has solutions x_1 and x_2, where $x_1 > x_2$. Find $x_1 - x_2$.
[Answer: 27]

T-3 Consider isosceles trapezoid $FCML$ with bases $\overline{FL}$ and $\overline{CM}$ and point E on $\overline{CM}$ such that the inscribed triangle ELF is equilateral. If $FL = 18$ and the triangle comprises 60% of the trapezoid's total area, then the perimeter of $FCML$ can be written as $x + y\sqrt{z}$ where $x. y$, and z are positive integers and z has no perfect square factors greater than 1. Find $x + y + z$.
[Answer: 49]

T-4 If A and B are positive numbers such that the system $\begin{cases} Ax + By = 6 \\ 9x + Ay = 10 \end{cases}$ has infinite solutions for (x, y), then B can be written in simplest form as $\frac{p}{q}$ where p and q are relatively prime integers. Find the value of $p + q$.
[Answer: 106]

T-5 Consider right triangle ABC with right angle B, D on $\overline{BC}$, and E on $\overline{AC}$ such that $\overline{DE} \perp \overline{AC}$. If $AB = 16$, $\tan(\angle CAB) = 3\tan(\angle DAB)$, and $\sin(\angle ACB) = \frac{8}{17}$, then $DE = \frac{x}{y}$ where x and y are integers with no common factors greater than 1. Find $x - y$.
[Answer: 143]

T-6 A line with equation $Ax + By = 120$ where A and B are integers has the properties that it contains the point (3,5) and it crosses the $x-$ and $y-$axes respectively at $(c, 0)$ and $(0, d)$ where c and d are positive integers. Find the sum of all possible values of B.
[Answer: 33]